Bucket transport for control of runaway electrons

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-Since it's unclear that proposed approaches (such as MGI or SPI) for preventing damage due to runaway electrons (REs) in ITER and subsequent large tokamaks will be adequate, here we consider a possible alternate approach:

-"Bucket transport", or "frequency sweeping", [1] a velocity-selective method for nonstochastically moving a target class of particles from one minor radius and energy [r,E] to another. Originally considered for transport of energetic ions for ash removal or profile control.

-Basic idea simple: Capture the target particles (in this case, REs) in the potential well ("bucket") of an applied perturbation, with frequency & spatial dependence [ω,m,n] making it resonant with the target, via

$$\omega = k_\parallel \bar{v}_\parallel = n\Omega_\zeta - m\,\Omega_b \approx n\Omega_\zeta[1- q_{mn}/q(r)]\,, \qquad (1)$$

with $\Omega_\zeta \equiv$ toroidal transit freq $\approx \bar{v}_\parallel/R$, $\Omega_b \equiv$ poloidal transit freq $\approx \Omega_\zeta/q$, $q_{mn}=m/n$. Then, by slowly reducing ω, particles initially trapped in the bucket stay in the bucket, allowing one to reduce $\bar{v}_\parallel$, draining energy from the REs, &/or moving them from one r to another (eg, sweeping them back in).

-To use this mechanism, one needs waves which can resonate with the REs, & which can penetrate into the plasma. Here, we consider using low-n fast compressional waves.

[1] Mynick, Pomphrey, *Nuclear Fusion* **34**, 1277 (1994).

(1) Particles:

-Parametrize real-space by straight field-line coordinates $\mathbf{x}$=(r or ψ, θ, ζ), with minor radial variable r(ψ), ψ=toroidal flux, (θ, ζ) = (poloidal,toroidal) angles.

-Guiding-center (gc) Hamiltonian H($\mathbf{z}$):

$$H(\mathbf{x},p_\parallel, \mu,t)=Mc^2[1+2\mu B/Mc^2 + (p_\parallel/Mc)^2]^{1/2} + e\Phi \approx Mc^2 + \mu B + p_\parallel^2/2M + e\Phi \,, \quad (2)$$

with $p_\parallel = v_\parallel \gamma_r$, $\gamma_r \equiv 1/[1-(v/c)^2]^{½}$. Can neglect eΦ term for REs.

-To the Hamiltonian H_0 of the unperturbed gc motion, add a single-helicity, t-dependent perturbation $h = h_{mn}(r,\mu)\sin(n\zeta- m\theta- \omega t)$.

Then from Hamilton's eqns, for axisymmetric H_0, have

$$\dot{E}_0=\partial_t H_0=0, \quad \dot{p}_{\zeta 0}=-\partial_\zeta H_0=0 \,, \quad \text{so} \quad \dot{E}/\dot{p}_\zeta=\partial_t h/(-\partial_\zeta h)= \omega/n \,. \qquad (3)$$

Thus, E-(ω/n)p_ζ (= energy in wave frame) is a constant of the motion, for constant ω.

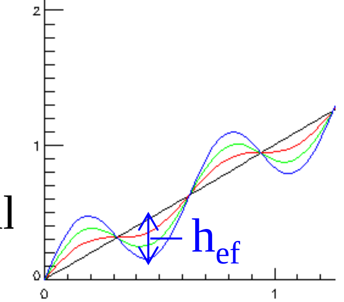
-Perturbation produces island of half-width $\delta p_\parallel=2(Mh_{mn})^{½}$, (4a)

in which resonant particles slosh at trapping freq $\omega_{tr}= (k_\parallel^2 h_{mn}/M\gamma_r^2)^{½}$. (4b)

-For compressional wave, have $h(\mathbf{x},t,\mu)=\mu\, \delta B(\mathbf{x},t)$, $h_{mn}(r,\mu)= \mu B_{mn}(r)$.

-Have $v_\parallel \approx c$ for REs, while $v_{\alpha 0} \approx$1.3e7 m/sec, $v_{\alpha 1}=v_\alpha(.1E_0) \approx$4e6 m/sec ≈c/73.2, so need much larger ω for REs than for the ash removal mechanism, implying (energy change/radial change) will be much larger for the RE application.

-Process is described with 1 radial & 1 v-space variable, using 1 each of $[\bar{r}, \psi(\bar{r}),q([\bar{r}), p_\zeta= -(e/c)\psi_p(\bar{r})]$ & $[v_\parallel, p_\parallel, g_\parallel \equiv p_\parallel/Mc, E]$.



-Constraints on B_{mn}, $\dot\omega$:

-Include 2 effects, which manifest themselves in the same way mathematically:

(a)The loop voltage $V_{lp}= 2\pi R E_\zeta$ can be modeled by taking $\Phi(s)=-E_\zeta s$, with s=distance along a field line. If this is too large, the slope to the overall potential $\mu\,\delta B(s)-eE_\zeta s$ will no longer have wells/buckets from δB.

(b)For $\dot\omega\neq 0$, phase velocity $\dot u_\parallel \equiv \dot\omega/k_\parallel$ also $\neq 0$. Treat motion in $v_\parallel$ by going to accelerated frame, with $M\dot v'_\parallel \equiv M(\dot v_\parallel - \dot u_\parallel) = -\partial_s eV_{ef}$, with effective potential

$$eV_{ef}(s) \equiv \mu\,\delta B - eE_{ef}s, \text{ with } eE_{ef} \equiv eE_\zeta - M\dot u_\parallel . \qquad (5)$$

-The well depth h_{ef} for wells on a slope is given by

$$h_{ef} = h_{mn}\{[1-z_h^2]^{1/2}-z_h[\pi/2-\arcsin z_h]\}, \qquad (6)$$

with $z_h \equiv |eE_{ef}/ k_\parallel h_{mn}|$. $h_{ef} \rightarrow [h_{mn},0]$ for $z_h \rightarrow [0,1]$.

Dropping $M\dot u_\parallel$ in eE_{ef} yields a minimum value for B_{mn} (K=kinetic energy):

$$1> z_h = |eV_{lp}/(2\pi R k_\parallel h_{mn})| = |eV_{lp}/(K\sin^2\xi)(B_0/B_{mn})/(2\pi R k_\parallel). \qquad (7a)$$

Dropping eE_ζ yields a maximum value for $\dot\omega$:

$$1> z_h = |M\dot u_\parallel/ k_\parallel h_{mn}| = |\dot\omega/ \omega_{tr}^2| . \qquad (7b)$$

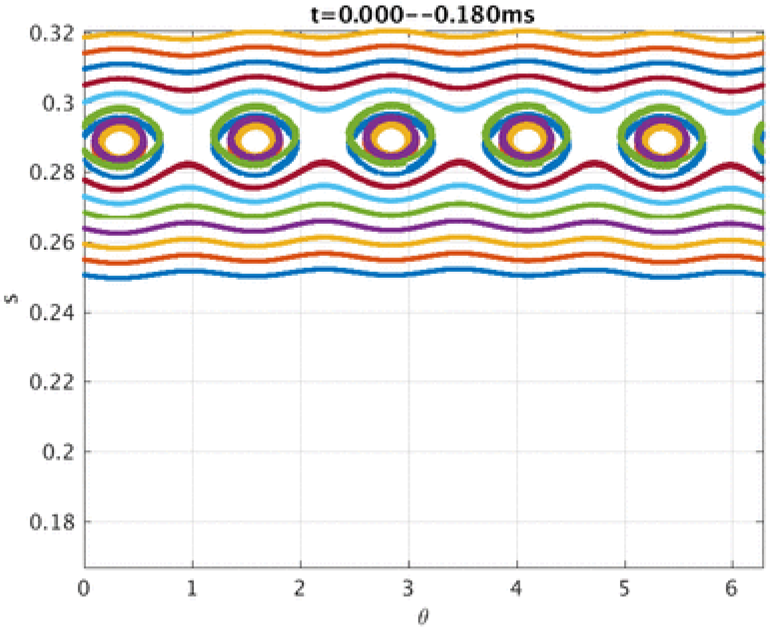
-Physically, to trap a particle in a bucket, it must bounce in the bucket more rapidly than the time for E_{ef} to accelerate it over the top of the well it is bouncing in.

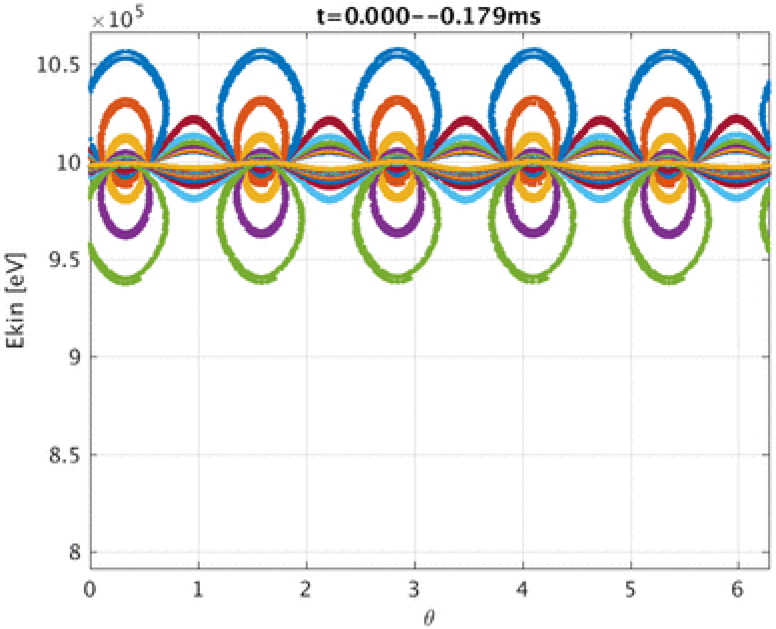
-For V_{lp}=1 volt, K=1 MeV, $<\sin\xi>= 0.2$, (7a) gives $B_{mn}/B_0 \gtrsim 0.4e\text{-}5/(Rk_\parallel)$, where $(Rk_\parallel)= \omega/\Omega_\zeta \sim 7.9$ MHz/7.7 MHz = 1.03.

-For this $[\omega, Rk_\parallel]$, $\omega_{tr} \approx \omega \sin\xi[(B_{mn}/B_0)/(\gamma_r+1)]^{1/2} \approx 57e3$/sec , & (7b) gives $\nu_{sw} \equiv \dot\omega/\omega < \omega_{tr}^2/\omega \approx 200$/sec for B_{mn}/B_0=1e-4.

-Test numerically , using VENUS[2] relativistic gc code (Hirvijoki):

-ITER equilibrium, $[q_0,q_a]=[1.65,3.25]$, K_0= 1 MeV, cosξ=0.8, $[m,n,B_{mn}/B_0]=[5,3,1.3e\text{-}3]$, sweep ω= 1 → 0 MHz in 10 msec:

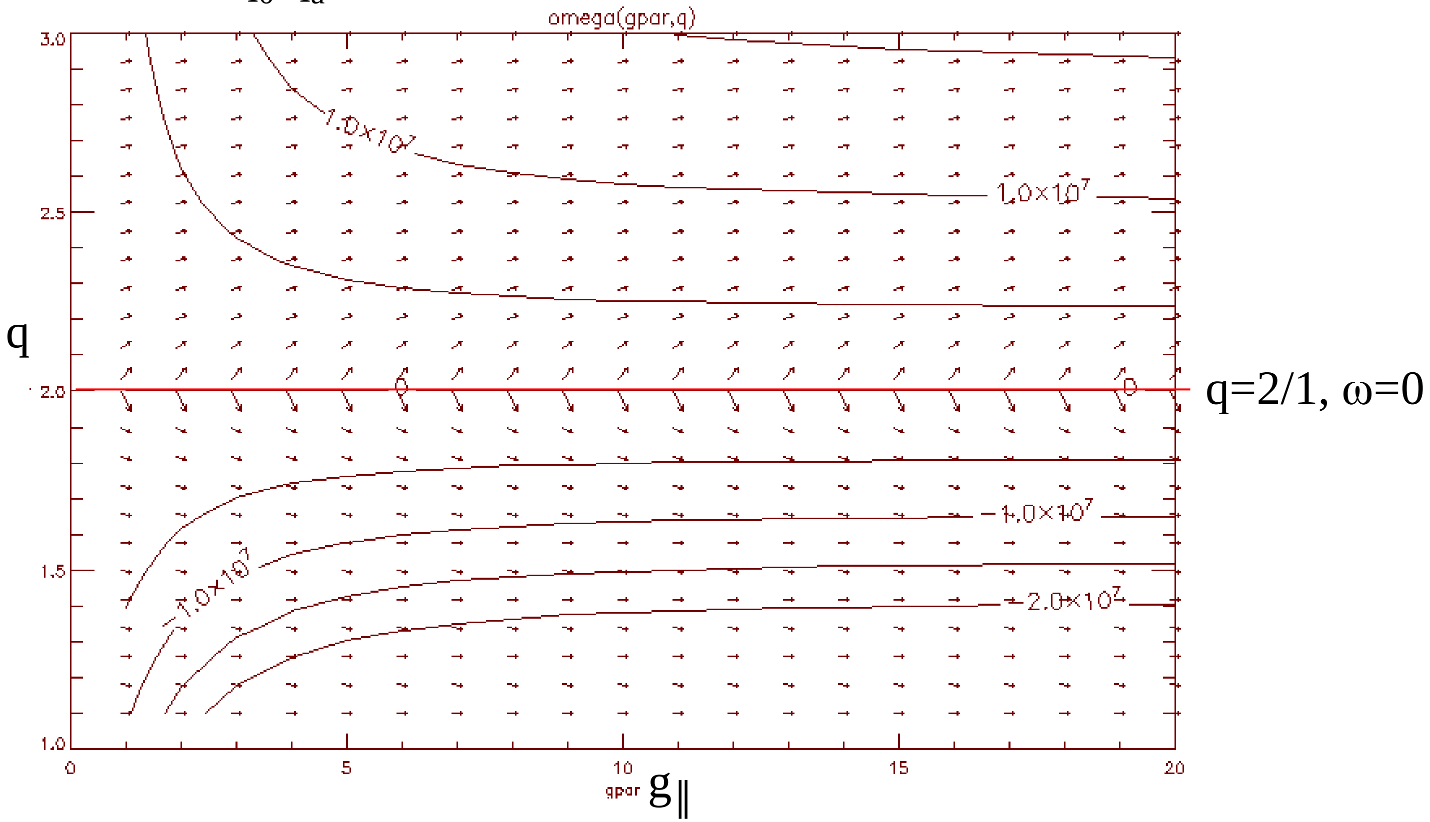




[2] Pfefferle, Cooper, Graves, Misev, *Computer Physics Commun*. **185 ,** 3127 (2014).

-Summarize the effect of the ω–sweeping by plotting contours of $\omega = k_\parallel \bar{v}_\parallel$ and direction of $\dot{E}/\dot{p}_\zeta = \omega/n$ in $[g_\parallel \equiv p_\parallel/Mc,\ q]$ plane:

-For ITER, $[q_0, q_a] = [1,3]$, with $[m,n] = [2,1]$:



-Bucket width $\delta g_\parallel \approx |g_0 \sin\xi|(2B_{mn}/B_0)^{1/2} \approx .056$ for $B_{mn}/B_0 = 1\text{e-}4$, so # sweeps needed ~ $g_0/\delta g_\parallel \approx 20/.056 \approx 357$.

(2)Applied waves:

-For ITER [$R \approx 6.2$ m, $B \approx 5.3$ T], have $\Omega_\zeta \approx c/R \approx 48.3e6$/sec ≈ 7.7 MHz, $\Omega_{gD} \approx 40.1$ MHz, so expect resonant ω in the IC range.

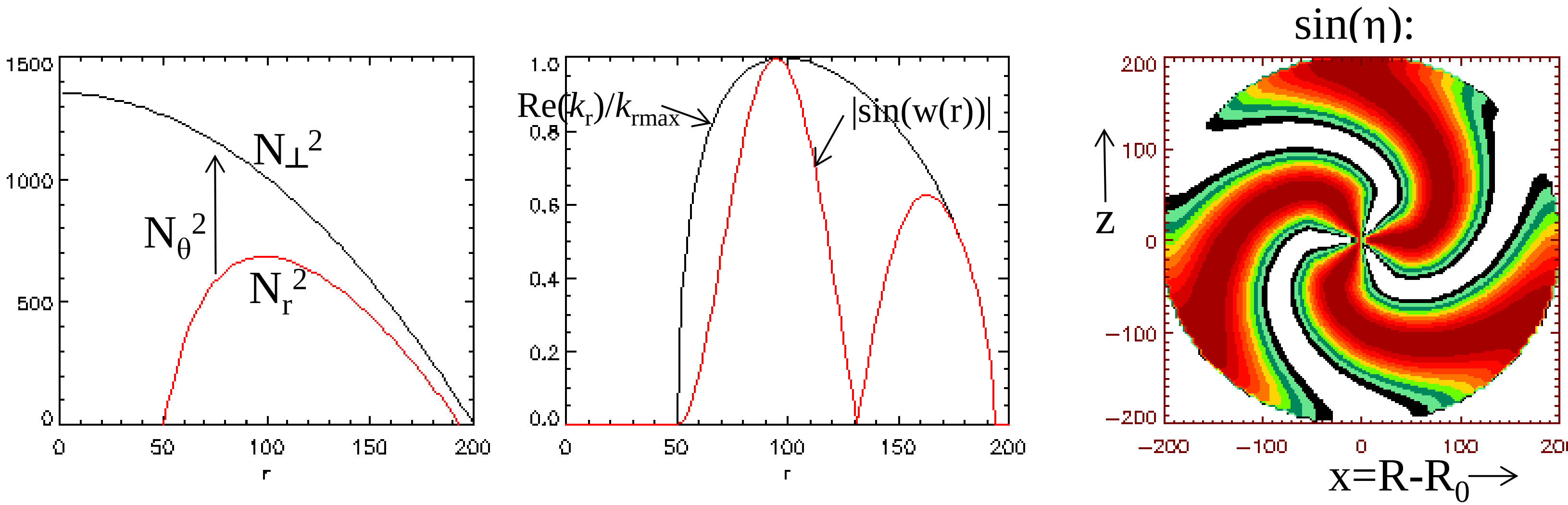
-Drive fast MS waves with RF antenna. In cold plasma approximation, these have $\omega \approx k v_A$, with v_A = Alfven velocity, $v_A/c = (\Omega_{gD}/\omega_{pD}) \approx 1/36 \sim (M_e/M_D)^{1/2} \ll 1$. Along with resonance condition $\omega = k_\parallel \bar{v}_\parallel$, implies $= k_\parallel / k = v_A / \bar{v}_\parallel \ll 1$, ie, near-perp propagation.

-These have dispersion reln $N_\perp^2 = (R - N_\parallel^2)(L - N_\parallel^2)/(S - N_\parallel^2)$, (8)

with refractive indices $N_{\parallel,\perp} \equiv k_{\parallel,\perp} c/\omega$, $N_\perp^2 = N_r^2 + N_\theta^2$, $N_\theta \approx k_\theta c/\omega = mc/\omega r$, and $S \equiv (R+L)/2$, $R \equiv 1 - \omega_p^2/[(\omega - \Omega_{gi})(\omega - \Omega_{ge})]$, $L \equiv 1 - \omega_p^2/[(\omega + \Omega_{gi})(\omega + \Omega_{ge})]$.

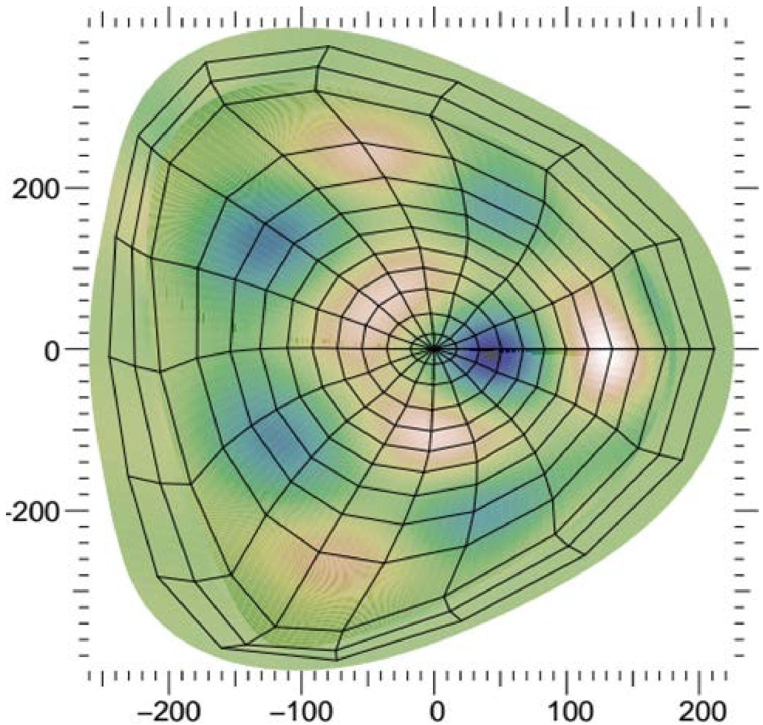
Solve this for $N_r^2(r\,|m,n,\omega) = N_\perp^2 - N_\theta^2$, which must be > 0 in r-range $[r_1, r_2]$ where mode is propagating. From this, compute $k_r(r|m,n,\omega)$, and from that, the radial eikonal $w(r) \equiv \oint_0^r dr' Re(k_r(r'))$, completing the mode phase $\eta(\mathbf{x},t|m,n,\omega) = w(r) + n\zeta - m\theta - \omega t$.

Eg: For ITER, with $[m,n,\omega] = [3,1,50e6/\text{sec} \approx 7.9 \text{ MHz}]$:

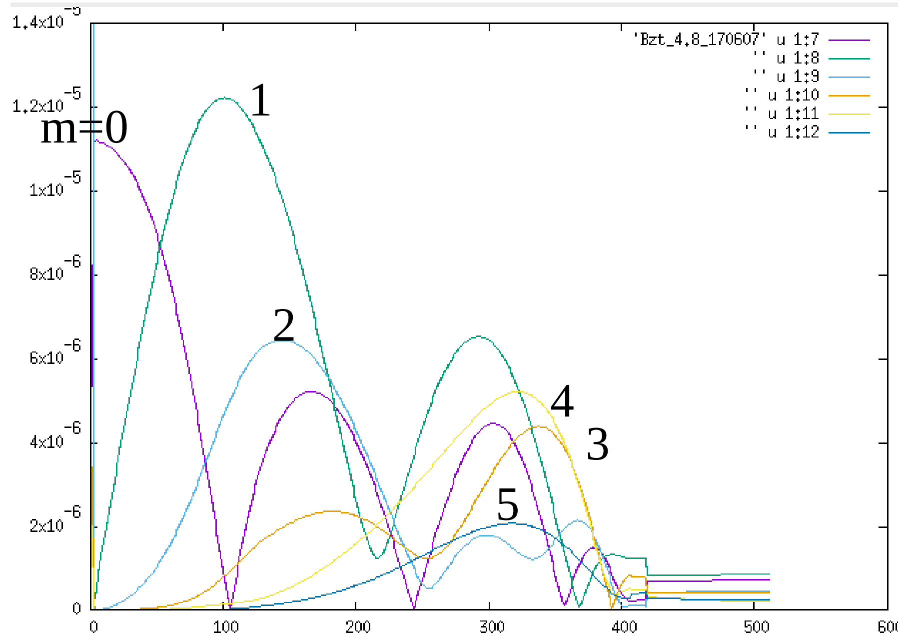


-For a given n, an RF antenna will produce a range of m-values.
From a TORIC simulation (Valeo) on ITER with $[q_0,q_a] = [0.9,3.7]$, $[n,\omega]=[1,4.8\text{ MHz}]$:

$\delta B(r,\theta;\ \zeta=0)$



Resolve into individual m's:



[$\delta B(-m) \approx \delta B(m)$, as expect from analytic disp.reln.]

-Power requirements:

-A given δB implies an RF power requirement $P_{rf} \approx (2\pi^2)aRV_g(\delta B^2/8\pi)/Q_{rf}$ (9), with group velocity $V_g \approx v_A$, cavity quality factor Q_{rf}, so $\delta B(\text{gauss}) \approx [(2/\pi)Q_{rf}P_{rf}/aRv_A]^{1/2}$, ≈ 0.27 gauss or $\delta B/B_0 \approx .55e\text{-}5$ for P_{rf}= 1 MW, Q_{rf}=1, in rough agreement with Toric simulation.

-Since scaling $\delta B \sim P_{rf}^{\,1/2}$ fairly weak, this may present the main drawback of this approach.

-Things which may help :

-Multiple simultaneous sweeps, from both multiple m's, as well as sideband buckets satisfying resonance condition $\omega = n\Omega_\zeta + \ell_b\Omega_b$, with $\ell_b \equiv (\ell\text{-}m)$, $\ell=0, \pm 1, \pm 2,\ldots$.

-Mechanisms which enhance $\sin\xi$, such as possible scattering due to nonlinear interaction with whistler waves[3], since $h_{mn} \sim \sin^2\xi(B_{mn}/B_0)$.

-Larger [m,n,ω] than considered here: Eg, if scale $[m,n,\omega] \text{-}> \alpha\,[m,n,\omega]$, leaves $\delta g_{\|}$, needed δB, $[r_1,r_2]$ range unchanged, & enhances $\omega_{tr} \sim k_{\|} \sim \alpha$, $\nu_{sw} = \omega_{tr}^2/\omega \sim \alpha^2/\alpha$, permitting faster sweeps at same δB.

-Antennas better for this purpose, & other possible waves, providing larger $\delta B(\mathbf{x})$ where buckets needed.

-Collisions: $\nu_\perp$ very small for REs, but frequency to scatter out of bucket much larger. However, as some particles scatter out, other particles scatter in, so overall effect may not be too much degraded. Numerical simulations seem good way to check.

[3]C.Liu, Ph.D. Dissertation, Princeton University (2017).