On the Physics of Onset of Nonlinear Disruptive Instabilities

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Nonlinear Simulations Find Faster-Than-Exponential Growth: Scaling with Heating Rate in Good Agreement with Theory

- Impose heating source proportional to equilibrium pressure profile

\[
\frac{\partial P}{\partial t} = \ldots + \gamma_H P_{eq} \Rightarrow \beta_N = \beta_{Nc} (1 + \gamma_H t)
\]

- Follow nonlinear evolution through heating, destabilization, and saturation
- Simulation starts below marginal stability point in beta
- Simulation results with different heating rates are well fit by \( \xi \sim \exp[(t-t_0)/\tau]^{3/2} \)
- Time constant scales as

\[
\tau \sim \gamma_{MHD}^{-0.72} \gamma_H^{-0.28}
\]

- Compare with theory:

\[
\tau = (3/2)^{2/3} \gamma_{MHD}^{-2/3} \gamma_h^{-1/3}
\]

- Discrepancy possibly due to non-ideal effects

![Graph showing log of magnetic energy vs. \((t-t_0)^{3/2}\) for 2 different heating rates with \(S = 10^6\), \(Pr = 200\), \(\gamma_H = 10^3\, s^{-1}\).]
Issues of boundary conditions were understood in early free-boundary simulations – remains a subject of intense debate and focus

- Boundary conditions are applied at the vacuum vessel, **NOT** the experiment limiter.
  - Vacuum vessel is conductor
  - Limiter is an insulator

- This is accurate for magnetic field:
  - \( B_n \) = constant at conducting wall
  - \( B_n \) can evolve at graphite limiter

- No boundary conditions are applied at limiter for velocity, temperatures, or density:
  - This allows fluxes of mass and heat through limiter
  - Normal heat flux is computed at limiter boundary

- From Kruger PP 2004:
  "... applying the natural boundary conditions at the limiter would give no density flux across the limiter, which is less physical than our current method of applying the natural boundary conditions at the vacuum vessel and allowing a mass flux across the limiter."

- Large viscosity added in region between limiter and vacuum vessel to make Neumann-like B.C.
  - More accurate models can be implemented.
NIMROD Simulation of DIII-D shot 87009 shows quantitative features of tokamak disruptions

Plasma loses 60% of magnetic energy in ~200 microseconds. Agreement within 20%.

Time scale set by heat transport in complex topological field as the magnetic field becomes stochastic

Similar levels of agreement from Izzo, Whyte
The drag term has not been used in the NIMROD computation; the plasma column distorts into the wall.

Contours of constant pressure at $t=0, 3 \tau_A$ from maximum displacement, and $1 \tau_A$ from maximum displacement.

- Computationally, this case ‘exercises’ NIMROD’s ability to advect sharp fronts in $n$ and to solve linear systems with very strong variation in coefficients over the periodic coordinate.

Helical surface currents flow parallel to $J_0$ along the phase of the column that moves inward and anti-parallel along the phase that moves outward.
Nonlinear Dependence of $\beta(w)$ and $\gamma(w)$ on Heating Rate Captured by NIMROD and Rutherford Modeling

- Timescales: $\tau_R > \tau_H > \tau_{\text{MHD}}$
- Thermal anisotropy: temperature flattening, fast parallel transport included
- Neoclassical and Polarization Effects necessary for NTM threshold and low heating rate limit: not yet in simulations.
- For low heating rates, island growth rate decreases at minimum detected size, reducing ECCD power required for stabilization
Energetic Particles have been shown to be important to the stability of the m>1 resistive and ideal modes

Energetic particles included into these cases cause significant damping and stabilization to the 2/1 modes, for slowing-down distributions similar to experiment. -> can be critical to modeling the threshold to nonlinear disruptive instability

As the fraction of $\beta$ that the energetic particles make up, $\beta_{\text{frac}}$, increases to experimental values, modes are strongly damped. DIII-D~0.16 JET~0.3 Varies with discharge

Nonlinear evolution also strongly affected
Detailed studies suggest both trapped and passing particles are responsible for stability properties

Damped and stabilized 2/1 mode conjectured to be responsible for qualitative differences between experiments (eg. DIII-D, vs JET).

R. Takahashi, D. Brennan, C.C. Kim, PRL 2009

Understanding effect in NSTX currently underway.

Simple Rutherford Modeling of Energetic Particle Effects not Viable, need Kinetic - MHD model such as NIMROD or M3D.

The Eigenfunction of $V_r$, the n=1 Spatial Projection of $\delta f$ in Phase Space Shows Trapped Cone, but also significant passing activity, non-trivial interaction for realistic cases.
Radiation driven islands are cooled by magnetic insulation, increasing resistivity -> increased drive

- The island is magnetically insulated from its surroundings, cooling the island.
- Lower temperature leads to increased resistivity, enhancing helical island current.
- The island then grows causing the process to continue.

\[ P_{\text{rad}} < \eta J^2 \rightarrow n_e < \sqrt{\frac{m_e}{e^2 E_{\text{eff}}}} \frac{\nu_{ei}}{\nu_{(eZ)_{\text{eff}}}} J \]

Radiation drive can change sign – eg. when island cools.

- Auxiliary power is shunted around the island by parallel conduction, consistent with density limit being independent of heating power.
- Quantity in square root is nearly independent of temperature*.
- Reminiscent of the Greenwald limit.

Modeling Active Stability Control Show Stable Windows of Operation – Simple Example: RFP with no flow

Control model
\[ \tilde{B}_r(r_c) = -G\tilde{B}_r(r_w) + K\tilde{B}_\phi(r_w) \]

For resistive wall and fixed plasma resistivity, four limits defined. With two walls: six limits.

Stable window with feedback for \( \lambda \) above resistive limit.

Possible to stabilize near ideal plasma, ideal wall limit.

Stable window varies in shape and size significantly with \( \lambda_0 \)

Richardson, Finn, Delzanno, “Control of ideal and resistive MHD modes in RFPs with a resistive wall”, PoP 17, 112511 (2010).
Finn, “Control of MHD modes with a resistive wall above the wall stabilization limit”, PoP 13, 082504 (2006).
Summary and Looking Forward

What are the foci for additional physics in the core plasma?

- Two-fluid terms: Do we have to resolve the electron skin depth locally? If so, it will be impossible within our lifetimes
- Models for anisotropic heat conduction are inadequate
  - Using Braginskii even in collisionless or semi-collisional regimes
- More accurate heat flux: In progress.
- Energetic particle effects can be critical – nonlinear simulation current focus
- Flow and flow shear – not mentioned but important
- Radiation driven island theory provides a testable quantitative prediction of the density limit -> possible solution to long standing puzzle
  - Theory predicts exponentially growing islands with a sudden robust onset condition
  - Consistent with a robust density limit and observed **rabidly** growing 2/1 tearing mode
- Need to directly verify local power balance in experiments and include in sims
Summary and Looking Forward

What are the foci for additional physics in the boundary?

• An external kink in cylindrical geometry demonstrates nonlinear free-boundary capability.
  • With a drag term, this case will provide a nonlinear benchmark.
  • The solver is the primary computational challenge.
• Nonlinear evolution across stability boundary to disruption has been simulated, but there is much more to do to address slow approach quantitatively.
  • Resistive wall is crucial to modeling disruptions -> not discussed here, but nearly complete in NIMROD
• Active Feedback Control modeling will add a new layer of necessary complexity in understanding experimental outcome in these cases.

• Others?
Theory of Non-Axisymmetric Vertical Displacement Events

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Effect of $n = 1$ Mode

- The $n = 1$ mode also requires moderation to prevent it from growing on Alfvenic timescale. This leads to significant increase in vertical force exerted between vacuum vessel and plasma, and, hence, in halo current.

- As $n = 1$ mode grows in amplitude, region of contact between plasma and vacuum vessel becomes toroidally asymmetric. Eventually, contact region becomes toroidally localized. This implies toroidal localization of halo current, and halo current force. However, net force still has to be large enough to moderate $n = 0$ and $n = 1$ modes. Assuming that net force remains approximately constant, this implies significant increase in force density in contact region.
Critical Questions

- What is peak vertical force exerted on vacuum vessel during VDE?
- What is maximum toroidal peaking factor of this force?
- What is maximum sideways force exerted on vacuum vessel during VDE?
Sharp-Boundary Plasma Model

- Strongly elongated, large aspect-ratio, high beta (i.e., $\beta \sim \epsilon$) plasma with uniform internal pressure. All equilibrium currents flow on plasma boundary.

- Model allows fairly realistic treatment of $n = 0$ and $n = 1$ external modes.

- $n = 0$ and $n = 1$ stability calculations involve matching of vacuum-like solutions at plasma boundary. Can explicitly include effect of halo current force in matching process. Allows self-consistent calculation of effect of halo currents on $n = 0$ and $n = 1$ stability.
Halo Current Patterns

Type 1

Contact region

Halo current

Type 2
Circuit Equations

- Model allows for two types of halo current pattern. Type 1 gives rise to no halo current force. Type 2 maximizes force.
- Can calculate mean resistance of halo current circuit associated with each pattern (for specified SOL and v.v. resistivities), as well as mean emf generated by plasma shrinkage. (Assuming that circuit path covers LCFS ergodically.) Associated circuit equations determine relative mix of Type 1 and Type 2 patterns.
- Net halo current force adjusted such that \( n = 0 \) (and \( n = 1 \), when it is unstable) mode marginally stable. Circuit equations then give \( n = 0 \) (and \( n = 1 \)) growth-rate.
Determination of Poloidal Extent of Contact Region

- Vacuum vessel
- Contact points
- Halo
- Plasma
Inputs to Model

- Vacuum vessel shape and thickness.
- Initial edge-\( q \).
- Plasma shape, current, and beta as plasma shrinks.
- SOL and vacuum vessel resistivities.
- Critical halo current fraction that triggers current quench.
Example Calculation: Edge-$q$, Current

$q_p, 10\hat{\beta}, \hat{I}_{\phi_p}$
Example Calculation: Toroidal Peaking Factor
**Example Calculation: Vertical and Sideways Forces**

For typical ITER parameters, peak vertical and horizontal forces are 31 MN and 12 MN, respectively.
Ansatz that current quench triggered when halo current faction exceeds critical value reproduces inverse relation between TPF and poloidal halo current fraction seen experimentally.
Simulations of asymmetric VDEs with M3D: model validation and comparison with experimental cases

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Some of the previous results

- From NF (2009):
  \[ I_{\text{halo}}(\phi) = \frac{1}{2} \int |\mathbf{n} \cdot \mathbf{J}| R dl, \]
  \[ \text{TPF} = \frac{2\pi I_{\text{halo(max)}}}{\int I_{\text{halo}} d\phi}. \]

- From PoP (2010):
  Horizontal force is maximum for \( \gamma \tau_w = 1 \)

..what does this mean really in ITER?
ITER vs Simulations: (1)

Chapman et al., NF (2010)
Neoclassical tearing threshold

\[ \tau_{wall} / \tau_R = 10^{-4} - 10^{-3} \]

However after thermal quench with $T \rightarrow 100-10 \text{ eV}$ range:

\[ \tau_{wall} / \tau_R = 0.1 - 1 \]
ITER vs Simulations: (2)

* At high S it is very time consuming having long wall time constants

\[ \frac{\tau_{\text{wall}}}{\tau_R} \]

- **ITER**
- **TQ**
- **simulations**
- **actual experiments**

\( S, \) Lundquist number

\( 10^6 \) \( 10^{10} \) \( 10^{11} \)
Apart the arbitrariness of $\eta_{\text{out}}$, the halo region is self-consistently determined by the time evolution of temperature.
Other AUG cases have confirmed the observation done for the FED case i.e. higher resolution and higher $S$, contribute to produce more symmetric VDE's.

(2D VDE overtakes 3D effects -> due to unrealistically low $\tau_{\text{wall}}$ and S mode scaling)

- Non axi-symmetry can be obtained by enhancing the amplitude of an arbitrary initial perturbation to the plasma (at time $t=0$).

- Enhancing the plasma viscosity for a given resistivity (or S) (i.e. enhancing the Prandtl number) has also the effect to smooth out non axi-symmetric modes, and to produce more symmetric VDE’s.

- There is a clear competition in the system between the VDE time scale (mainly determined by the wall time constant), the current and temperature evolution in the plasma (determined by the transport and by the Lundquist number) and the evolution of the resistive modes, which determine the final TPF and halo fraction.
TPF = 1
(no applied initial perturbation)

S = 5e6

TPF = 1.3
(with initial perturbation)
Perturbed poloidal flux and pressure contours

Poincaré puncture plot

A 2/1 resistive mode is dominant in these simulations.
Several discrepancies with the experiment

- the thermal quench is well before the current quench (instead similar rate in simulation)
- high perpendicular transport, can reproduce a faster pressure decay: in this case however TPF and hcf can become unrealistic
- a 2/1 resistive kink responsible for asymmetry in simulations Experimentally unclear (role of pure ideal modes?)
- high resolution simulations resilient to asymmetry (init. pert. needed) What happens in experiments?
Conclusions (1)

• Progress in 3D disruptions simulations/validation have been achieved.

• Thermal quench (TQ) remains a big issue (simulation possible?)

• Plasma conditions after TQ crucial in determining evolution.

• Role of transport and transport scaling after TQ also crucial.

• Resistive instabilities seem to play the main role in simulations (2/1 mode and in some case 1/1 also).

• More data on relevant modes probably needed from experiments.

• ITER simulations need to be completed at high resolution.

• Force calculations can be refined by using 3D electromag. wall codes.