Surface Currents during a Major Disruption

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Abstract

The surface current on the plasma-vacuum interface during a disruption event involving kink instability can play an important role in driving current (Halo or Hiro current) into the vacuum vessel. However, there have been disagreements over the nature or even the sign of the surface current in recent theoretical calculations based on idealized step-function background plasma profiles. We revisit such calculations by replacing step-function profiles with more realistic profiles characterized by strong but finite gradient along the radial direction. Consequently, the resulting "surface current" is no longer a delta-function current density, but a finite and smooth current density profile with internal structure, concentrated within the region with strong plasma pressure gradient. Moreover, this current density profile has peaks of both signs, unlike the delta-function case with a sign opposite to, or the same as the plasma current. We show analytically and numerically that such current density can be separated into two parts, with one of them (convective current density) having the physical meaning of transporting the background current density by the displacement, and the other part called the residual current density, which can have important effects during disruption if it enters the vessel wall. We will compare our results with previous work carried out with step-function profiles, as well as simulation data.

Introduction

◊ During a major disruption in a tokamak, a kink instability can follow a vertical displacement event (VDE).

◊ The current (identified as Halo or Hiro current in the literature) flowing into the vacuum chamber walls can have significant damaging effects.

◊ There has been controversies over the nature, or even the sign, of the Halo/Hiro current.

◊ Step-function derivations were used in most theoretical studies. We will show that it is important to relax such an approximation to help clearing up the sign confusion, and understanding the physical nature of such current.

Outline

◊   Review of step-function results --- identify the problem

◊   Numerical results with step-function approximation removed

◊   Discuss the sign, and the physical nature of "surface current"

◊   Show that the current density and mass density profiles can generally evolve into different forms and that could affect the instability condition

◊   Conclusions

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Review of step-function results --- problem setup

◊ Following [*Strauss et al.* 2010] --- cylindrical (r, θ, ϕ), large aspect ratio (R_0/b), reduced MHD

magnetic field $\mathbf{B} = \nabla\psi \times \hat{\phi} + B_0\hat{\phi}$

density $\rho = 1$ for $r < a$, 0 for $a < r < b$

background current density $j_{\phi 0} = -\nabla_\perp^2 \psi_0 = \begin{cases} -2B_0/q_0 R_0 & \text{for } r < a \\ 0 & \text{for } r > a \end{cases}$

unstable mode with displacement $\boldsymbol{\xi} = \nabla\Phi \times \hat{\phi} \propto e^{i(m\theta + n\phi) + \gamma t}$

thin resistive wall at $r = b$, with resistivity η_w and thickness δ, (wall time: $\tau_w = \delta b/m\eta_w$)

$$B_{\theta 0} = -\frac{\partial \psi_0}{\partial r} = \frac{1}{r}\int_0^r j_{\phi 0}(r')r'dr' = \frac{rB_0}{R_0 q(r)}$$

Review of step-function results --- eigen equation

Start with equation of motion: $\gamma^2 \nabla \cdot (\rho \nabla_\perp \Phi) = \mathbf{B}_0 \cdot \nabla \nabla_\perp^2 \psi_1 + \mathbf{B}_1 \cdot \nabla \nabla_\perp^2 \psi_0$

using equation for magnetic potential:

$$\psi_1 = \mathbf{B}_0 \cdot \nabla \Phi = iB_0 k_\parallel \Phi, \text{ with}$$

$$k_\parallel = \frac{1}{R_0}\left[n - \frac{m}{q(r)}\right] = \frac{1}{R_0}\left[n + \frac{mR_0}{B_0 r^2}\int_0^r j_{\phi 0} r' dr'\right] = \begin{cases} \dfrac{-1}{q_0 R_0}\left[m - nq_0\right] & \text{for } r < a \\[3mm] \dfrac{1}{q_0 R_0}\left[nq_0 - \dfrac{ma^2}{r^2}\right] & \text{for } r > a \end{cases}$$

First order magnetic field: $\mathbf{B}_1 = \nabla \psi_1 \times \hat{\phi} = \dfrac{im\psi_1}{r}\hat{r} - \dfrac{\partial \psi_1}{\partial r}\hat{\theta}$

$$\Rightarrow \quad \gamma^2 \nabla \cdot \left[\rho \nabla_\perp \frac{\psi_1}{k_\parallel}\right] = -B_0^2 k_\parallel \nabla_\perp^2 \psi_1 + \frac{2mB_0^2}{q_0 R_0 r}\delta(r-a)\psi_1$$

Review of step-function results --- eigen mode solution

$$\nabla_\perp^2 \psi_1 = 0 \text{ for } 0 \le r < a, \ a < r < b, \ b < r$$

boundary conditions : $\psi_1 = 0$ at $r = 0$ and $r \to \infty$

$$\nabla_\perp^2 = \partial_r(r\partial_r)/r - m^2/r^2$$

$$\psi_1 = \begin{cases} \psi_p = \psi_{1\alpha}(r/a)^m \ , & \text{for } 0 \le r < a \\ \psi_v = \psi_{2\alpha}(r/a)^m + \psi_{3\alpha}(a/r)^m \ , & \text{for } a < r < b \\ \psi_x = \psi_{4\alpha}(b/r)^m \ , & \text{for } b < r \end{cases}$$

at $r = a$, $\psi_p = \psi_v$, $\dfrac{\gamma^2}{B_0^2}\psi_p' = k_\parallel^2(\psi_v' - \psi_p') - \dfrac{2mk_\parallel}{q_0 R_0 a}\psi_p$

at $r = b$, $\psi_x = \psi_v$, $\gamma\delta\psi_x = \eta_w(\psi_x' - \psi_v')$

solve $\psi_{2\alpha}$, $\psi_{3\alpha}$, $\psi_{4\alpha}$ in terms of $\psi_{1\alpha}$ and obtain equation for γ :

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Review of step-function results --- growth rate

$$\frac{\gamma^2}{2B_0^2} = -\frac{\left[1 + 2/(\gamma\tau_w)\right]k_\parallel^2}{1 - (a/b)^{2m} + 2/(\gamma\tau_w)} - \frac{k_\parallel}{R_0 q_0} = \frac{m - nq_0}{R_0^2 q_0^2}\left\{1 - \frac{(m - nq_0)\left[1 + 2/(\gamma\tau_w)\right]}{1 - (a/b)^{2m} + 2/(\gamma\tau_w)}\right\}$$

Third order algebraic equation --- can be solved exactly.

In the limit of $\tau_w \rightarrow \infty$ for a highly conducting wall,

for $nq_0 - m + 1 - (a/b)^{2m} > 0$, $\quad \dfrac{\gamma^2}{2B_0^2} \approx \dfrac{(m - nq_0)\left[nq_0 - m + 1 - (a/b)^{2m}\right]}{R_0^2 q_0^2\left[1 - (a/b)^{2m}\right]}$

for $nq_0 - m + 1 - (a/b)^{2m} < 0$, $\quad \gamma \approx \dfrac{2(nq_0 - m + 1)}{\left[m - nq_0 - 1 + (a/b)^{2m}\right]\tau_w} \rightarrow 0 \quad$ instability

condition: $m - 1 < nq_0 < m$

with large growth rate only for $m - 1 < m - 1 + (a/b)^{2m} < nq_0 < m$

Review of step-function results --- plasma surface current

There is a surface current on the plasma surface ($r = a$), according to [*Strauss et al*. 2010],

$$K = B_{1\theta}\big|_{r=a-0^+}^{r=a+0^+} = \psi'_p - \psi'_v = -\frac{2B_0}{q_0 R_0} \frac{(m-nq_0)[1+2/(\gamma\tau_w)]}{[1-(a/b)^{2m}+2/(\gamma\tau_w)]} \xi_r$$

where $\xi_r = im\Phi/r = im\Phi/a$

K is of the same direction as $j_{\phi 0}$ (negative) for positive ξ_r. However, [*Zakharov et al*. 2012] and [*Webster* 2010] have (equivalently):

$$I = K - j_{\phi 0}\xi_r = \frac{2B_0}{q_0 R_0}\left\{1 - \frac{(m-nq_0)[1+2/(\gamma\tau_w)]}{1-(a/b)^{2m}+2/(\gamma\tau_w)}\right\}\xi_r \propto \gamma^2$$

The sign of I, which is considered important to disruption physics, is the opposite to K. How to understand this difference?

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Relaxing the step-function approximation --- motivations

◊   Current density and mass density profiles in real plasmas do not have sharp (step-function like) boundaries, and are usually not of the same form.

◊   Direct 3D dynamical simulations have difficulties treating sharp boundaries --- difficult to compare theory with simulation.

◊   Instead of getting a δ-function solution, which is problematic in a linear treatment, now the first order current density is regular and well behaved.

◊   The surface currents may have internal structures within the sharp boundary --- even if either K or I is zero, the current density within the boundary region can be nonzero.

◊   Resolving such internal structures might help understanding the meaning of K or I, and the sign difference.

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Eigen equation --- solved numerically along r (1D)

$$\gamma^2\left[\rho\nabla_\perp^2\left(\frac{\psi_1}{k_\parallel}\right) + \frac{\partial\rho}{\partial r}\frac{\partial}{\partial r}\left(\frac{\psi_1}{k_\parallel}\right)\right] = -B_0^2 k_\parallel \nabla_\perp^2\psi_1 + \frac{mB_0}{r}\frac{\partial j_{\phi 0}}{\partial r}\psi_1,$$

"Boundaries" of $j_{\phi 0}$ and ρ are at a and a_ρ, with thickness $\sim 1/\kappa$ and $1/\kappa_\rho$. In general, a and a_ρ, κ and κ_ρ do not have to be the same.

$$j_{\phi 0} \rightarrow \begin{cases} -2B_0/q_0 R_0 & \text{for } \kappa(a-r) \gg 1 \\ 0 & \text{for } \kappa(r-a) \gg 1, \end{cases} \qquad \rho \rightarrow \begin{cases} 1 & \text{for } \kappa_\rho(a_\rho - r) \gg 1 \\ 0 & \text{for } \kappa_\rho(r - a_\rho) \gg 1 \end{cases}$$

$$k_\parallel = \frac{1}{R_0}\left[n + \frac{mR_0}{B_0 r^2}\int_0^r j_{\phi 0} r' dr'\right] \rightarrow \begin{cases} \dfrac{-1}{q_0 R_0}\left[m - nq_0\right] & \text{for } \kappa(a-r) \gg 1 \\ \dfrac{1}{q_0 R_0}\left[nq_0 - \dfrac{ma^2}{r^2}\right] & \text{for } \kappa(r-a) \gg 1 \end{cases}$$

$$\nabla_\perp^2 = \partial_r(r\partial_r)/r - m^2/r^2$$

Some profiles used

$$j_{\phi 0} = -\frac{B_0}{q_0 R_0} \operatorname{erfc}[\kappa(r-a)], \text{ with } a = 0.5,\ b = 1$$
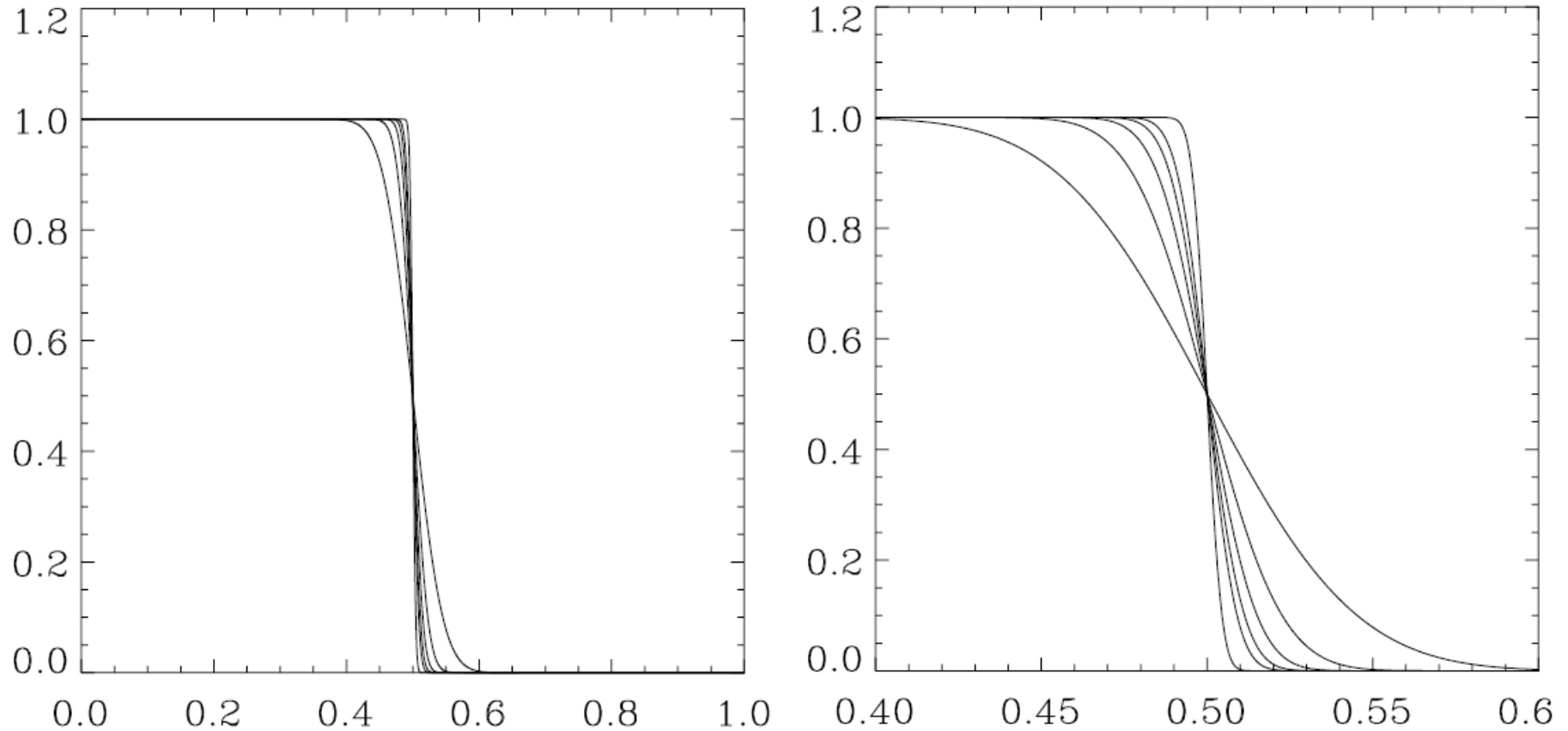
Normalized profiles with $\kappa = 20, 40, 60, 80, 100, 200$

The $k_\parallel$ function

For this profile, with $m = n = 1$, $R_0 = 3$, $\kappa = 40$, $q_0 = 0.1, 0.2, 0.25$ (dashed curve), $0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$:
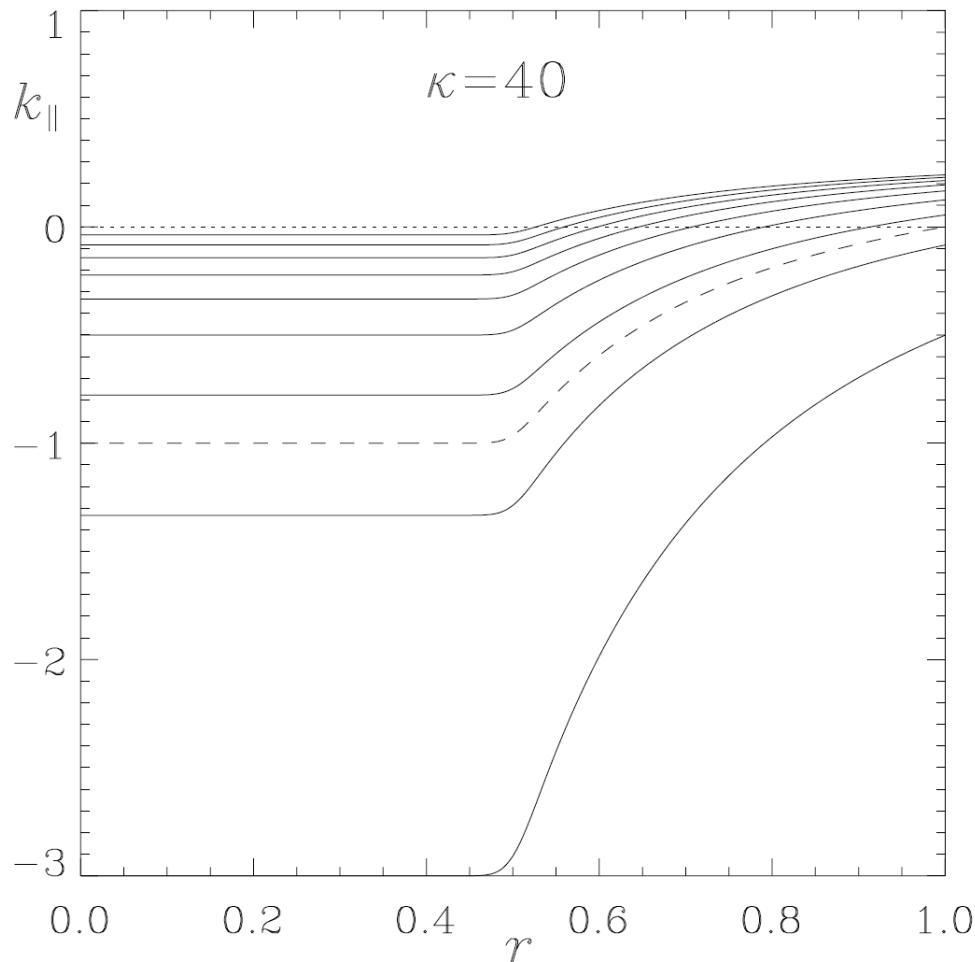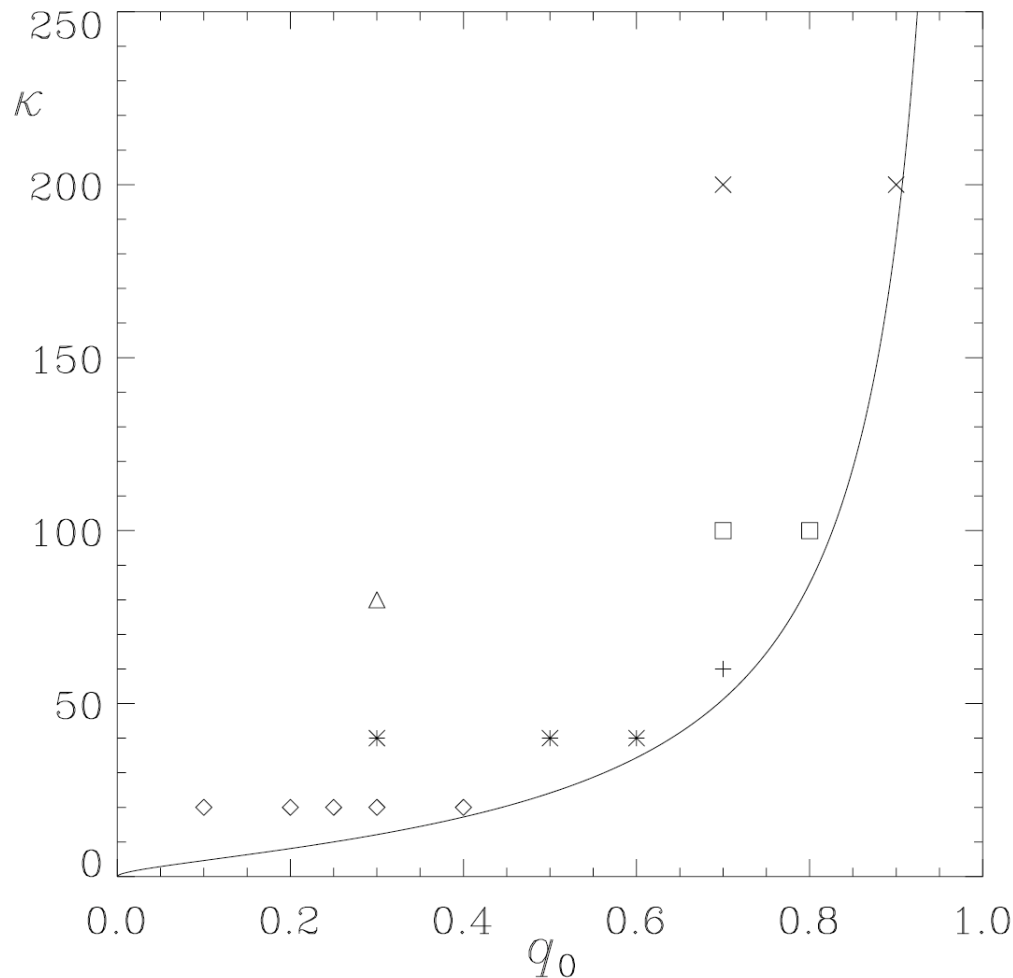

The dotted line is the line $k_\parallel = 0$.

Since $\psi_1 = iB_0 k_\parallel \Phi$, Φ is singular at $k_\parallel = 0$ (unless ψ_1 is zero there) --- problematic unless $\rho \sim 0$. This happens only for $q_0 > 0.25$. The step-function approximation also has this issue.

$d^2 k_\parallel / dr^2$ is large near $r = a$ for large κ.

q_0 and κ for unstable modes

For this profile, with $a = a_\rho$, $\kappa = \kappa_\rho$, $m = n = 1$, $R_0 = 3$, $B_0 = 1$, $\tau_w = 1000$, q_0 and κ for all runs with an unstable mode are shown:

Each run is indicated by a symbol. Same symbol indicates same κ.

All κ are larger than the solid curve, meaning that ρ at the point $k_\parallel = 0$ is less than $0.5\mathrm{erfc}(5) \sim 7.7\mathrm{e}{-13}$.

Runs with smaller κ for the same q_0 do not find unstable modes.

Growth rate
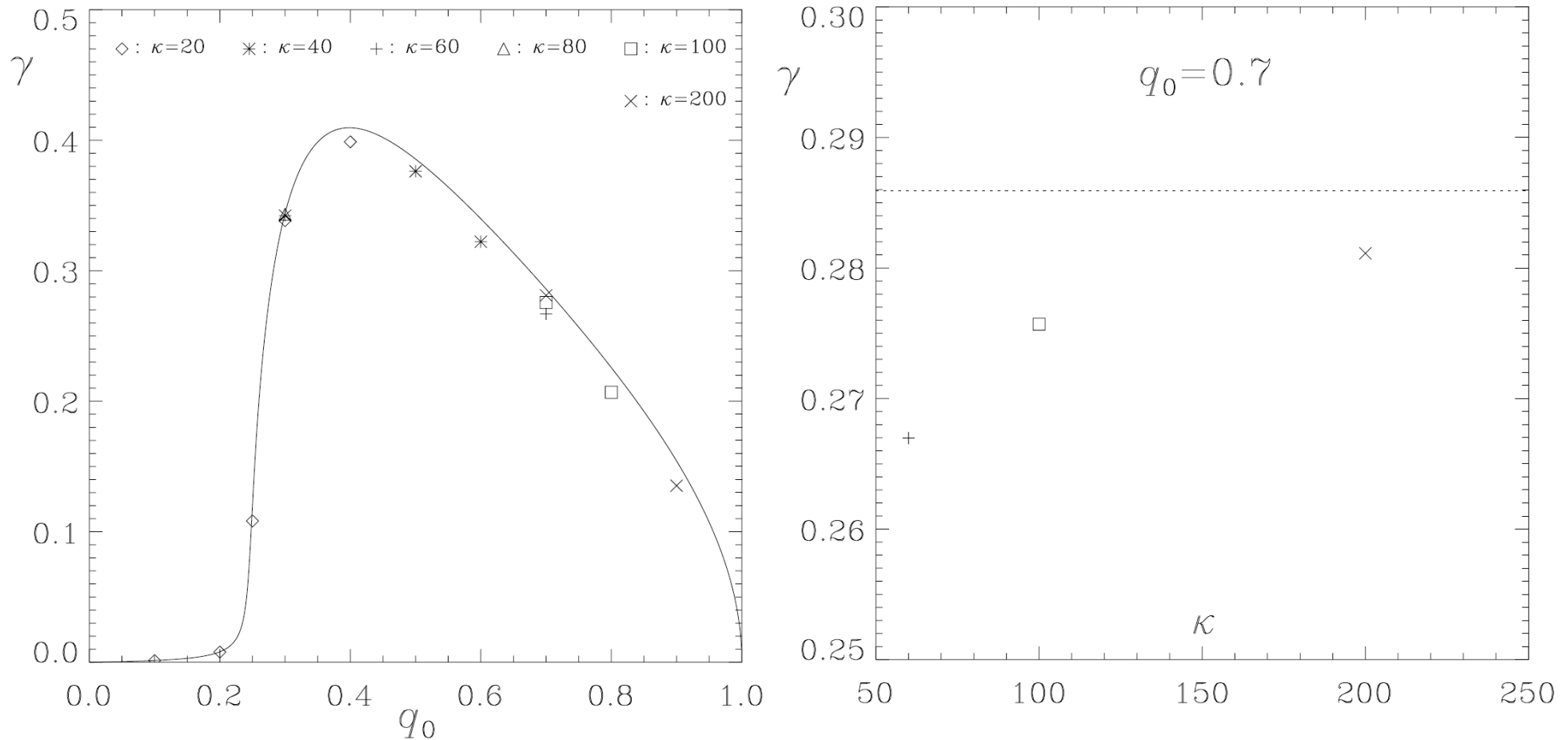
The solid curve in the left figure (and the dotted line in the right) is γ for the step-function problem. γ tends to the value for the step-function problem in the large κ limit, as expected. γ is small for $q_0 < (a/b)^2 = 0.25$, as discussed above, which is uncommon in practice.

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First order current cf. surface current
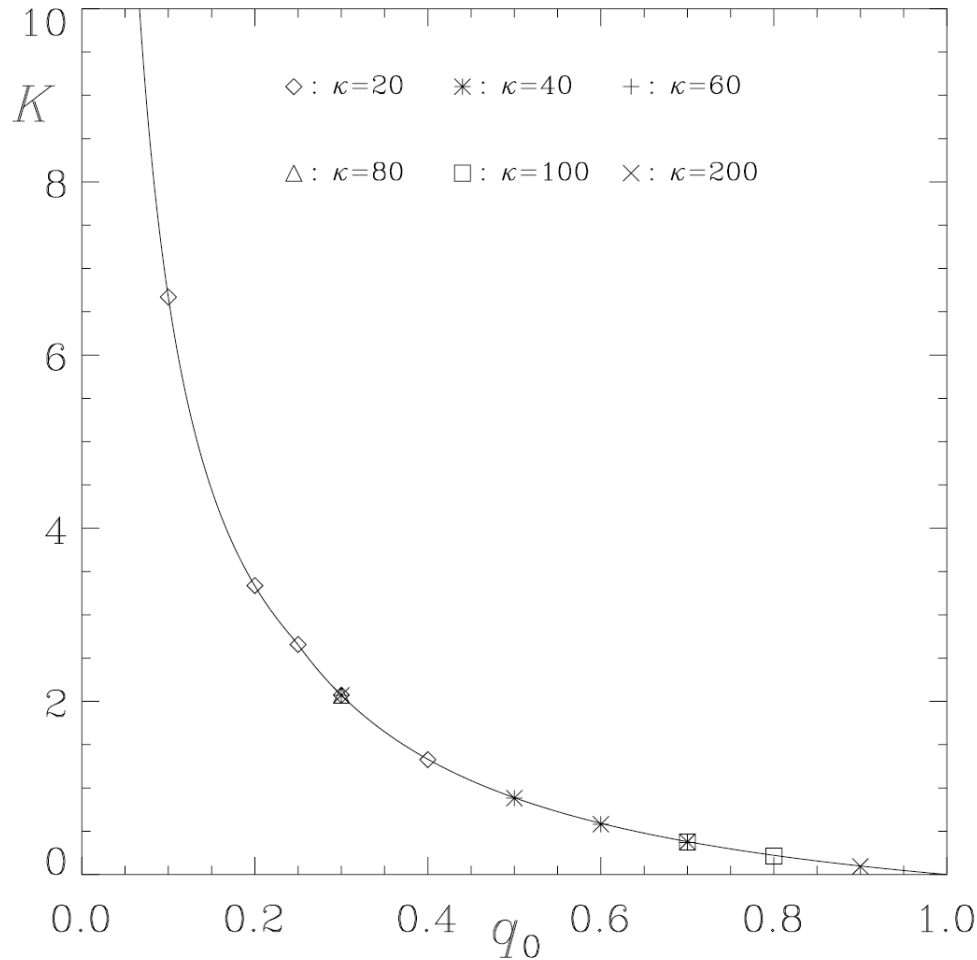
Solid curve is the step-function analytical form:

$$K = \frac{2B_0}{q_0 R_0} \frac{(m - nq_0)\left[1 + 2/(\gamma\tau_w)\right]}{\left[1 - (a/b)^{2m} + 2/(\gamma\tau_w)\right]}$$

Symbols are integrated first order current:

$$K = \left|\int_0^b j_{\phi 1} dr\right| = \left|\int_0^b \nabla_\perp^2 \psi_1 dr\right|$$

normalized for unit ξ_r.

The integrated first order current tends to the surface current found in [*Strauss et al*. 2010] in the large κ limit.

I as a definition of "residual current"

Back to the step-function problem, *I* can be regarded as *defined* by:

$$I = K - j_{\phi 0}\xi_r = \frac{2B_0}{q_0 R_0}\left\{1 - \frac{(m - nq_0)[1 + 2/(\gamma\tau_w)]}{1 - (a/b)^{2m} + 2/(\gamma\tau_w)}\right\}\xi_r$$

In the form of current density with $j_I = I\delta(r\text{-}a)$, $j_{\phi 1} = K\delta(r\text{-}a)$:

$$j_I = j_{\phi 1} - j_{\phi 0}\xi_r\delta(r - a) = j_{\phi 1} + \xi_r\frac{\partial j_{\phi 0}}{\partial r} \equiv j_{\phi 1} - j_c$$

j_c (convective current) has a physical meaning of moving $j_{\phi 0}$ in ξ_r:

$$j_{\phi 0}(r) + j_c(r) = j_{\phi 0}(r) - \xi_r\frac{\partial j_{\phi 0}}{\partial r} \approx j_{\phi 0}(r - \xi_r)$$

j_I (or *I*) is then defined as the residual current by $j_{\phi 1} - j_c$.

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Analytical forms of *K* and *I*

First order current $j_{\phi 1} = -\nabla_\perp^2 \psi_1$, or *K*, can be solved from:

$$\gamma^2 \left[\rho \nabla_\perp^2 \left(\frac{\psi_1}{k_\parallel} \right) + \frac{\partial \rho}{\partial r} \frac{\partial}{\partial r} \left(\frac{\psi_1}{k_\parallel} \right) \right] = -B_0^2 k_\parallel \nabla_\perp^2 \psi_1 + \frac{mB_0}{r} \frac{\partial j_{\phi 0}}{\partial r} \psi_1$$

In the large κ limit, only terms with large gradient contribute:

$$j_{\phi 1} \approx j_c + j_I$$

where $j_c = -iB_0 \dfrac{\partial^2 k_\parallel}{\partial r^2} \Phi \approx -\xi_r \dfrac{\partial j_{\phi 0}}{\partial r}$ is the convective current,

and $j_I = iB_0 k_\parallel \gamma^2 \dfrac{\partial \rho}{\partial r} \dfrac{\partial \Phi}{\partial r} \bigg/ \left(\gamma^2 \rho + B_0^2 k_\parallel^2 \right)$ is the residual current (*I*).

◊ j_c and j_I do not have to have the same radial profile.

◊ $j_I \propto \partial \Phi / \partial r \propto \xi_\theta$, rather than ξ_r as in the expression of *I*.

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A special example: $I = 0$ for the uniform density case ($\rho = 1$)

Since $j_I \propto \partial \rho / \partial r$, it is zero if ρ is uniform.
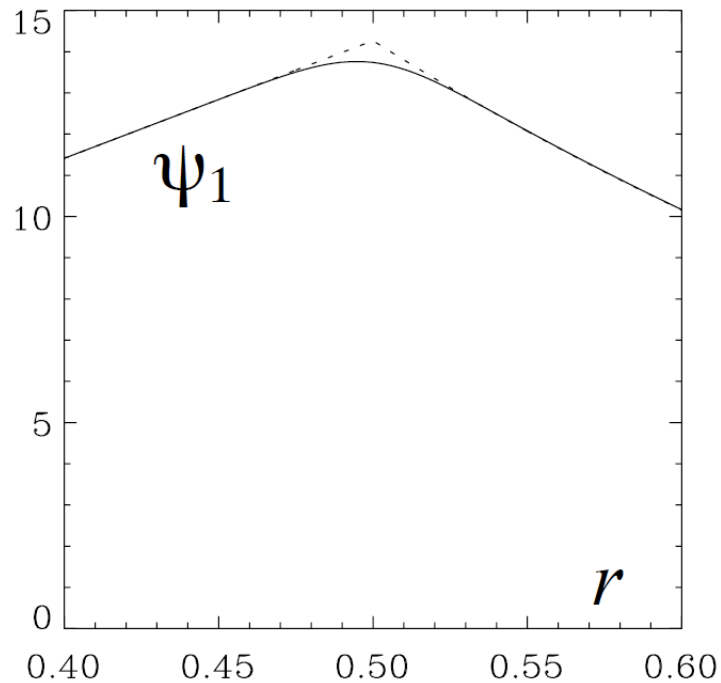
In particular, for $m = n = 1$, the eigen equation is solved by

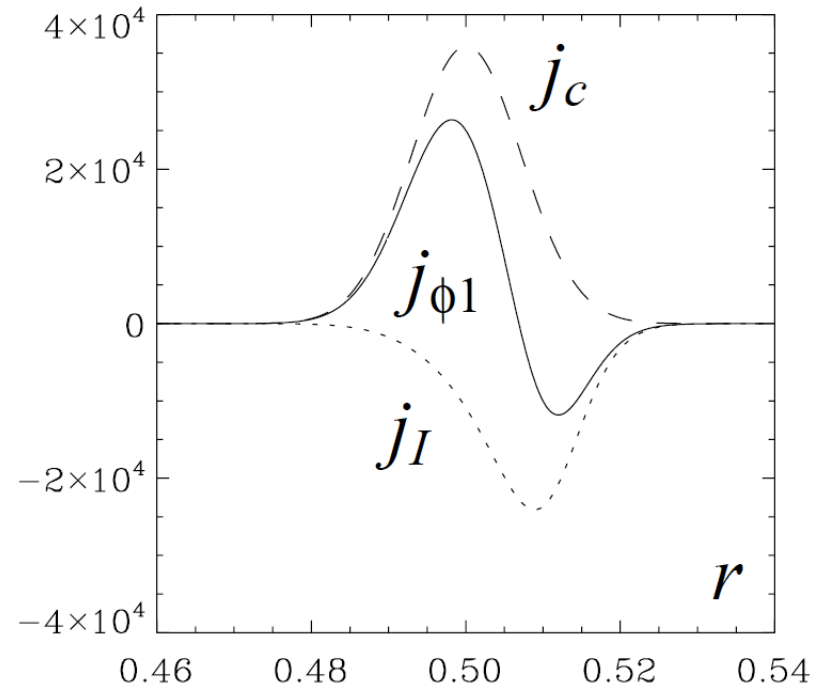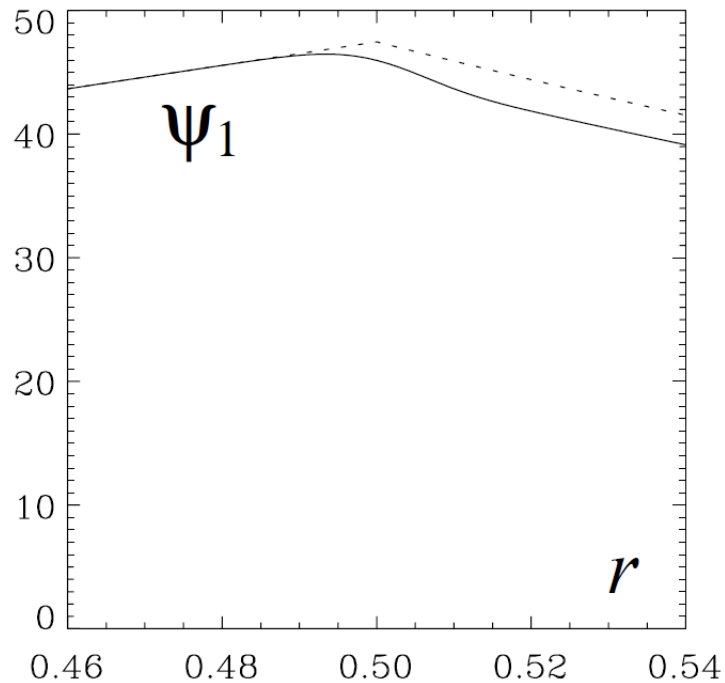$\nabla_\perp^2 \Phi = 0$, or $\Phi \propto r$, $\psi_1 = i B_0 k_\parallel \Phi \propto k_\parallel r$
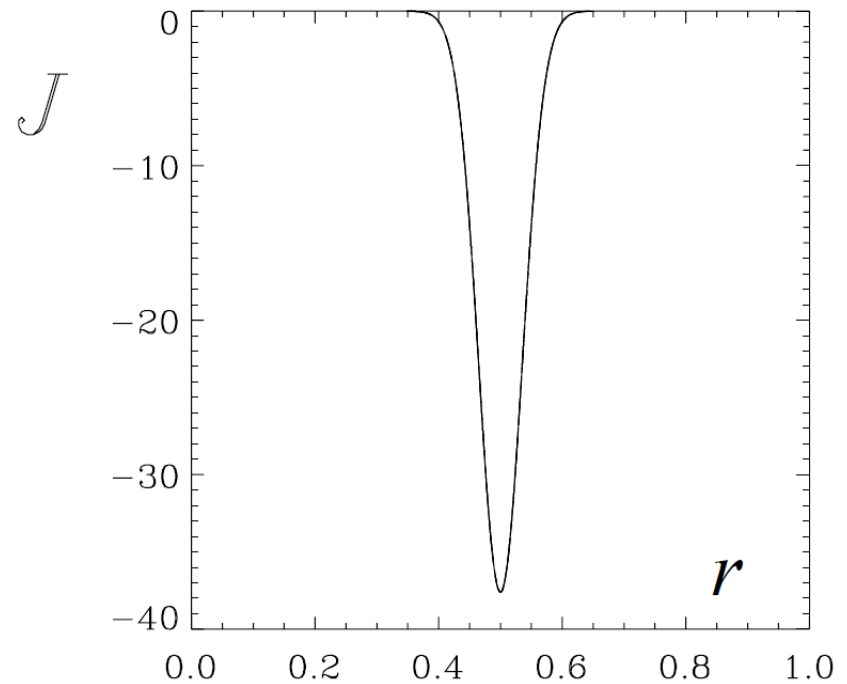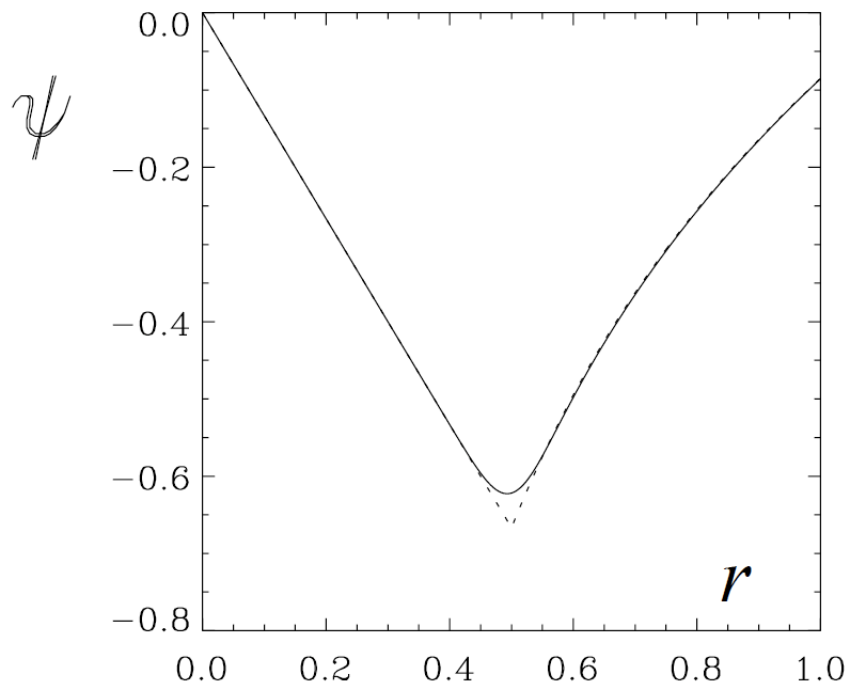
However, unstable modes exist only for $q_0 < (a/b)^2 = 0.25$ in this case ($q_0 = 0.2$ shown; dotted curve is for step-function), which is uncommon.

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Internal structures for $q_0 = 0.3$, $a = a_\rho = 0.5$, $\kappa = \kappa_\rho = 40$
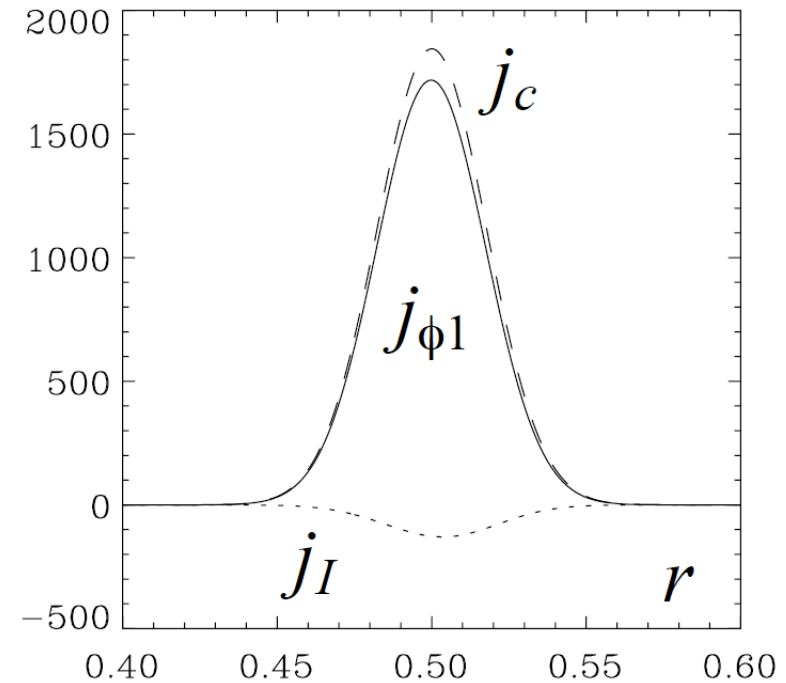
◊   growth rate is not small, but $m - nq_0$ is also not small.

◊ j_I has a similar structure as $j_{\phi 1}$ ($\sim j_c$), but is much smaller.

Internal structures for $q_0 = 0.7$, $a = a_\rho = 0.5$, $\kappa = \kappa_\rho = 100$


◊ $j_{\phi 1}$ has a clear internal structure with both signs, and is zero at a point in between.

◊ j_I is getting larger and shifting to the right of j_c.

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Internal structures for $q_0 = 0.9$, $a = a_\rho = 0.5$, $\kappa = \kappa_\rho = 200$
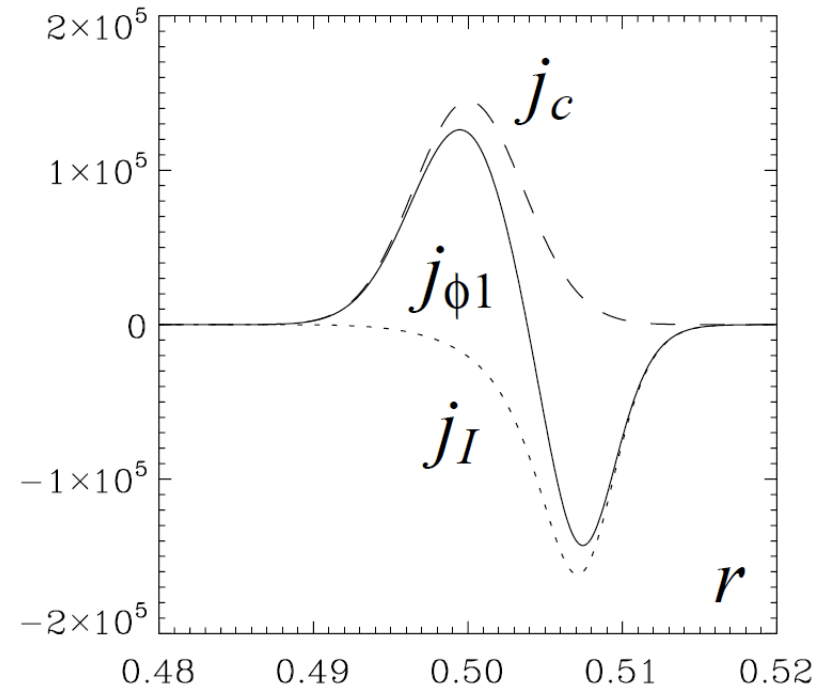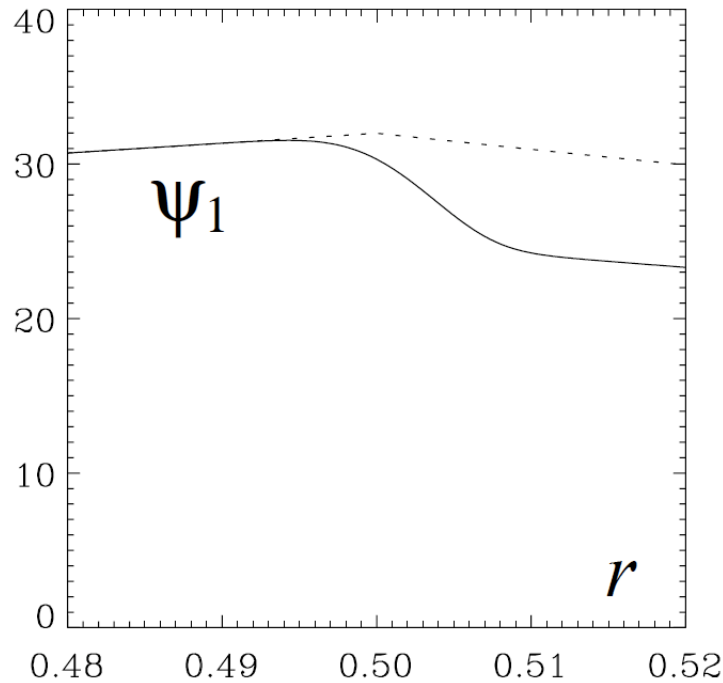
◊ $j_{\phi 1}$ has a negative peak stronger than the positive peak, but the total integrated current is still positive but small ($K \sim 0$).

◊ j_I is large and shifted much to the right of j_c so that the two have only a small overlap portion.

◊   In the quasi-linear sense, the $j_{\phi 0}$ profile will be modified to be very different from the ρ profile, which is carried by the flow.

Shifted density case: $q_0 = 0.7$, $a = 0.5$, $a_\rho = 0.55$, $\kappa = \kappa_\rho = 200$
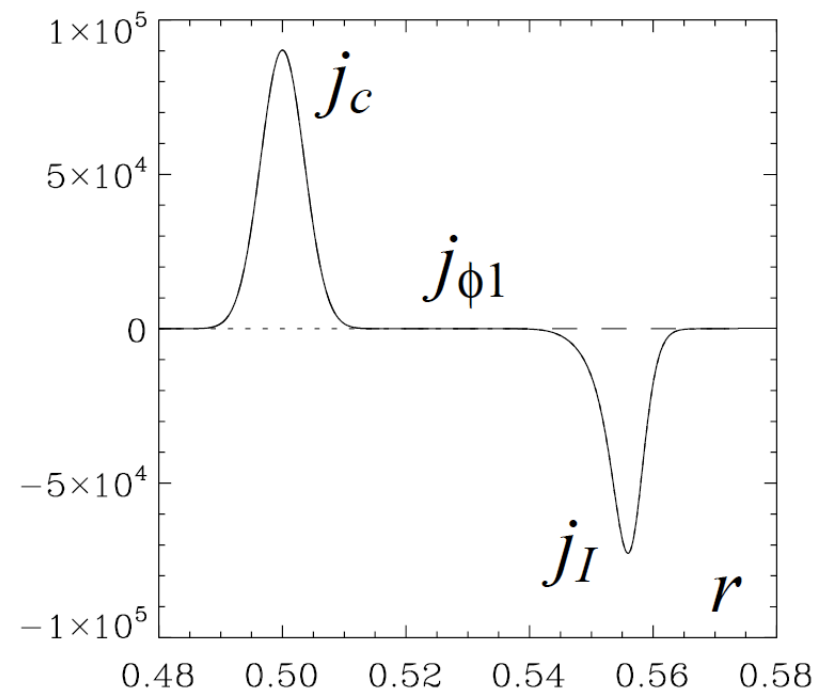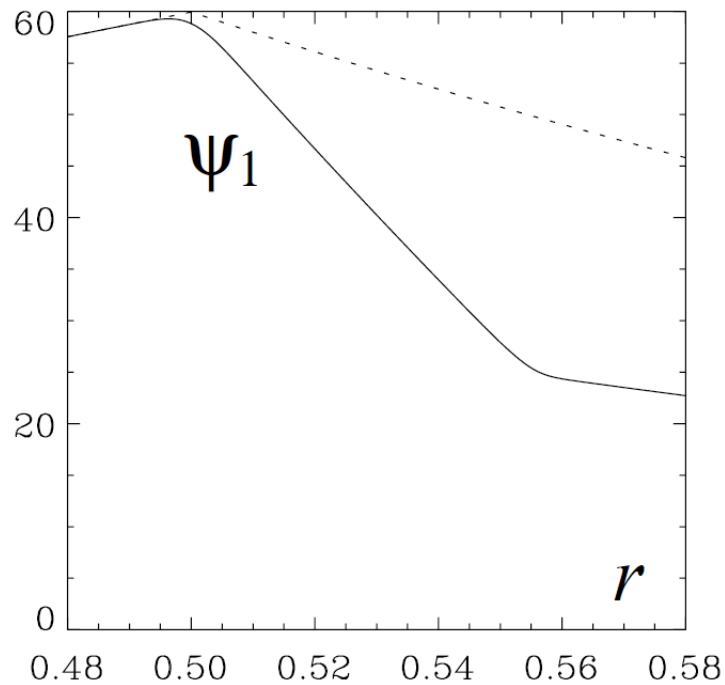
◊ $j_{\phi 1}$ has two distinct structures: a positive one $\sim j_c$, and a negative one $\sim j_I$.

◊ The two structures carry similar magnitude of opposite current.

◊ j_I is strong at location where $j_{\phi 0} \sim 0$. Nonlinear effects needed?

◊ $q_0 = 0.8, 0.9$ has no unstable mode for this profile.

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Step-function model for the shifted density case

Analytic solution for $m = n = 1$, with growth rate:

$$\frac{\gamma^2}{2B_0^2} = \frac{\left(\dfrac{a^2}{a_\rho^2} - q_0\right)}{R_0^2 q_0^2}\left\{1 - \frac{\left(\dfrac{a^2}{a_\rho^2} - q_0\right)\left(1 + \dfrac{2}{\gamma\tau_w}\right)}{1 - \dfrac{a_\rho^2}{b^2} + \dfrac{2}{\gamma\tau_w}}\right\}$$

◊   reduces to the previous expression when $a = a_\rho$.

◊   instability boundary shifted to $q_0 < (a/a_\rho)^2 < 1$ (for $a = 0.5$, $a_\rho = 0.55$, $q_0 < 0.826$).

◊   since the presence of j_I would generally make $j_{\phi 0}$ and ρ evolving into different profiles, the stability boundary can change as well.
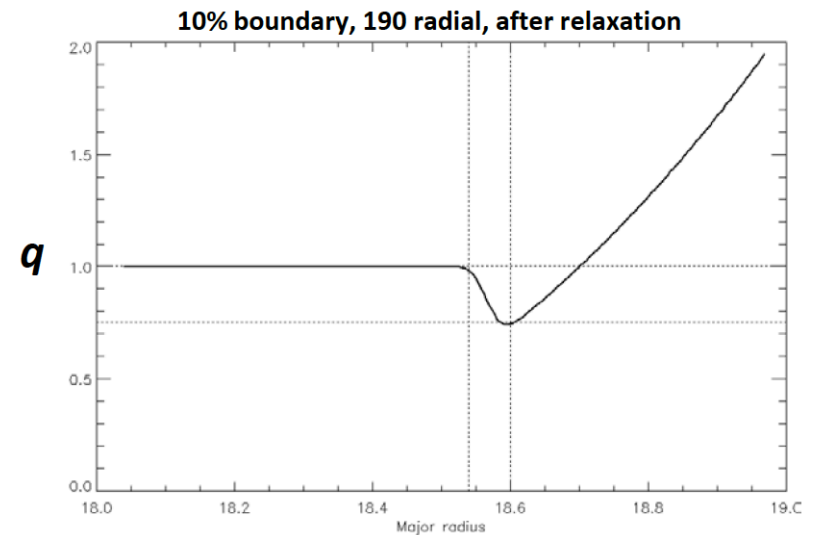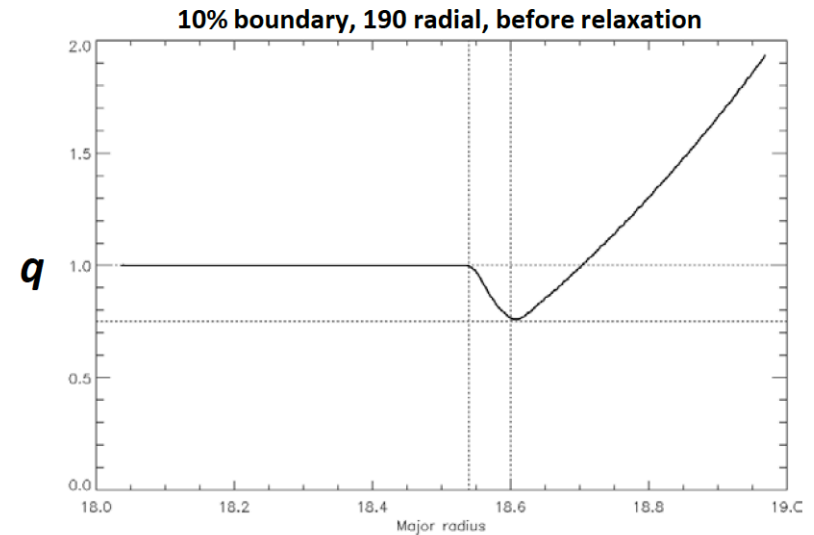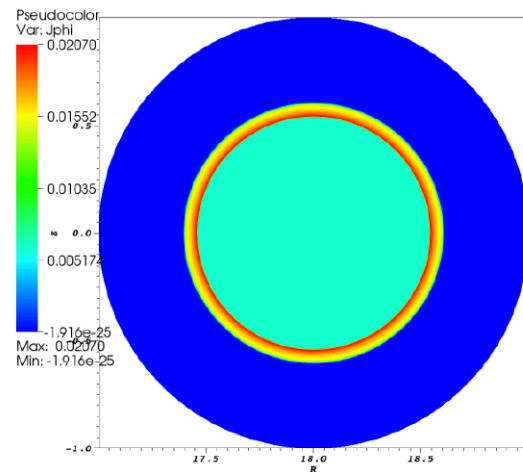
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Comparison with M3D data --- Initial equilibrium

- R_0=18, δ/a=0.1, p_0/ε=100, n_0/n_{vac}=1 $\rightarrow$ T_0/T_{vac} = 100
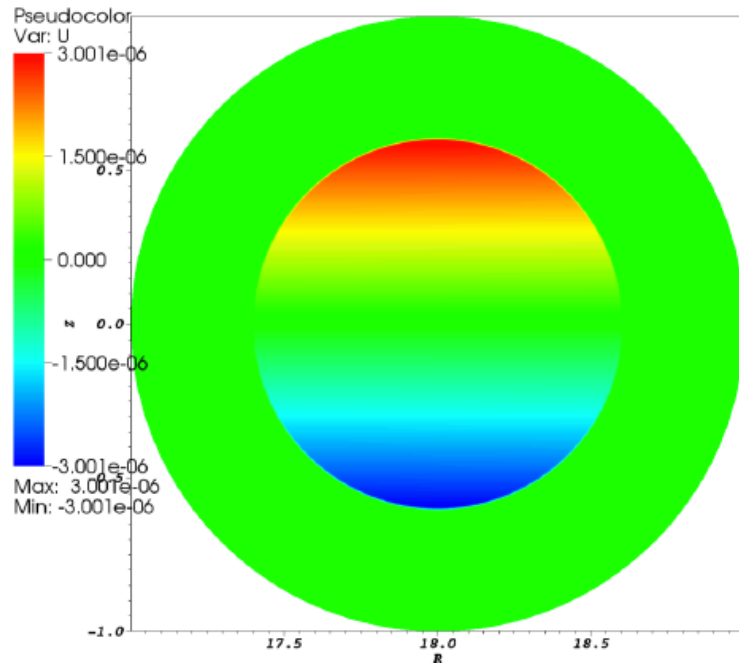- 190 radial zones
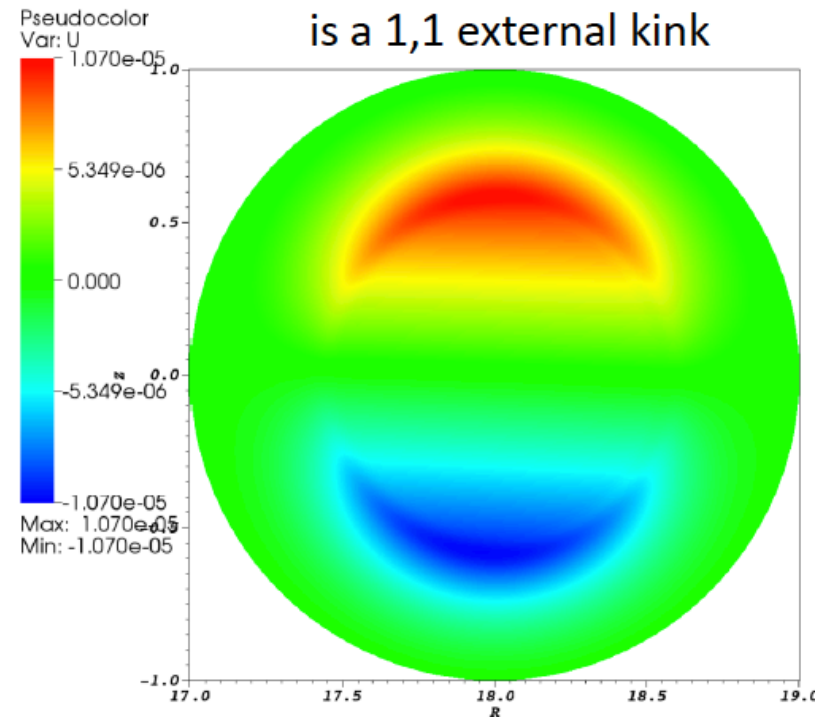

pressure
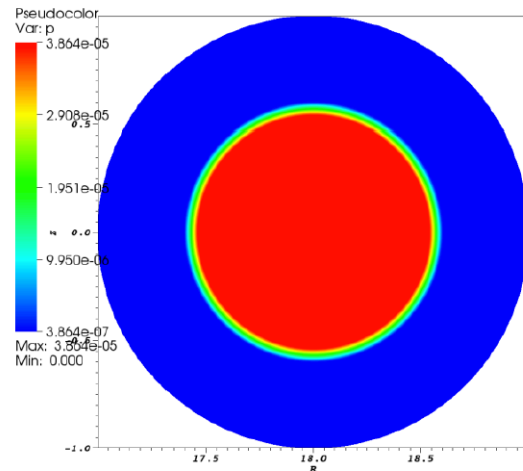
toroidal current density

Comparison with M3D data --- linear eigenmode

Initial perturbation

$n=1$ eigenmode is a 1,1 external kink

Details:

$\eta_{plas} = 10^{-6}$; $\eta_{vac} \approx 9.5 \times 10^{-4}$; $\eta_{wall} = 0$
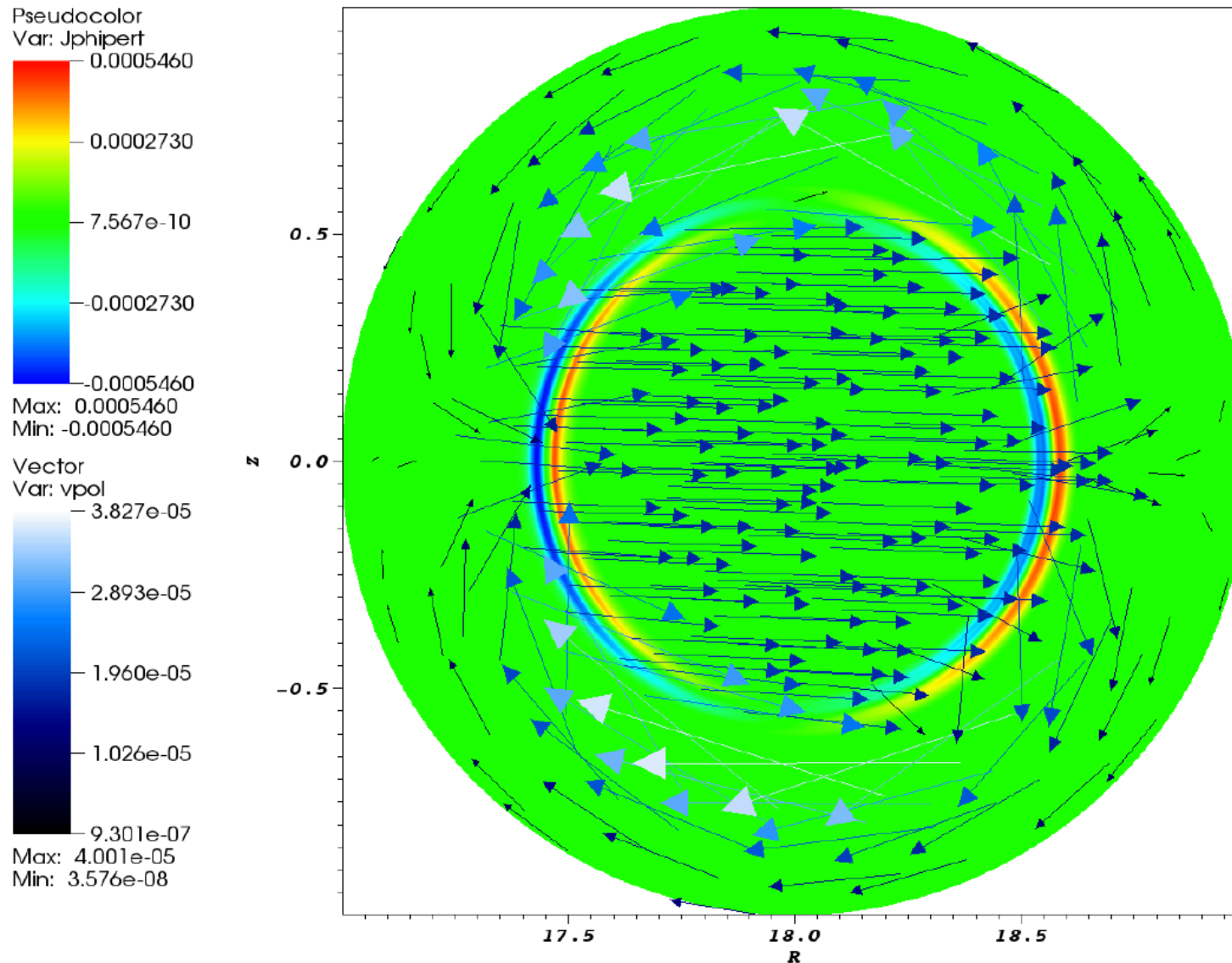
$\mu = 10^{-5}$; $\mu_{H\_tor} = 10^{-3}$

$\kappa_\perp = 10^{-6}$; $\kappa_{||} = 5 \times 10^2$

density evolution off

ohmic heating on

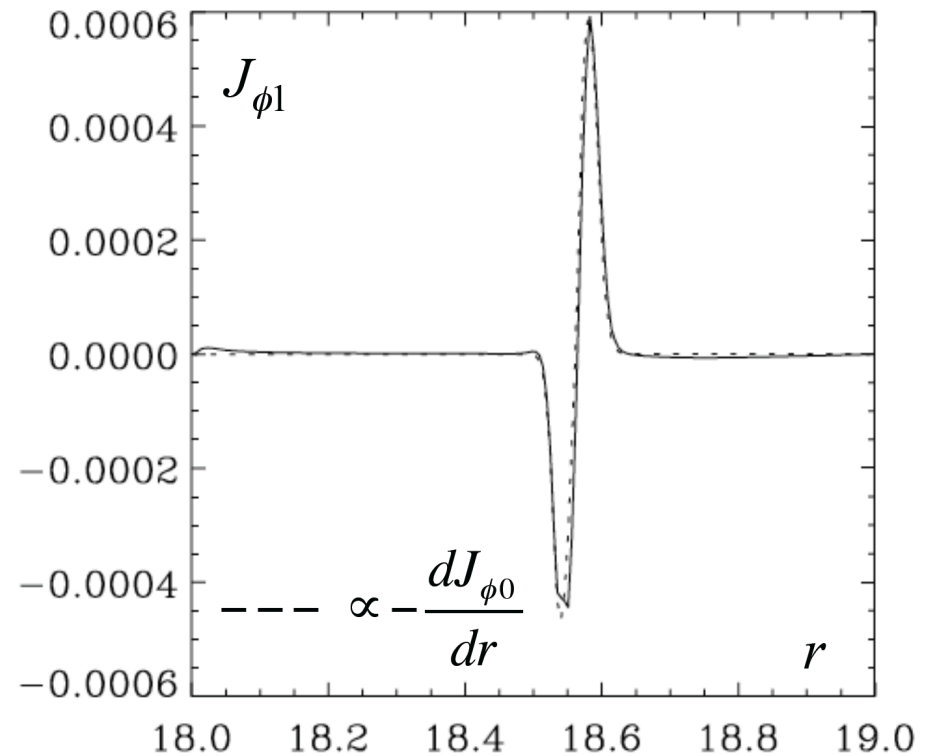$\gamma\tau_A = 0.0210 \pm 0.00105$

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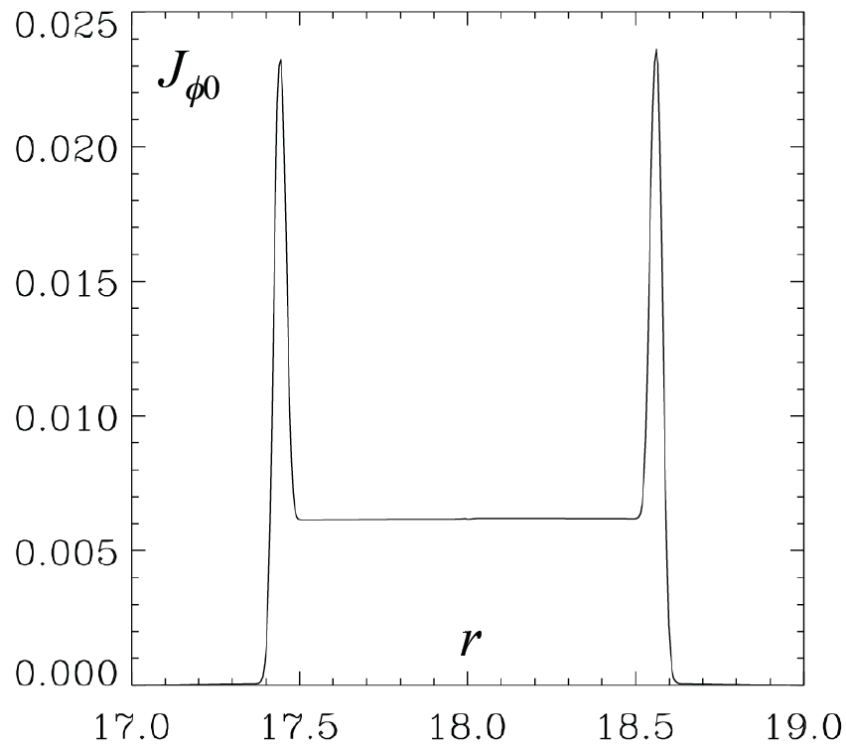
Comparison with M3D data --- first order current density

- Rigid displacement of plasma column

- Rearrangement of "vacuum" to avoid compression

- 1,1 toroidal current sheets of both signs at plasma boundary

Comparison with M3D data --- current profile

◊ cylindrical geometry with large aspect ratio, but not RMHD, uniform ρ

◊ unstable mode with a main $m = n = 1$ component

◊ ξ_r is positive, $j_{\phi 1}$ is mainly consistent with $-\partial j_{\phi 0}/\partial r$ implying very weak residual current.

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Conclusions

◊ The step-function derivation of kink-mode instability has been generalized to arbitrary background current and density profiles.

◊ The confusion over the sign of the surface current in the literature has been resolved by identifying the analytic forms of the convective current and residual current, which are the two components of the first order current but have opposite signs.

◊ The presence of the residual current can evolve the plasma current into a profile different from that of the density. The instability condition changes sensitively to the change of such profiles.

◊ The residual current can be important if instability occurs for density profile extends much beyond the current density profile and has sharp gradient.

◊ Direct 3D nonlinear simulations are needed. Our linear studies can serve as benchmarks of 3D MHD codes.

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