Disruption Current Asymmetry and Boundary Conditions

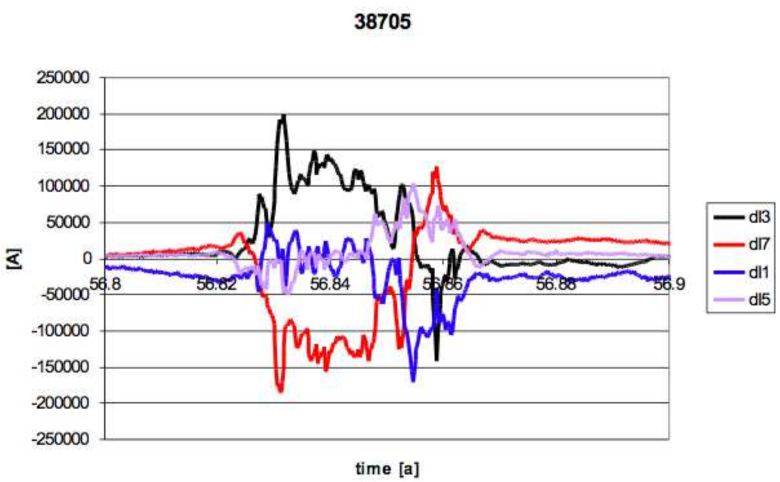
H. Strauss, *HRS Fusion*

PPPL Disruption Theory Workshop 2014

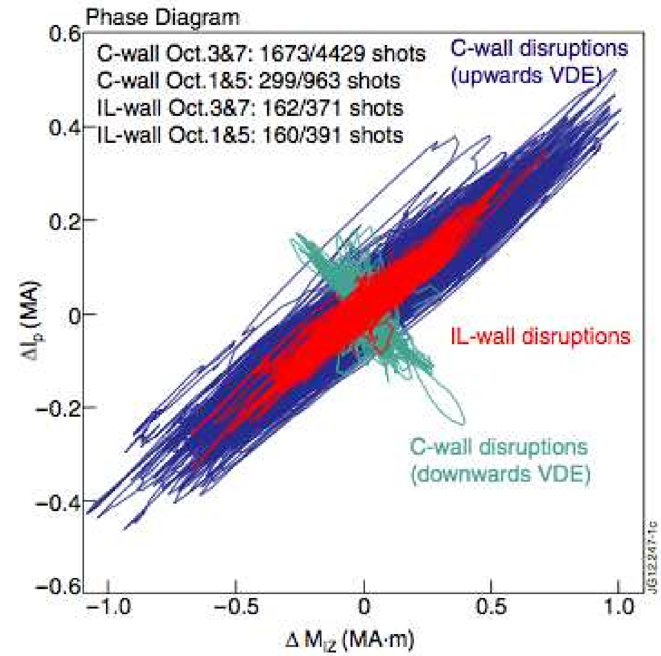
Outline

- Disruption Current Asymmetry
  - theory
  - numerical simulations
- MHD boundary conditions
  - theory of electromagnetic sheath
  - wall gaps
- Is JET a good predictor for ITER?

Toroidal variation of toroidal current in JET



(a)



(b)

(a) Current I_ϕ measured in quadrants of JET, showing $n = 1$ toroidal variation. (b) Toroidal current variation $\Delta I_\phi = \int \tilde{J}_\phi dRdZ$ vs. the vertical moment $\Delta M_{IZ} = \int Z\tilde{J}_\phi dRdZ$ of the current variations. [Gerasimov *et al.* N.F. 2014]

This was interpreted by the Hiro current model [Zakharov *et al.* 2012]. It was shown analytically [Strauss *et al.* 2010] that the slope is proportional to VDE displacement. This is verified by M3D simulations.

Relation of toroidal current perturbation to vertical current moment

Analytical model Phys. Plasmas (2010)] showed the relation of ΔI to M_{IZ} is caused by VDE displacement of a kink mode. It is basically a kinematic effect. It does not require Hiro current. The following is a new derivation.

The vertical current moment is the perturbed current multiplied by $r \sin\theta$,

$$\tilde{M}_{IZ} = \int_0^a \tilde{J}_\phi r^2 \sin\theta dr d\theta = -\oint \frac{\partial\tilde{\psi}}{\partial r} a^2 \sin\theta d\theta \tag{1}$$

in a circular cross section where the boundary is $r = a$, noting that

$$\tilde{J}_\phi = -\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\tilde{\psi}}{\partial r} - \frac{1}{r^2}\frac{\partial^2\tilde{\psi}}{\partial\theta^2} \tag{2}$$

integrating by parts, and assuming that the wall is a good conductor, so that $\tilde{\psi} \approx 0$ at $r = a$.

The toroidal current is

$$\tilde{I}_\phi = \int_0^a \tilde{J}_\phi r dr d\theta = -\oint \frac{\partial\tilde{\psi}}{\partial r} a d\theta \tag{3}$$

Note that (1) and (3) differ by a factor of $\sin\theta$.

The flux change $\delta\tilde{\psi}$ produced by an axisymmetric displacement potential Φ is

$$\delta\tilde{\psi} = \nabla\Phi \times \nabla\tilde{\psi} \cdot \widehat{\phi}. \tag{4}$$

The VDE displacement potential has the form $\Phi = \xi_{VDE}(r)\cos\theta$. Iterating $\tilde{\psi} = \psi_1 + \psi_2 + \psi_3 + \ldots$ and taking the radial derivative, imposing a rigid wall boundary condition $\xi_{VDE}(a) = 0$, gives

$$\tilde{\psi}'_{k+1} = \frac{\xi'_{VDE}}{r}\left(\frac{\partial}{\partial\theta}(\tilde{\psi}'_k\cos\theta) + 2\tilde{\psi}'_k\sin\theta\right) \tag{5}$$

where the prime denotes a radial derivative. Summing (5) over k and integrating over θ, using (1),(3) gives

$$\tilde{I}_\phi = 2\frac{\xi'_{VDE}}{a^2}\tilde{M}_{IZ}. \tag{6}$$

• $\tilde{\psi}$ can consist of an arbitrary sum of (m, n) modes, not just a $(1, 1)$.

• Plasma current does not have to touch the wall.

• For an upward VDE, $\xi'_{VDE}(a) > 0$.

M3D simulations

ITER FEAT15MA equilibrium was modified by setting toroidal current and pressure to zero outside the $q = 2$ surface, keeping the total toroidal current constant (MGI model) [Izzo *et al.* 2008]. Plasma was evolved in 2D to an initial VDE displacement, then evolved in 3D. In other cases, toroidal current was set to zero outside the $q = 1.5$ surface (JET?).

The perturbed current and vertical displacement were measured as

$$\Delta I_\phi = \frac{1}{2\pi}\left(\oint d\phi < \tilde{J}_\phi >^2\right)^{1/2} \quad (7)$$

$$\Delta M_{IZ} = \frac{1}{2\pi}\left(\oint d\phi < Z\tilde{J}_\phi >^2\right)^{1/2} \quad (8)$$

$$G(R_*, \xi) = \mathrm{min(G)} \quad (9)$$

$$(10)$$

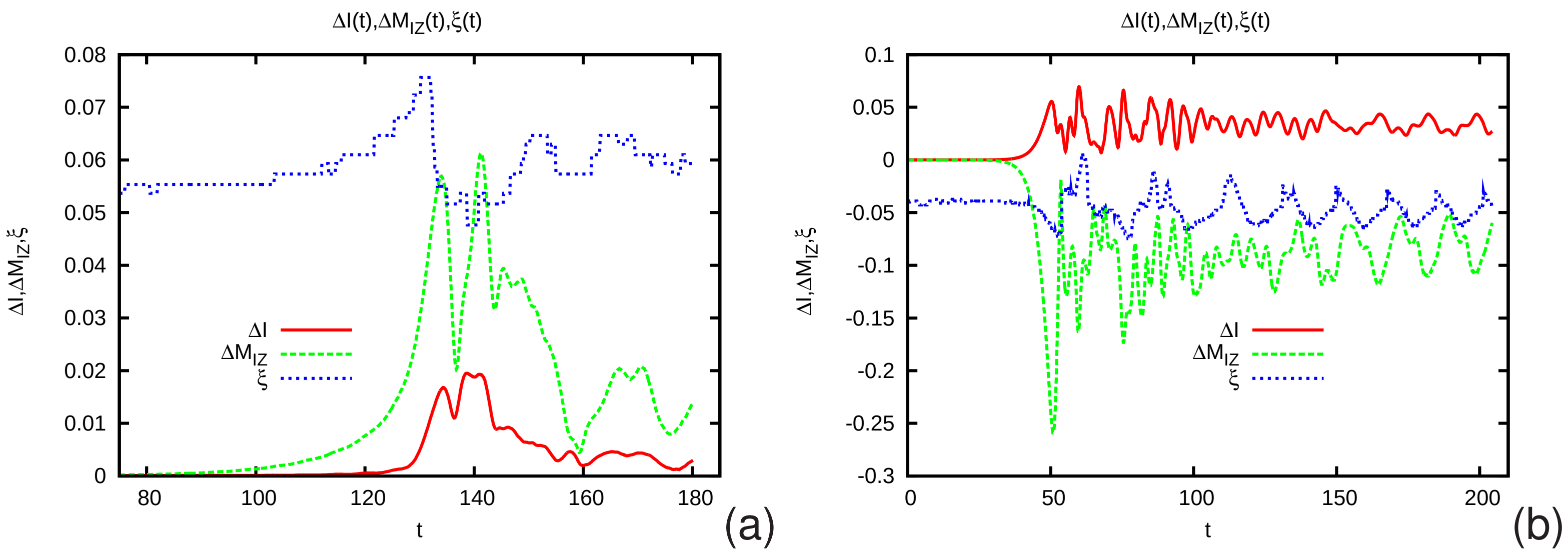
where $G = RB_\phi$ is toroidal flux and

$$\tilde{J}_\phi = J_\phi - \frac{1}{2\pi}\oint d\phi J_\phi \quad (11)$$

$$< \tilde{J}_\phi > = \int dRdZ\tilde{J}_\phi \quad (12)$$

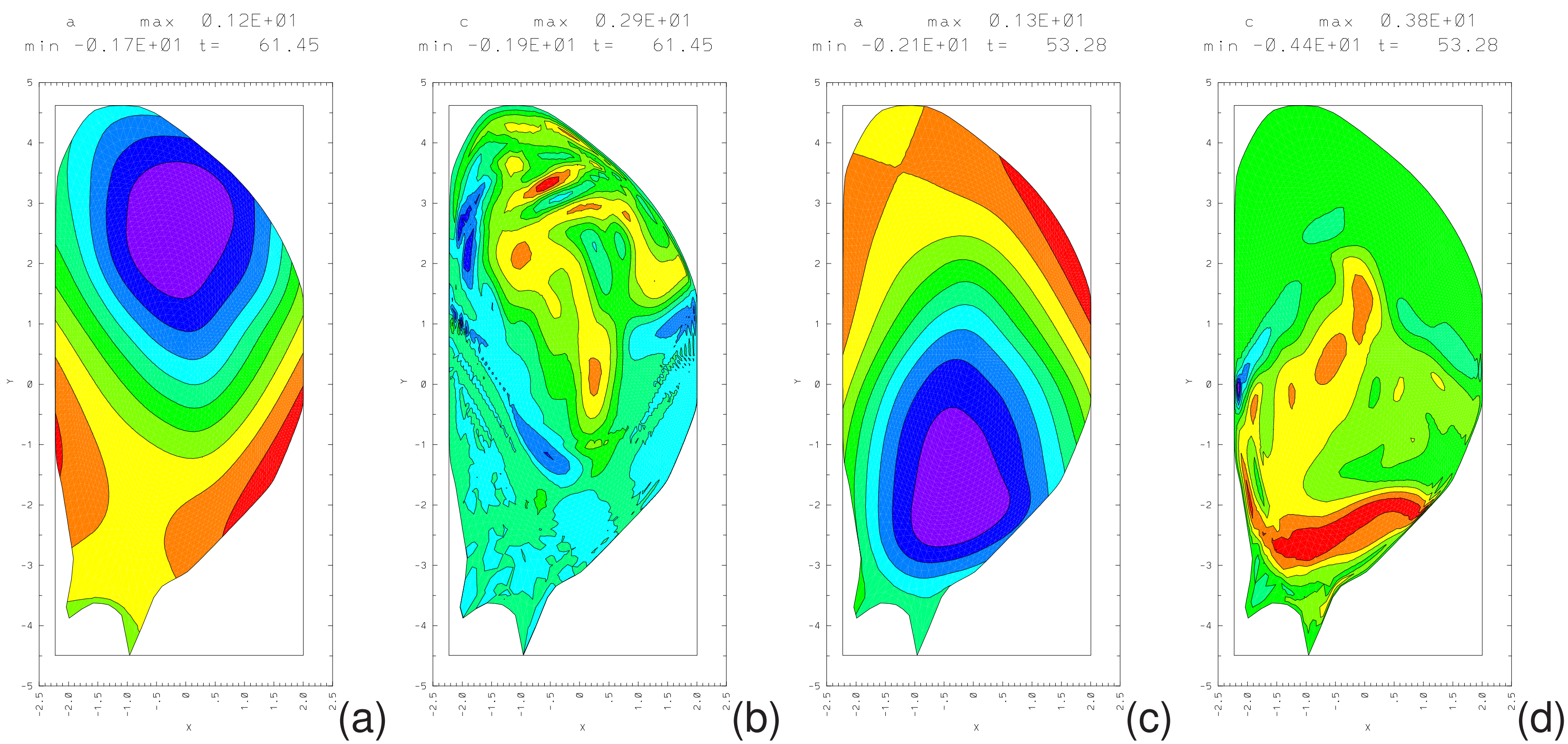
Time history of perturbed current, vertical current moment, and vertical displacement

M3D simulations were done with $S = 10^6$, wall penetration time $\tau_{wall} = 10^4 \tau_A$. Velocity boundary condition $v_n = 0$.



Time history of $\Delta I_\phi, \Delta M_{IZ}, \xi$. (a) upward VDE (b) downward VDE

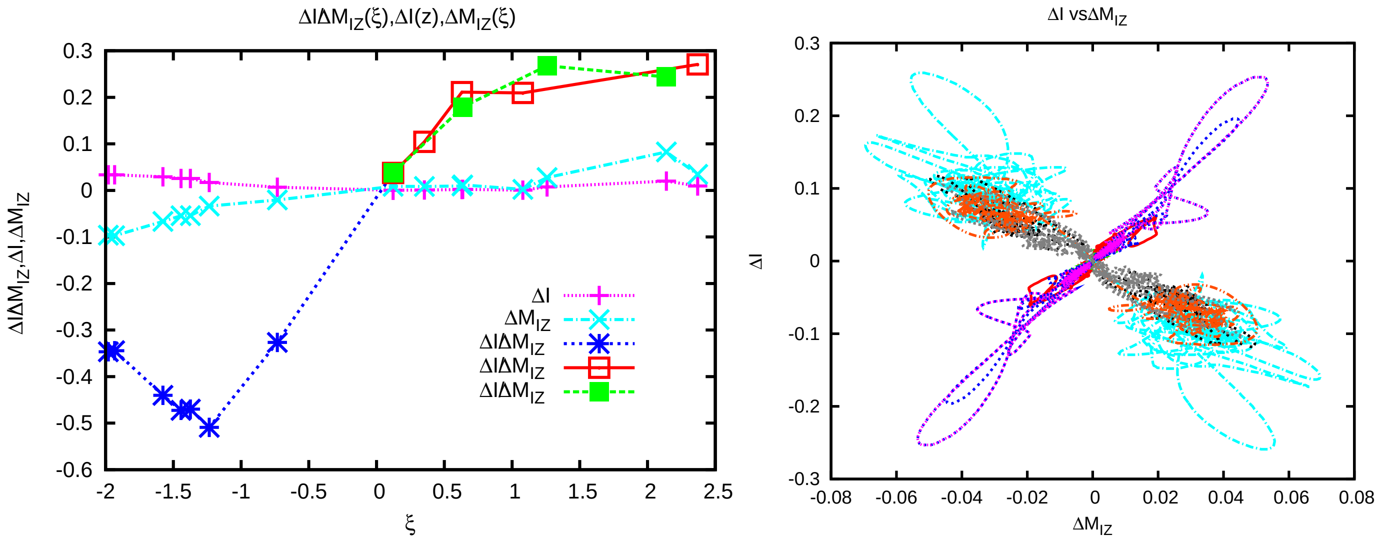
magnetic flux and toroidal current



Upward VDE (a) ψ (b) J_ϕ **Downward VDE** (c) ψ (d) J_ϕ

Plasma is turbulent, not an equilibrium with surface current.

Time averaged $\Delta I_\phi/\Delta M_{IZ}$ and time histories $\Delta I_\phi, \Delta M_{IZ}$



(a) Time averages of $\Delta I_\phi, \Delta M_{IZ}, \xi$. Showing $\Delta I_\phi/\Delta M_{IZ} \propto \xi$, for $|\xi| \lesssim 1$, when plasma current channel reaches the wall. (b) Time histories of $\Delta I_\phi, \Delta M_{IZ}$ the cases in (a).

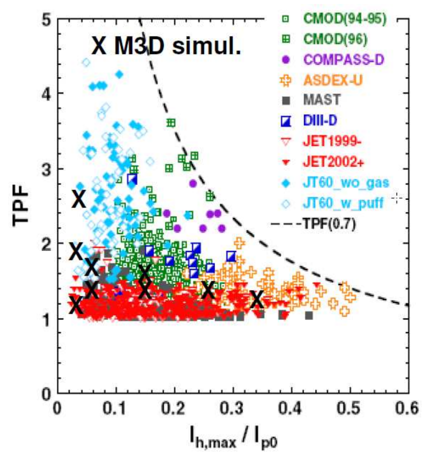
ΔI_ϕ, wall current, halo current

In the previous runs, $\Delta I \approx 0.01 I$. Apply $\nabla \cdot \mathbf{J} = 0$ in the plasma up to the wall, and in the wall: Plasma current and wall current asymmetries have opposite sign, connected by 3D halo current.

$$\frac{dI_{plasma}}{d\phi} + I_{halo3D} = I_{halo3D} - \frac{dI_{wall}}{d\phi} = 0$$

$$I_{halo3D} = \oint J_n R dl, \qquad J_n = \frac{1}{R}\frac{\partial^2 \psi}{\partial n \partial \phi}.$$

The magnitude of the variation of the toroidal current can be expressed $\Delta I_\phi / I_\phi = (1/2\pi) C_{halo} \times TPF \times HF$



It was shown [Strauss *et al.* 2012] that

$$C_{halo} \leq \frac{\pi}{2}.$$

ITER database plot of TPF vs. HF. $HF \times TPF < 0.7 \Rightarrow \Delta I_\phi < 18\%$ This agrees with JET data [Gerasimov *et al.* 2014].

resistive wall boundary conditions

The magnetic field components in the plasma are matched to the vacuum field at a thin resistive shell of thickness δ. Surrounding this is an outer vacuum region. The wall current is given by

$$\mathbf{J}_{wall} = \frac{1}{\delta}\hat{\mathbf{n}} \times (\mathbf{B}_{vac} - \mathbf{B}_{plas}). \tag{13}$$

$\mathbf{B}_{vac}$ is the magnetic field outside the wall, $\mathbf{B}_{plas}$ is the magnetic field on the plasma side of the wall.

$$\hat{\mathbf{n}} \cdot \mathbf{B}_{vac} = \hat{\mathbf{n}} \cdot \mathbf{B}_{plas}. \tag{14}$$

This gives a boundary condition to determine the vacuum field, which is done with GRIN Green's function code. In the wall, the normal component of the magnetic field satisfies

$$\frac{\partial B_n}{\partial t} = -\nabla \cdot (\eta_{wall}\mathbf{J}_{wall} \times \hat{\mathbf{n}}). \tag{15}$$

Recent M3D simulations have been concerned with calculation of the force on the wall caused by plasma disruptions. The total wall force is given by

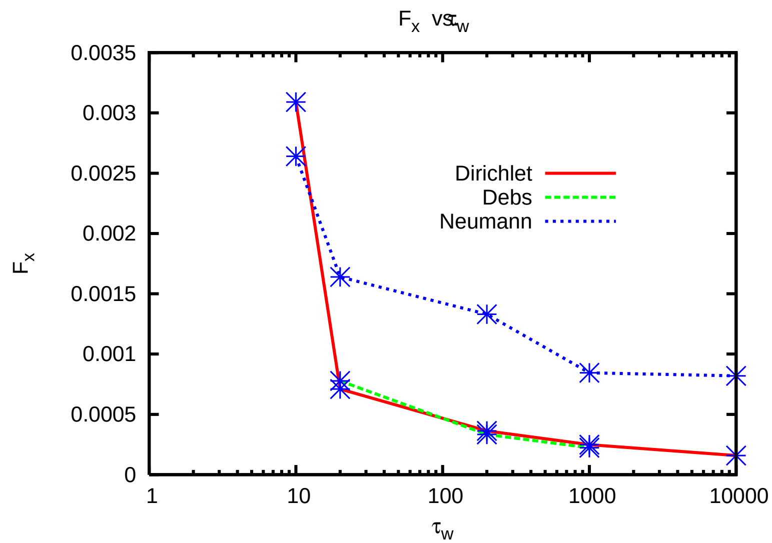
$$\mathbf{F} = \int dl R d\phi \mathbf{J}_{wall} \times \mathbf{B}_{wall}. \tag{16}$$

Of particular importance is the net horizontal force, F_x. Here F_x is obtained by taking the horizontal components of $\mathbf{F}$.

Effect of velocity boundary conditions on F_x

The main effect on the force comes from the magnetic, resistive wall boundary conditions. The velocity boundary condition is less important. Comparison [Strauss, 2014] between:

- Dirichlet: $v_n = 0$,
- Neumann: $\partial v_n / \partial n = 0$,
- DEBS: $v_n = \eta_w (\mathbf{J}_{wall} \times \mathbf{B}_{wall} \cdot \hat{\mathbf{n}})/B^2$, $\mathbf{E} = \eta_w \mathbf{J}_{wall}$



$F_x(\tau_{wall})$ for Neumann, Dirichlet and DEBS velocity boundary conditions.

Sheath compatible velocity boundary conditions

[H. Strauss, 2014]. The velocity is

$$\mathbf{v} = \mathbf{v}_\perp + v_\parallel \frac{\mathbf{B}}{B}, \quad \mathbf{v}_\perp = c\frac{\mathbf{E}\times\mathbf{B}}{B^2}, \quad \mathbf{E} = \nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \tag{17}$$

The sheath potential accelerates ions to the sound velocity [Stangeby, 2000] $c_s = (T_e/M_i)^{1/2}$ at which they strike the wall. The parallel velocity $v_\parallel$ does not affect the magnetic field, hence does not affect halo current or wall force. Near the wall $c_s/v_A \approx 10^{-2}$, so $v_\parallel \approx 0$ is a reasonable approximation.

The electrostatic potential Φ at the sheath entrance is approximately [Stangeby, 2000]

$$\Phi \approx 3\frac{T_e}{e}. \tag{18}$$

The perpendicular velocity normal to the wall, from (17),(18), is approximately

$$v_{\perp n} = k_\perp \frac{c}{B}\Phi \approx 3k_\perp \rho_s c_s = 3r\omega_* = \mathcal{O}(\rho_s) \tag{19}$$

where ρ_s is the gyroradius using c_s. For modes with MHD scale length, $k_\perp \rho_s \ll 1$, so

$$v_{\perp n} = 0 \tag{20}$$

is a good approximation.

$\partial \mathrm{A}/\partial t$ contribution to $\mathbf{E}$

The sheath is described by the usual electrostatic approximation, in the case of interest $k_\perp \Delta_{sheath} << 1$, where Δ_{sheath} is the sheath thickness. This has been shown for radio - frequency electromagnetic waves [D'Ippolito,2006] and it is easily verified for MHD.

The magnetic field in leading order does not vary in the sheath; $(1/c)(\partial A_{||}/\partial t)$ varies on the $k_\perp^{-1}$ length scale. Otherwise magnetic perturbations of $\mathcal{O}(\Delta_{sheath}^{-1})$ would be produced. Similarly b_n is constant.

The $\mathbf{b}$ component of (17) can be integrated to give $E_{||} = \nabla_{||}\tilde{\Phi}$ where

$$\tilde{\Phi} = \Phi - (1/c)(\partial A_{||}/\partial t)s, \tag{21}$$

and s is a local coordinate such that $b_n \partial s/\partial n = 1$, and $s = \mathcal{O}(\Delta_{sheath})$. Effectively $(1/c)(\partial A_{||}/\partial t)$ is absorbed into Φ by a gauge transformation.

In the MHD limit in which Δ_{sheath} and ρ_s are neglected, this implies that $v_{\perp n}$ satisfies a Dirichlet boundary condition,

$$v_{\perp n} = 0 \tag{22}$$

and the total normal velocity is

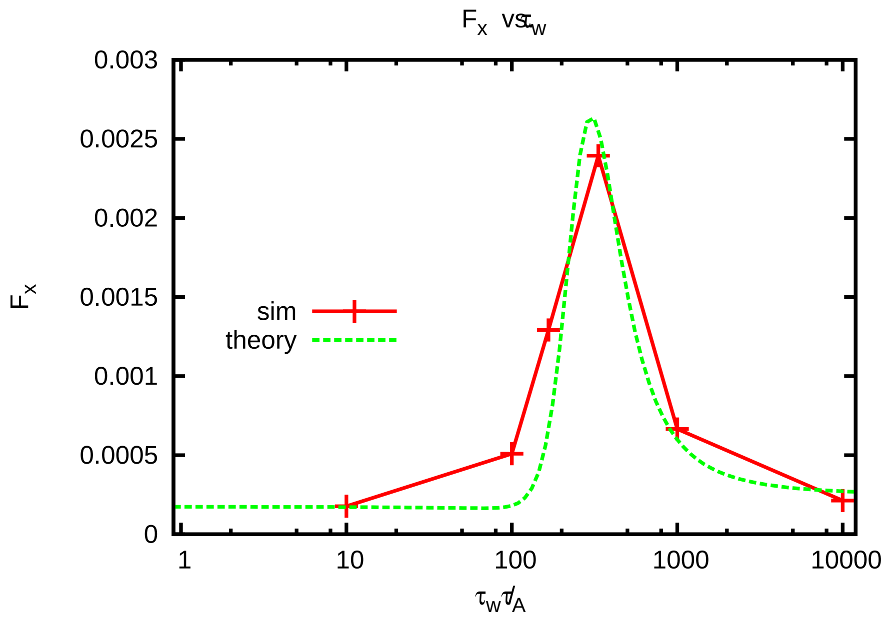
$$v_n = c_s |b_n| \tag{23}$$

directed from the plasma into the wall.

M3D results on wall force

In AVDE disruptions, magnetic flux is scraped off at the resistive wall, causing q at the last closed flux surface to drop to $q = 2$. Plasma becomes kink unstable to $(2, 1)$ mode [J. Manickam *et al.* 2012; H. Strauss *et al.* (2013)]

The wall force depends on $\gamma\tau_w$, where γ is the nonlinear growth rate of $n = 1$ MHD instabilities and τ_w is the resistive wall penetration time. Sideways wall force is maximum when $\gamma\tau_w = \mathcal{O}(1)$.



The wall force can be less than 10% of the maximum.

Hiro current boundary condition model

Plasma and magnetic field flow through the wall, as if it were a highly porous dielectric. Where the plasma penetrates the wall, it turns into an ideal conductor [Zakharov *et al.* 2012].

This is supposed to model a wall which consists of tiles separated by gaps. When plasma fills the gaps between tiles, current can be conducted between them.

The plasma and magnetic field flow at the same rate, modeling the maximal force condition $\gamma\tau_{wall} = \mathcal{O}(1)$. The model does not contain τ_{wall}.

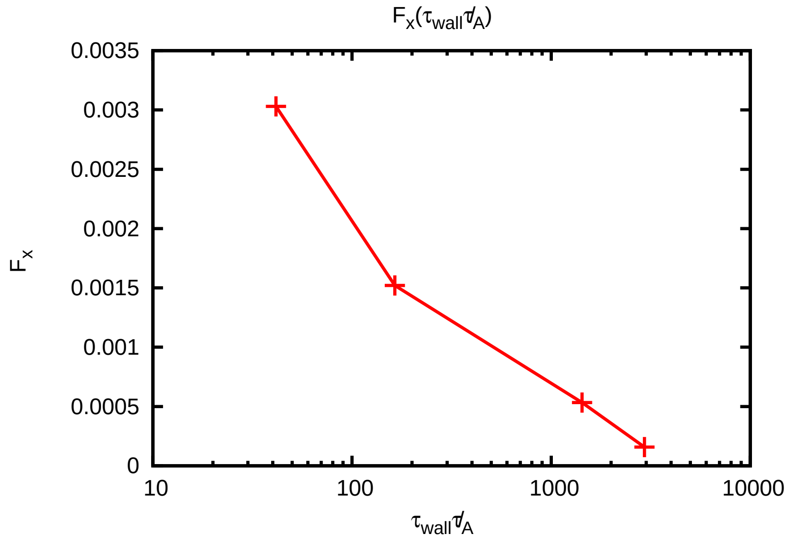
Plasma skin current is carried into the wall, which produces a wall force.

The plasma stops moving before bulk current penetrates the wall.

(Plasma opening switch - electrical conduction depends on plasma flow between electrodes [Strauss *et al.* 2007]. Simulation with conventional velocity boundary conditions.)

Effect of wall gaps on τ_w

Wall gaps were modeled with M3D as resistive strips on the wall. The averaged wall resistivity was $\eta_{wall} \approx (\Delta_{gap}/L)\eta_{gap} + \eta_{w0}$, where Δ_{gap} is gap width, L is wall length. $\tau_{wall} = r\delta/\eta_{wall}$, $\tau_{w0} = r\delta/\eta_{w0}$, $\tau_{gap} = r\delta/\eta_{gap}$, where δ is wall thickness.



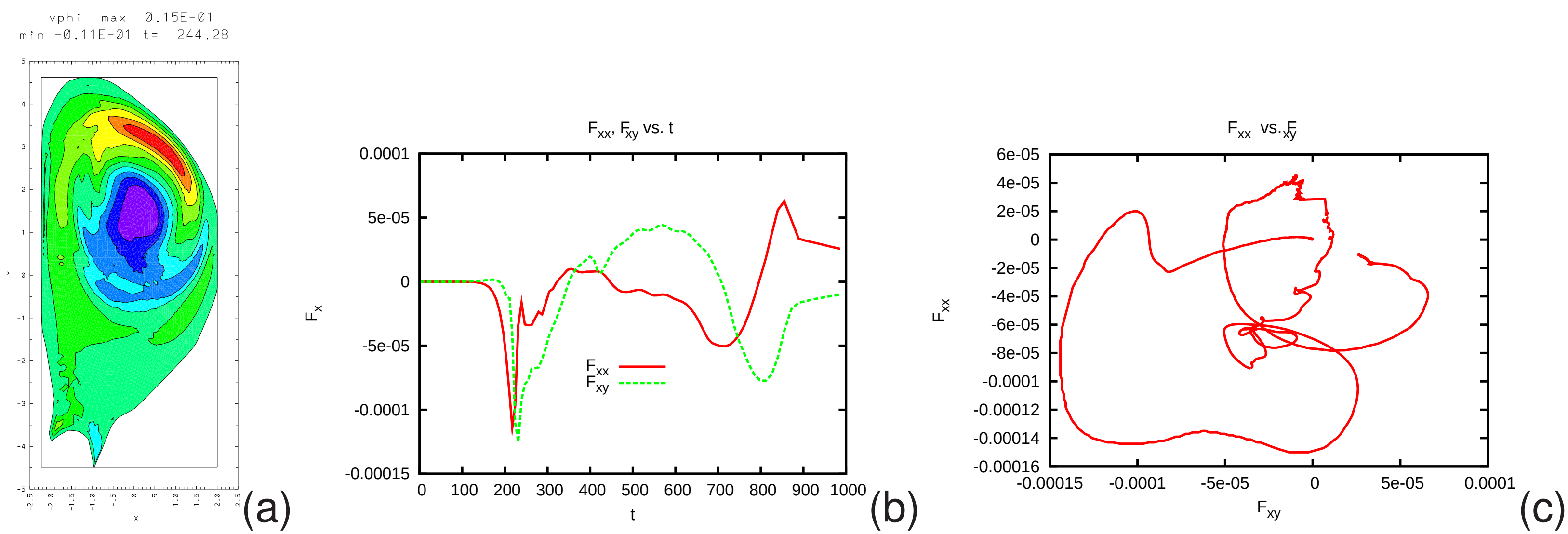
Wall force F_x on first wall with toroidal and poloidal gaps as a function of τ_{wall}/τ_A. Here $\tau_{w0} = 10^4\tau_A$, $\tau_{w0} \geq \tau_{gap} \geq \tau_A$, $\Delta_{gap}/L = 0.01$.

What is τ_{gap}? Let edge plasma fill the gap: $\eta_{gap} = r^2/(\tau_A S_{edge})$, $\tau_{gap} = (\delta/r)S_{edge}\tau_A$. $\tau_{wall} = \tau_{w0}/[1 + (r\Delta_{gap}/L\delta)\tau_{w0}/(\tau_A S_{edge})]$. Let $\tau_A = 10^{-6}s$, $S_{edge} = 10^6$. If $\tau_{w0} < 1s$, the effect of gaps is negligible.

Plasma does not make the wall a perfect condutor.

M3D results on rotation

Disruptions can generate torque. If the magnetic field penetrates the wall, and there is vertical asymmetry, then there is a net rotation. It has a zonal flow structure. [H. Strauss *et al.* 2014]



(a) Toroidally averaged toroidal velocity $\overline{v}_\phi$. Between the plasma center and the wall, the flow reverses sign. (b) Time history of sideways force F_x, and the projections of the force in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions, $F_x = \mathbf{F} \cdot \hat{\mathbf{x}}$, and $F_y = \mathbf{F} \cdot \hat{\mathbf{y}}$. It can be seen that the direction of sideways force is rotating, with period $\tau_\phi \approx 600\tau_A$. (c) different plot of same data.

Are JET disruptions predictive for ITER?

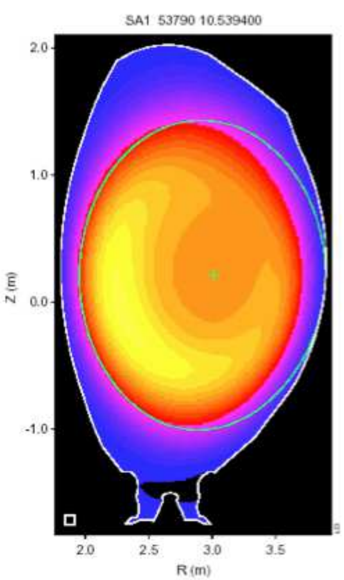
JET has short wall time: $\tau_{w-JET} = 3ms, \tau_{w-ITER} = 300ms.$

JET and ITER τ_A are comparable, $\tau_A \approx 1\mu s.$

For MHD instabilities, $\gamma\tau_w = \mathcal{O}(1)$ in JET, $\gamma\tau_w = \mathcal{O}(100)$ in ITER

Hence the scaled sideways force is much larger in JET.

JET toroidal rotation in disruptions $= 100Hz$, NSTX and C-Mod, $1kHz$.



Radiation from a JET disruption, which looks like an $m, n = 1, 1$ island, suggesting $q \approx 1$. From Plyusnin *et al.* , IAEA 2004.

JET disruptions have $q_{LCFS} \approx 1$, "most" disruptions occur when $q_{LCFS} \approx 2$.

how is a state with $q_{LCFS} \approx 1$ produced?

Summary

- Relation of toroidal current asymmetry to perturbed vertical current moment [Gerasimov *et al.* 2014] can be calculated with M3D
  - It does not require Hiro current model
  - It does not require only $(m, n) = (1, 1)$ modes.
- $v_n = 0$ boundary condition was used in the simulations
  - Magnetic resistive wall boundary condition is more important
  - Consistent with sheath
  - Wall gaps filled by plasma are negligible
- JET may overestimate the wall force
  - $\gamma\tau_w = \mathcal{O}(1)$
  - In ITER $\gamma\tau_w = \mathcal{O}(100)$
  - JET may underestimate rotation