Relativistic runaway electrons in a near-threshold electric field

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Outline

- Summary of numerical modelling
  - Non-monotonic RE distribution function
- Historical background & RE Fokker-Planck Equation
- New Kinetic Theory for RE in a near threshold electric field
  - Analytical Solution of the Fokker-Planck Equation
  - Physics details of the theory
    - Non-monotonic distribution function
    - Sustainment electric field
    - Avalanche onset electric field
    - New avalanche growth rate
    - Hysteresis
- Mitigation/current decay regime
- Summary

Numerical calculations

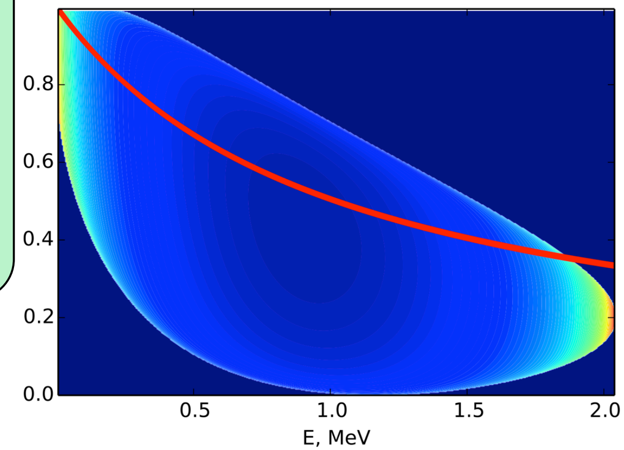
Bounce averaged Fokker-Planck + current channel modeling (0D & 1D)*

- Kinetic model:

Scattering and friction accounts for an interaction with not fully ionized impurities (significantly higher scattering on High-Z impurities) Z_{Ar}(1.6eV, 5 MeV)~12

Synchrotron and bremsstrahlung (enhanced scattering and radiation limits the RE energy)

Non-simplified (conservative) knock-on source (significantly affects avalanche rate)



- Current channel modeling:
  - 0D chain equation $2\pi RE = -L\frac{dI}{dt}$ and radiation balance $P_{rad}(T) = P_{\Omega}(T)$
  - GTS. 1D current diffusion, heat diffusion, densities diffusion equations

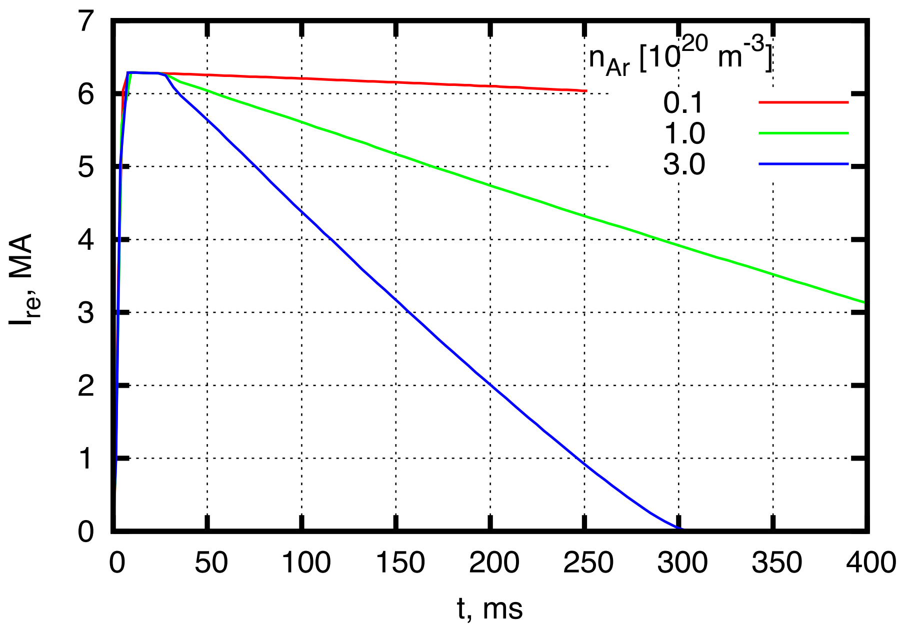
* [P. Aleynikov, K. Aleynikova, B. Breizman, G. Huijsmans, S. Konovalov, S. Putvinski, and V. Zhogolev, 25th IAEA Fusion Energy Conference, St. Petersburg, Russian Federation, 2014, pp. TH/P3–38.]

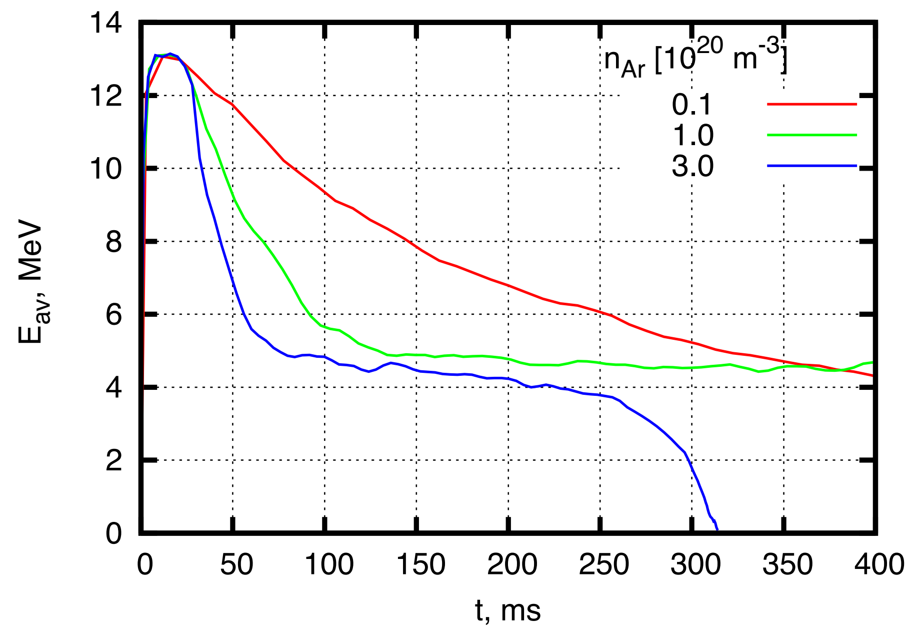
Mitigation in ITER

- Start from a given value of RE seed current (≈ 70kA) after the TQ
- Ar density required for TQ mitigation is $\approx 10^{19} m^{-3}$

- Red curves – not mitigated RE decay
- Green and Blue– Ar density is introduced at 30ms

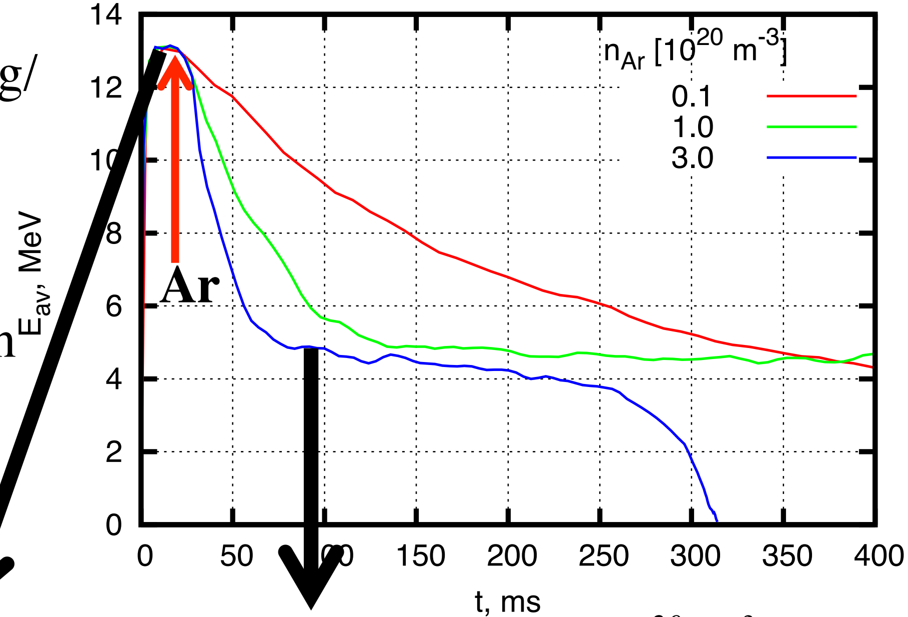
Mitigation of the RE current and energy with different Ar MGI.



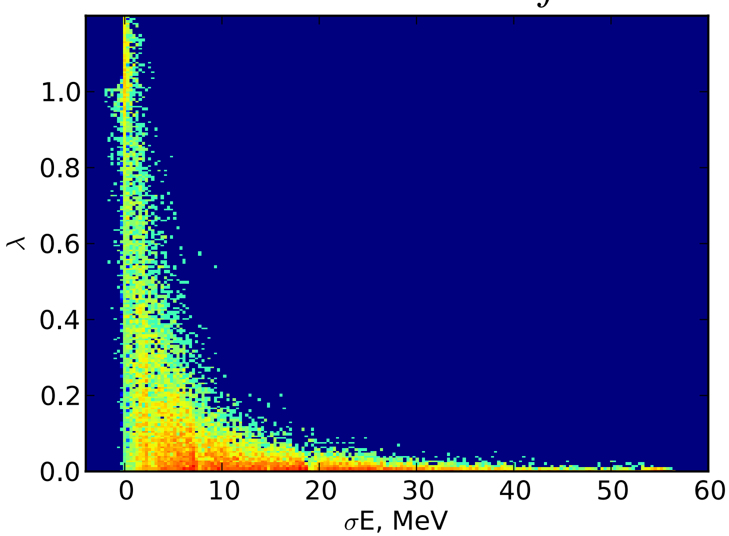


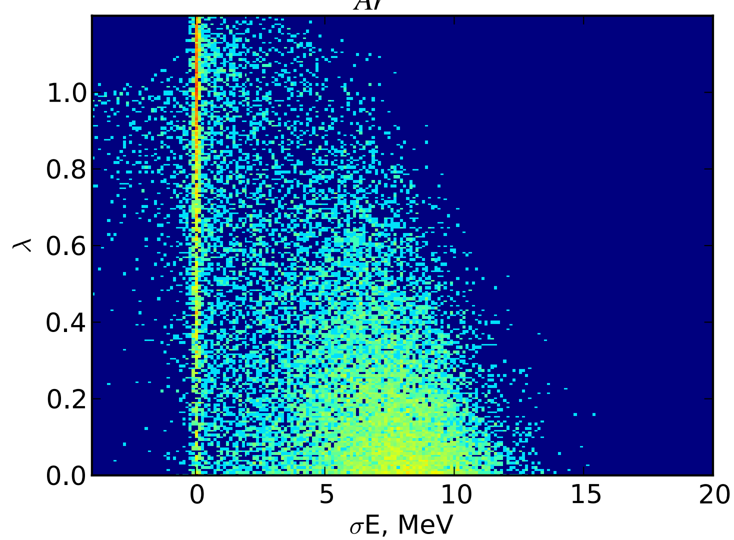
RE distribution function

- Exponential distribution function during/ after avalanche (agrees with [RP])
- Fast energy drop after MGI
- Plateau on the average energy evolution
- Peculiar distribution during the plateau
- High E ~ 2E_c during the decay



RE 2D distribution function at t=30ms and t=100ms. $n_{Ar} = 3 \cdot 10^{20} m^{-3}$





Solutions of RE kinetic equation

Genesis

- The effect was identified in 1925 by Charles Wilson (inventor of the cloud chamber)
  [C.T.R. Wilson, Proc. Cambridge Philos. Soc. **42**, (1925) 534]
- Early experimental observation in tokamaks in 50^{th} and 60^{th} and later studied in
  [Bobrovski 1970, Vlasenkov 1973, TFR group 1973, Alikaev 1975]

Diffusive collisions

- The first analysis of runaway phenomena has been carried our by Harry Dreicer
  [Proceedings of 2^{nd} Geneva conf 1958, **31**, 57; Phys Rev., 1959, **115**, 238]
- Frequently cited theory has been derived by Alexander Gurevich
  [JETF 1960, **39**, p1296]
- Relativistic case by Jack Connor and Jim Hastie
  [J. W. Connor, R. J. Hastie, Nucl. Fusion **15**, 415 (1975)]

Avalanche

- Discovery of the avalanche phenomena by Yuri Sokolov
  [Yu. A. Sokolov, JETP Lett., **29**, No. 4 (1979)]
- Marshall Rosenbluth, Sergei Putvinski "Theory for avalanche of runaway electrons in tokamaks"
  [M.N. Rosenbluth, S.V. Putvinski, Nucl. Fusion **37**, 1355, 1997]
- Studies with "Rosenbluth-Putvinski" avalanche source and synchrotron
  [Martin-Solis (1998); Andersson, Helander and Eriksson, (2001-…); Stahl *et al*., PRL (2015)]

Kinetic equation

$$\frac{\partial F}{\partial t} + eE\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2\cos\theta F - \frac{1}{p\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta F\right) = \hat{C}F + \hat{R}F + \hat{S}F$$

Small-angle collisions:

$$\hat{C}F = \frac{mc}{\tau}\left(\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 + m^2c^2\right)F + \frac{(Z+1)}{2\sin\theta}\frac{mc\sqrt{p^2+m^2c^2}}{p^3}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}F\right)$$

Synchrotron radiation reaction:

$$\hat{R}F = \frac{mc}{\tau_{rad}}\left[\frac{1}{m^2c^2p^2}\frac{\partial}{\partial p}p^3\sqrt{m^2c^2+p^2}\sin^2\theta F + \frac{1}{p\sin\theta}\frac{\partial}{\partial\theta}\frac{p\cos\theta\sin^2\theta}{\sqrt{m^2c^2+p^2}}F\right]$$

Large-angle collisions (Möller source): $\hat{S}F$

$$\hat{S}F = \int n_{cold}c\frac{\sqrt{\gamma_0^2-1}}{\gamma_0}\left\langle\delta\left[\cos\theta - \cos\theta_p\right]\right\rangle\frac{2\pi r_e^2}{\gamma_0^2-1}\left\{\frac{(\gamma_0-1)^2\gamma_0^2}{(\varepsilon-1)^2(\gamma_0-\varepsilon)^2} - \frac{2\gamma_0^2+2\gamma_0-1}{(\varepsilon-1)(\gamma_0-\varepsilon)} + 1\right\}\frac{b}{2}F\frac{p^2dpd\lambda}{\sqrt{1-b\lambda}}$$

$$\left\langle\delta\left[\cos\theta - \cos\theta_p\right]\right\rangle = \frac{1}{\pi}\frac{1}{\sqrt{b^2\lambda\lambda_0 - \left(\sqrt{1-b\lambda}\sqrt{1-b\lambda_0} - \sqrt{\frac{\varepsilon-1}{\varepsilon+1}}\sqrt{\frac{\gamma_0+1}{\gamma_0-1}}\right)^2}}$$

Time scales:

$$\tau \equiv \frac{m^2c^3}{4\pi n_e e^4\ln\Lambda}$$

$$\tau_{rad} \equiv \frac{3m^3c^5}{2e^4B^2}$$

Kinetic equation in Rosenbluth-Putvinski

$$\frac{\partial F}{\partial t}+eE\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2\cos\theta F-\frac{1}{p\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta F\right)=\hat{C}F+\hat{R}F+\hat{S}F$$

Small-angle collisions:

$$\hat{C}F=\frac{mc}{\tau}\left(\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2+m^2c^2\right)F+\frac{(Z+1)}{2\sin\theta}\frac{mc\sqrt{p^2+m^2c^2}}{p^3}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}F\right)$$

Synchrotron radiation reaction:

$$\hat{R}F=0$$

Large-angle collisions (Möller source): $\hat{S}F$

$$S=\frac{n_r\delta(\lambda-\lambda_2)\sqrt{1-\lambda b}}{\tau\ln\Lambda}\frac{1}{p^2}\frac{\partial}{\partial p}\left(\frac{1}{1-\sqrt{1+p^2}}\right). \quad (4)$$

Time scales:

$$\tau\equiv\frac{m^2c^3}{4\pi n_e e^4\ln\Lambda}$$

$$\tau_{rad}\equiv\frac{3m^3c^5}{2e^4B^2}$$

The equation in this form is solved analytically in [M. N. Rosenbluth, S. V. Putvinski, Nucl. Fusion **37**, 1355 (1997)]

However this source appropriate well above the avalanche threshold, but needs to be generalized in the near-threshold regime.

Kinetic equation

$$\frac{\partial F}{\partial t}+eE\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2\cos\theta F-\frac{1}{p\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta F\right)=\hat{C}F+\hat{R}F+\hat{S}F$$

Small-angle collisions:

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Large-angle collisions (Möller source): $\hat{S}F$

$$\hat{S}F=\int n_{cold}c\frac{\sqrt{\gamma_0^2-1}}{\gamma_0}\left\langle\delta\left[\cos\theta-\cos\theta_p\right]\right\rangle\frac{2\pi r_e^2}{\gamma_0^2-1}\left\{\frac{(\gamma_0-1)^2\gamma_0^2}{(\varepsilon-1)^2(\gamma_0-\varepsilon)^2}-\frac{2\gamma_0^2+2\gamma_0-1}{(\varepsilon-1)(\gamma_0-\varepsilon)}+1\right\}\frac{b}{2}F\frac{p^2dpd\lambda}{\sqrt{1-b\lambda}}$$

$$\left\langle\delta\left[\cos\theta-\cos\theta_p\right]\right\rangle=\frac{1}{\pi}\frac{1}{\sqrt{b^2\lambda\lambda_0-\left(\sqrt{1-b\lambda}\sqrt{1-b\lambda_0}-\sqrt{\frac{\varepsilon-1}{\varepsilon+1}}\sqrt{\frac{\gamma_0+1}{\gamma_0-1}}\right)^2}}$$

Time scales:

$$\tau\equiv\frac{m^2c^3}{4\pi n_e e^4\ln\Lambda}$$

$$\tau_{rad}\equiv\frac{3m^3c^5}{2e^4B^2}$$

1) Möller (avalanche) source ($\hat{S}F$) is weaker than electron drag by Coulomb logarithm.

2) The small parameter is $\varepsilon=\dfrac{E-E_a}{E_a}$, i.e. electric file is close to the threshold.

Solution of the Fokker-Planck equation

The separation of timescales between small-angle collisions and knock-on collisions suggests a two-step approach to the problems of runaway production:

1) Ignore the large-angle collisions and study the behavior of pre-existing runaways
2) Use the distribution function of the accumulated runaways to predict their production and loss

Dimensionless kinetic equation:

$$\frac{\partial F}{\partial s}+\frac{\partial}{\partial p}\left[\mathrm{E}\cos\theta-1-\frac{1}{p^2}-\frac{1}{\bar{\tau}_{rad}}p\sqrt{1+p^2}\sin^2\theta\right]F=$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\left[\mathrm{E}\frac{\sin\theta}{p}F+\frac{(Z+1)}{2}\frac{\sqrt{p^2+1}}{p^3}\frac{\partial F}{\partial\theta}+\frac{1}{\bar{\tau}_{rad}}\frac{\cos\theta\sin\theta}{\sqrt{1+p^2}}F\right]$$

[P. Aleynikov, B. Breizman, *Theory of Two Threshold Fields for Relativistic Runaway Electrons*, Phys. Rev. Lett., **114**, 155001 (2015)]

Fast pitch-angle equilibration

In the near-threshold case the time-scale for pitch-angle equilibration is much shorter than the momentum evolution time-scale.

Thus, the lowest order kinetic equation is:

$$\frac{\mathrm{E}}{p}F + \frac{(Z+1)}{2}\frac{\sqrt{p^2+1}}{p^3}\frac{1}{\sin\theta}\frac{\partial F}{\partial\theta} = 0 \quad \text{solution: } F = G(t;p)\frac{A}{2\sinh A}\exp[A\cos\theta]$$

$$A(p) \equiv \frac{2\mathrm{E}}{(Z+1)}\frac{p^2}{\sqrt{p^2+1}}$$

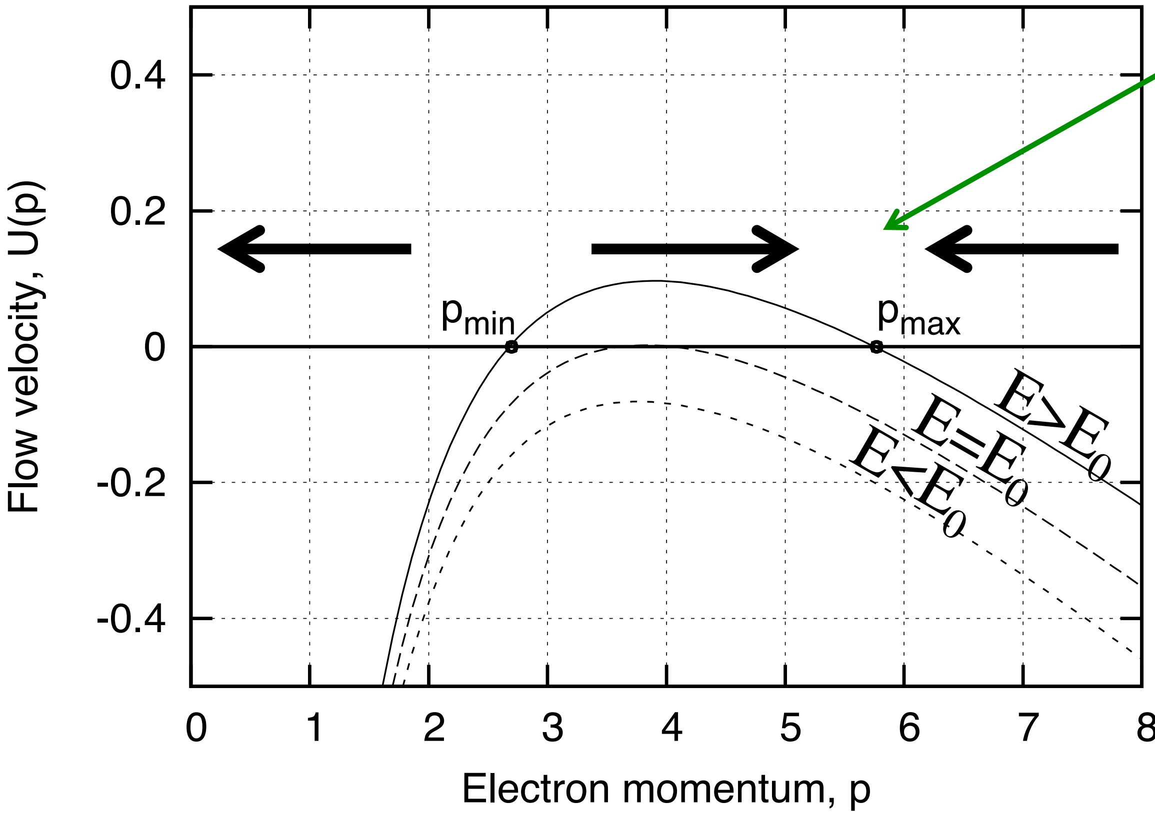
Integration of the exact kinetic equation over all pitch-angles eliminates the lowest order terms and gives a one-dimensional continuity equation for the momentum flow:

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial p}U(p)G = 0$$

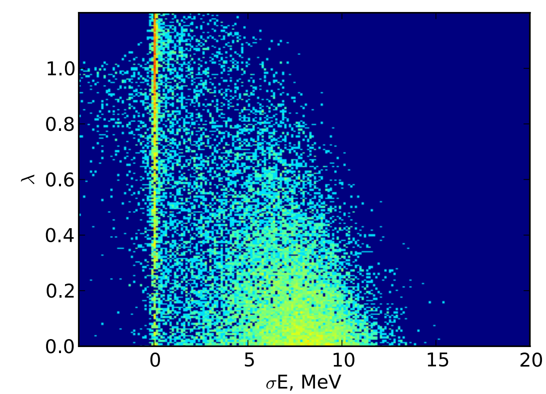
$$U(p) \equiv -\left[\frac{1}{A(p)} - \frac{1}{\tanh(A(p))}\right]\mathrm{E} - 1 - \frac{1}{p^2} + \frac{Z+1}{E\overline{\tau}_{rad}}\frac{p^2+1}{p}\left[\frac{1}{A(p)} - \frac{1}{\tanh(A(p))}\right]$$

The flow velocity

$$U(p) \equiv E\cos\theta_{av} - 1 - \frac{1}{p^2} - \frac{Z+1}{E\bar{\tau}_{rad}}\frac{p^2+1}{p}\cos\theta_{av};\ \cos\theta_{av} = \frac{1}{\tanh\left(A(p)\right)} - \frac{1}{A(p)}$$



Peaking of the distribution function around p_{max}



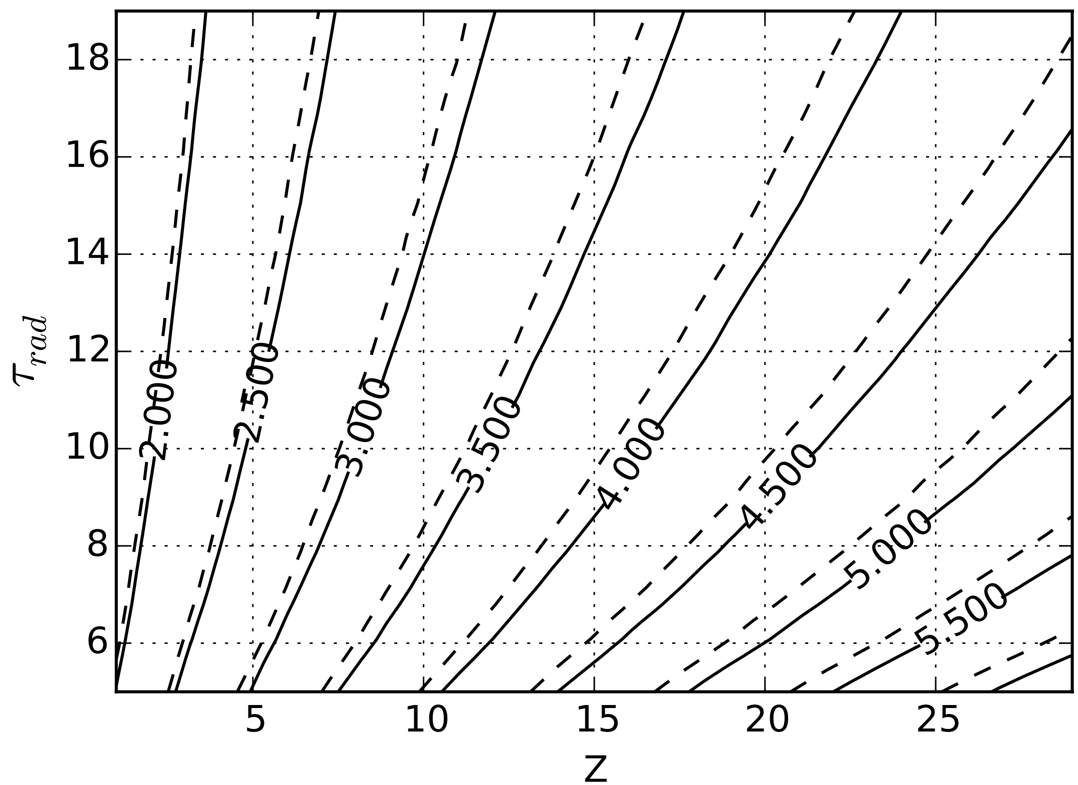
The roots p_{min} and p_{max} merge at a certain electric field $E=E_0$.
This is the minimal electric field required for RE sustainment.

The sustainment threshold field

The sustainment threshold field is always higher than Connor's E_c field.

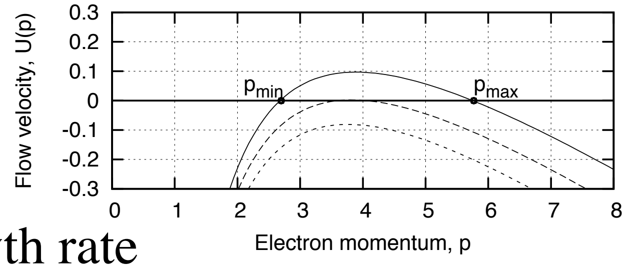
$$E_0 = E_c\left(1+\frac{(Z+1)}{\sqrt{\tau_{rad}/\tau}}\Big/\sqrt[6]{\frac{1}{8}+\frac{(Z+1)^2}{\tau_{rad}/\tau}}\right)$$

Contours of the sustainment threshold field (solid)



Avalanche growth rate

1) All primaries are at p_{max} with $\gamma_0 = \sqrt{p_{\max}^2 + 1}$
2) Both electrons have $p > p_{min}$ after the collision
3) Energy conservation requires $\gamma_{\min} < \gamma < \gamma_0 + 1 - \gamma_{\min}$
4) The cross-section for such collisions gives the growth rate



$$\Gamma \equiv \frac{1}{n_{re}} \frac{\partial n_{re}}{\partial t} = \frac{n_e c \tau}{2} \frac{\sqrt{\gamma_0^2 - 1}}{\gamma_0} \int_{\gamma_{\min}}^{\gamma_0 + 1 - \gamma_{\min}} \frac{d\sigma}{d\gamma} d\gamma$$

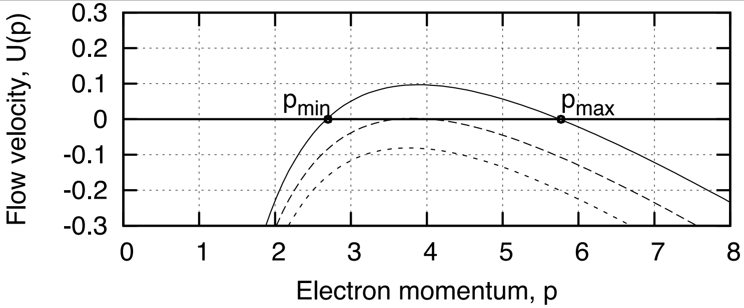
5) The integral is evaluated analytically for the Möller cross-section:

$$\Gamma = \frac{1}{4\Lambda \gamma_0 \sqrt{\gamma_0^2 - 1}} \left\{ \left(\gamma_0 + 1 - 2\gamma_{\min}\right)\left(1 + \frac{2\gamma_0^2}{(\gamma_{\min} - 1)(\gamma_0 - \gamma_{\min})}\right) - \frac{2\gamma_0 - 1}{\gamma_0 - 1} 2 \ln\left(1 + \frac{\gamma_0 + 1 - 2\gamma_{\min}}{\gamma_{\min} - 1}\right) \right\}$$

Avalanche onset field

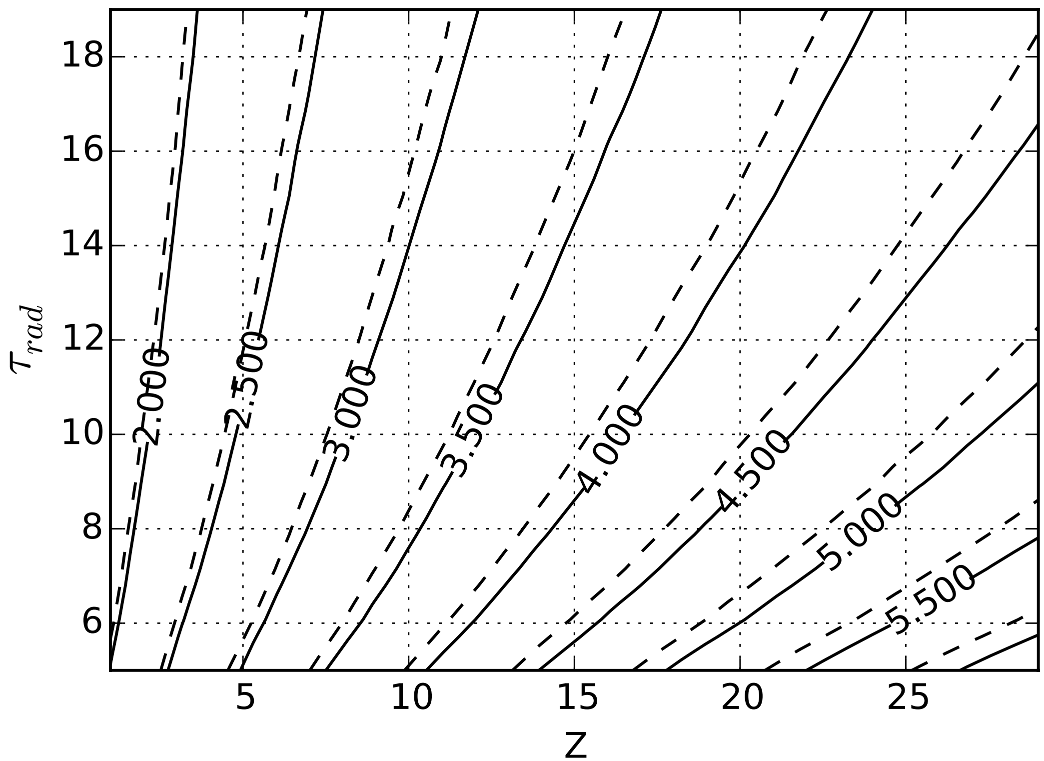
The avalanche onset condition:

$$\gamma_0 + 1 - 2\gamma_{min} = 0$$



The avalanche onset field E_a is greater than the runaways sustainment field E_0

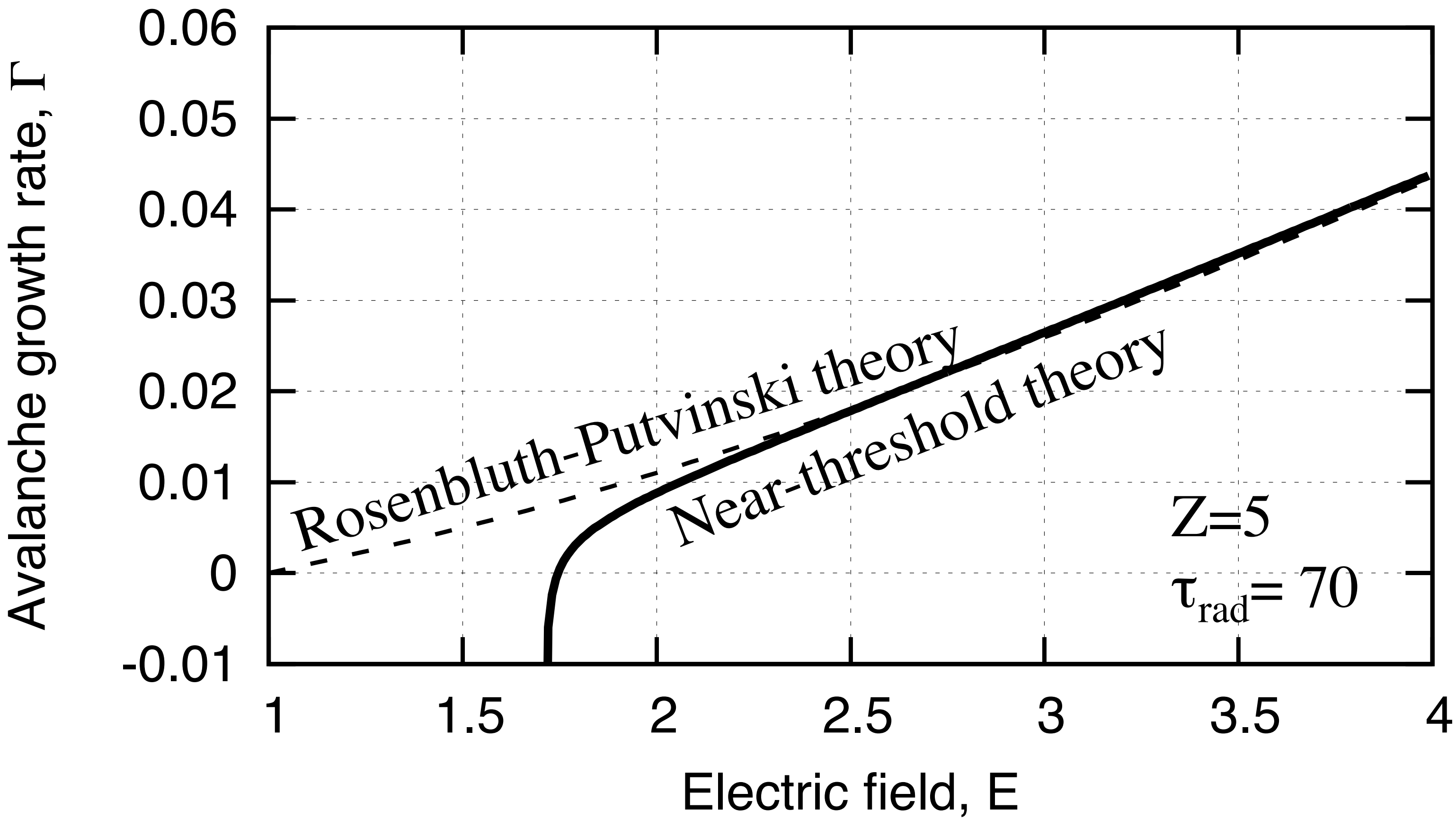
Contours of the avalanche onset field (dashed)



Near-threshold avalanche growth rate

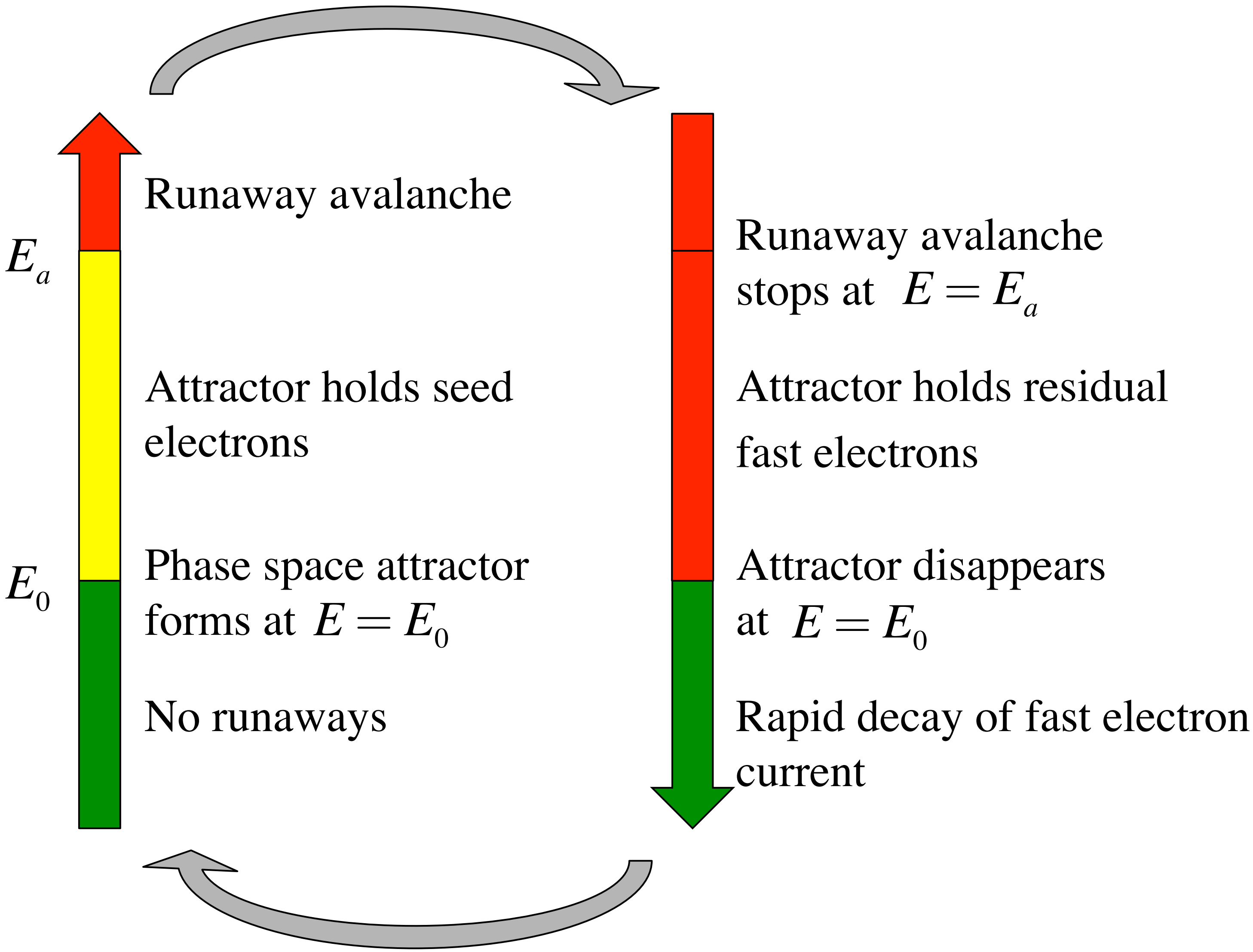
Rosenbluth-Putvinski

$$\frac{1}{j_{re}}\frac{\partial j_{re}}{\partial t} \approx \frac{1}{\tau \ln\Lambda}\sqrt{\frac{\pi}{3(Z+5)}}(E-1)$$



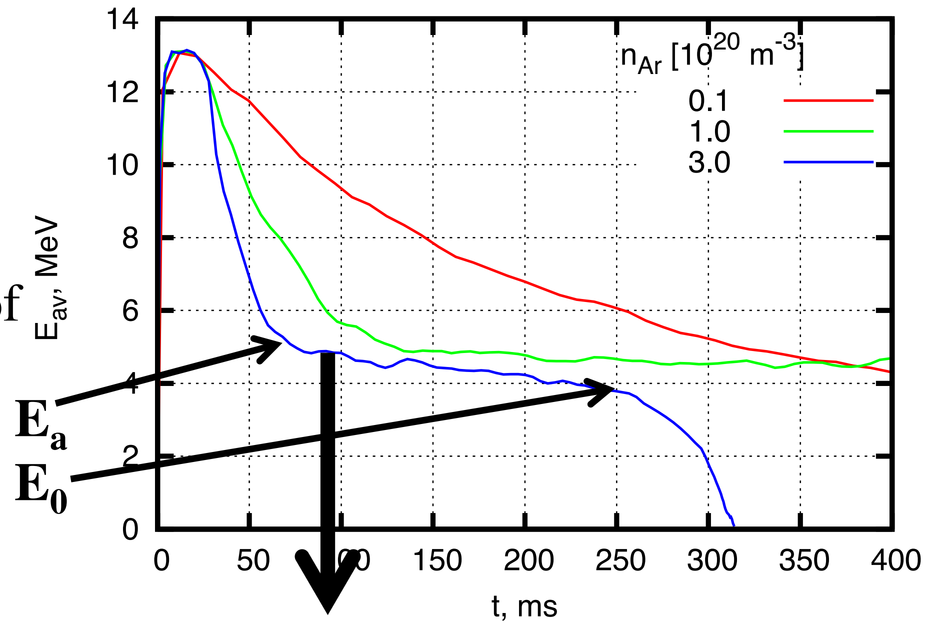
Hysteresis

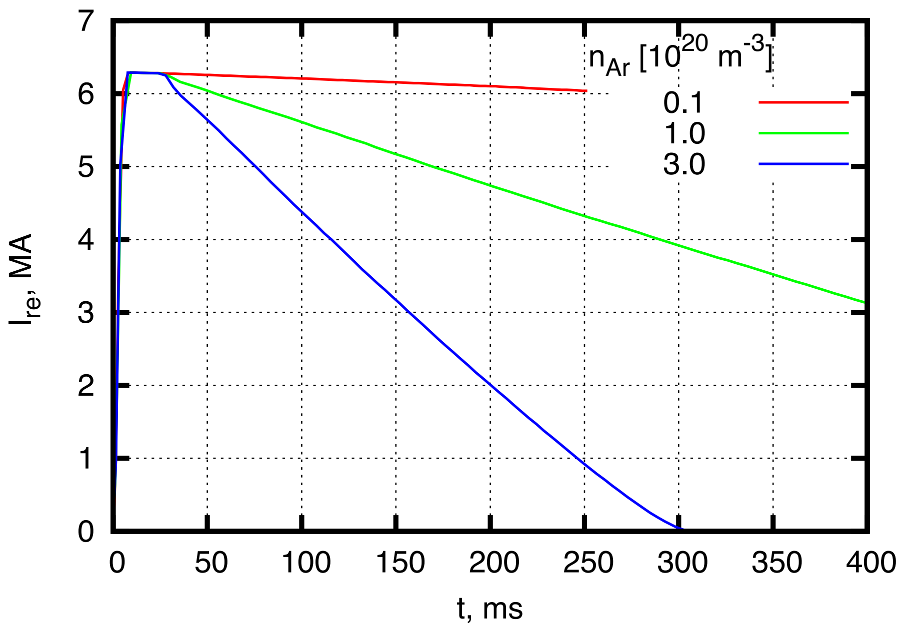
Growing inductive field | **Decaying inductive field**

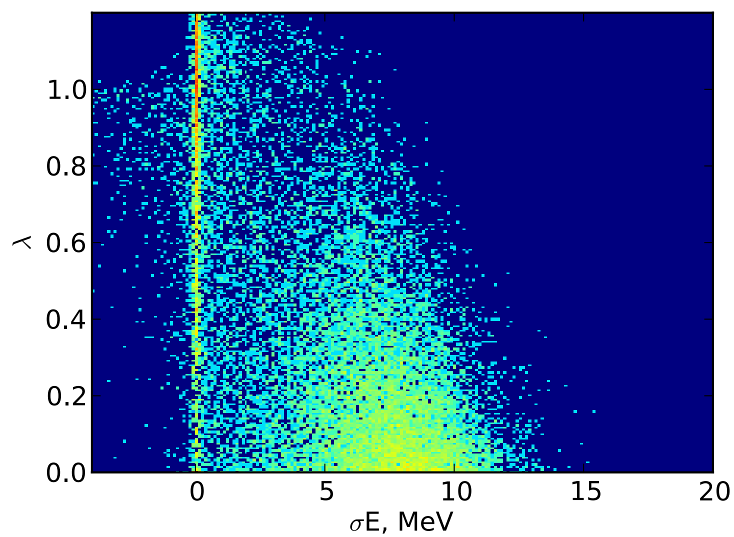


Mitigation regime

- The avalanche is switched-off
- The field stays between E_a and E_0
- The runaways remain peaked at p_{max}
- The runaway density and current decrease in step with the dissipation of the magnetic energy







Current decay time-scale

1) $E_0 \approx E_a$ - function only of plasma parameters

2) The total energy loss is $\sim (j_\Omega + j_{re})E_0 \approx (j_\Omega + j_{re})E_a$

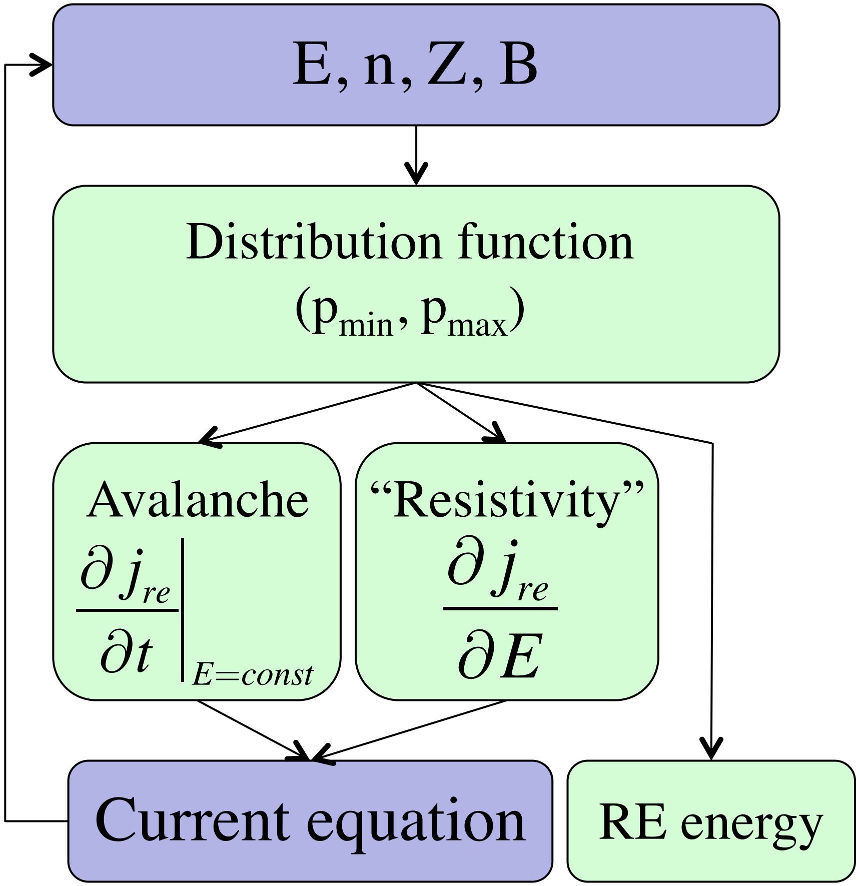
$$\frac{\partial j}{\partial t} = \frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial E_0}{\partial r}$$

3) The current decay is linear and the decay rate is given by

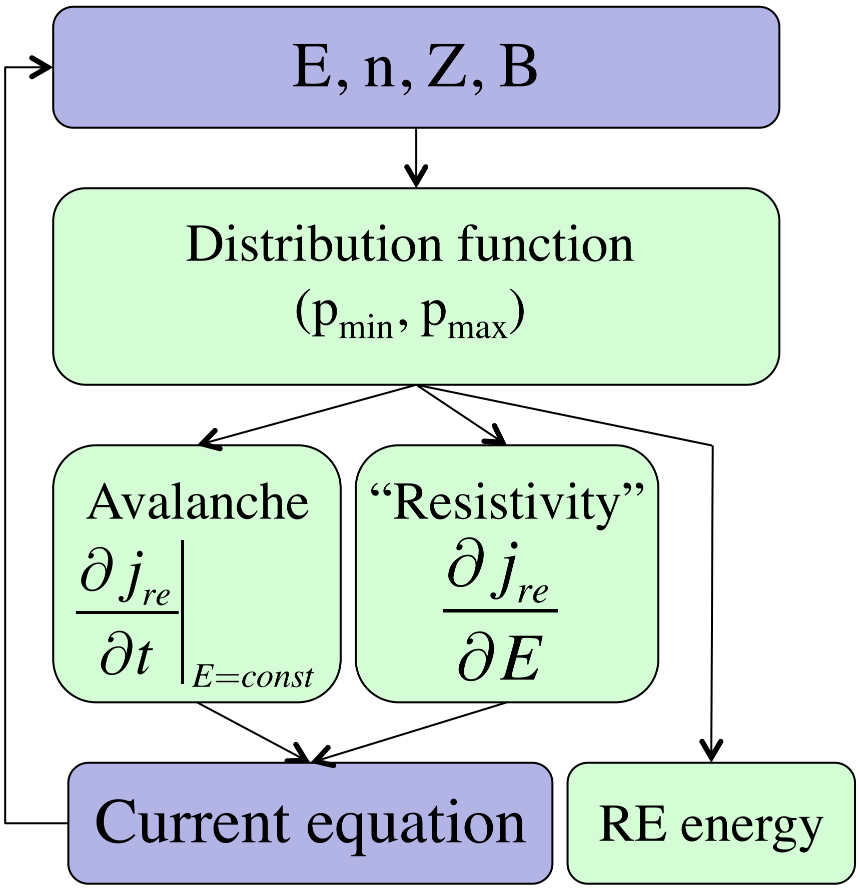
$$\frac{d(I_{re} + I_\Omega)}{dt} \approx -\frac{2\pi R}{(L - L_{wall})} E_0$$

4) This estimate is insensitive to the distribution function

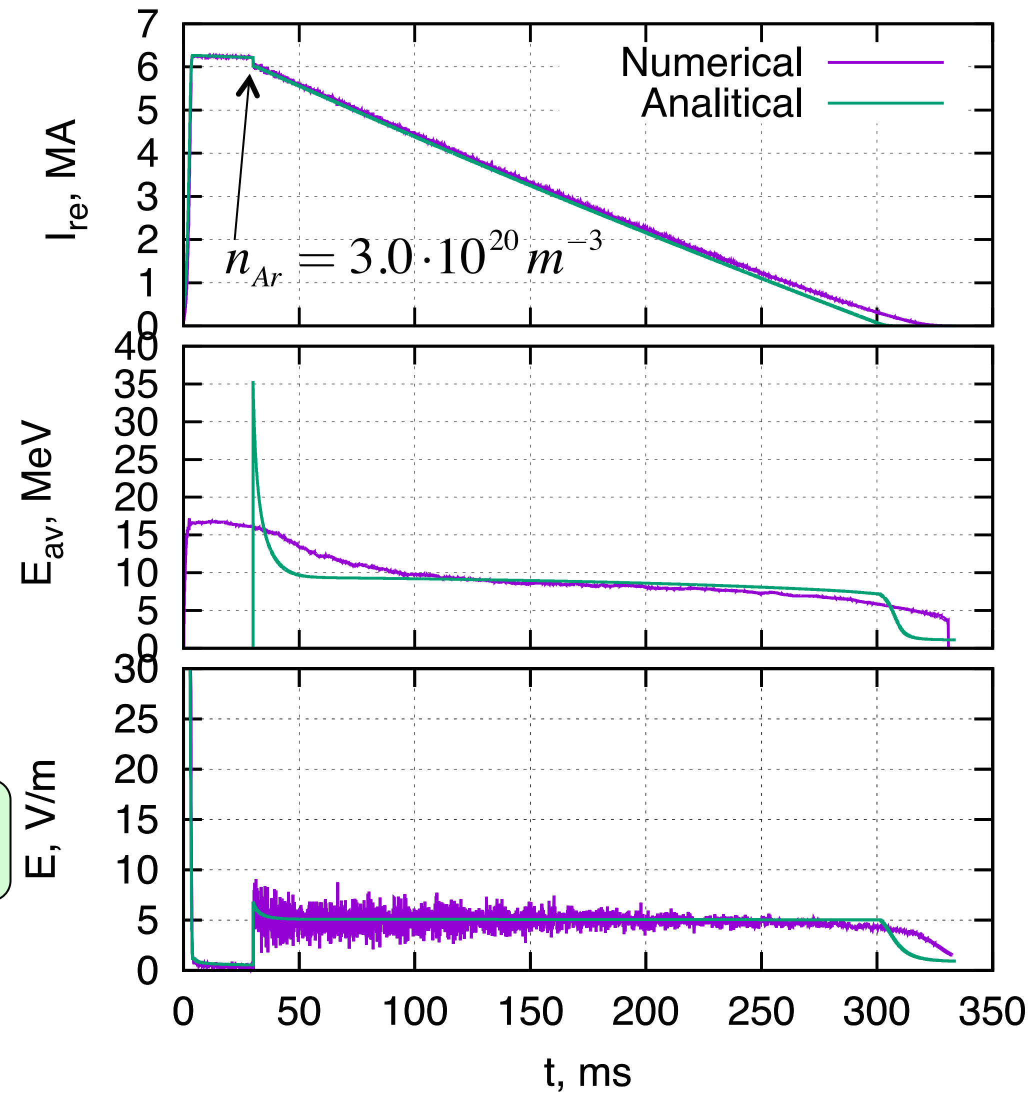
Reduced RE kinetic model for fluid-like codes



Reduced RE kinetic model for fluid-like codes



[IAEA_FEC_2014] results
with Reduced RE kinetic module.

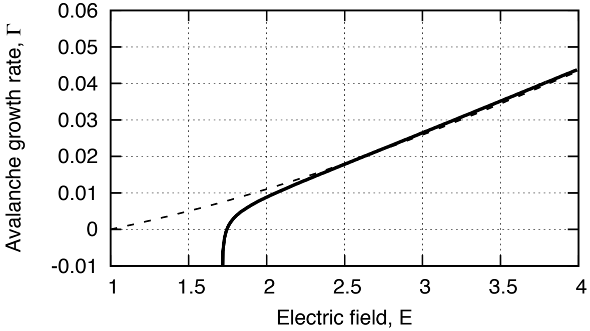


Summary

$$E_0 \approx E_c \left(1 + \sqrt[3]{\frac{(Z+1)^2}{\tau_{rad}/\tau}}\right)$$

1) Rigorous kinetic theory for relativistic RE in the near-threshold regime

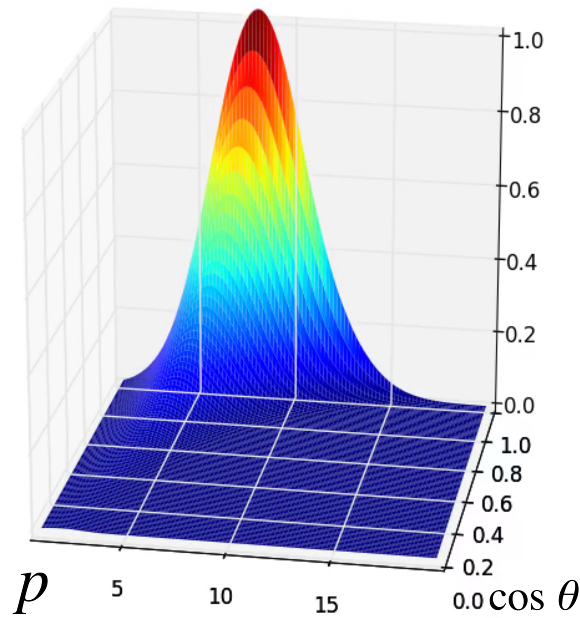
2) The electric field for runaway avalanche onset is higher and the avalanche growth rate is lower than previous predictions



3) The new theory predicts peaking of the runaway distribution function near p_{max}

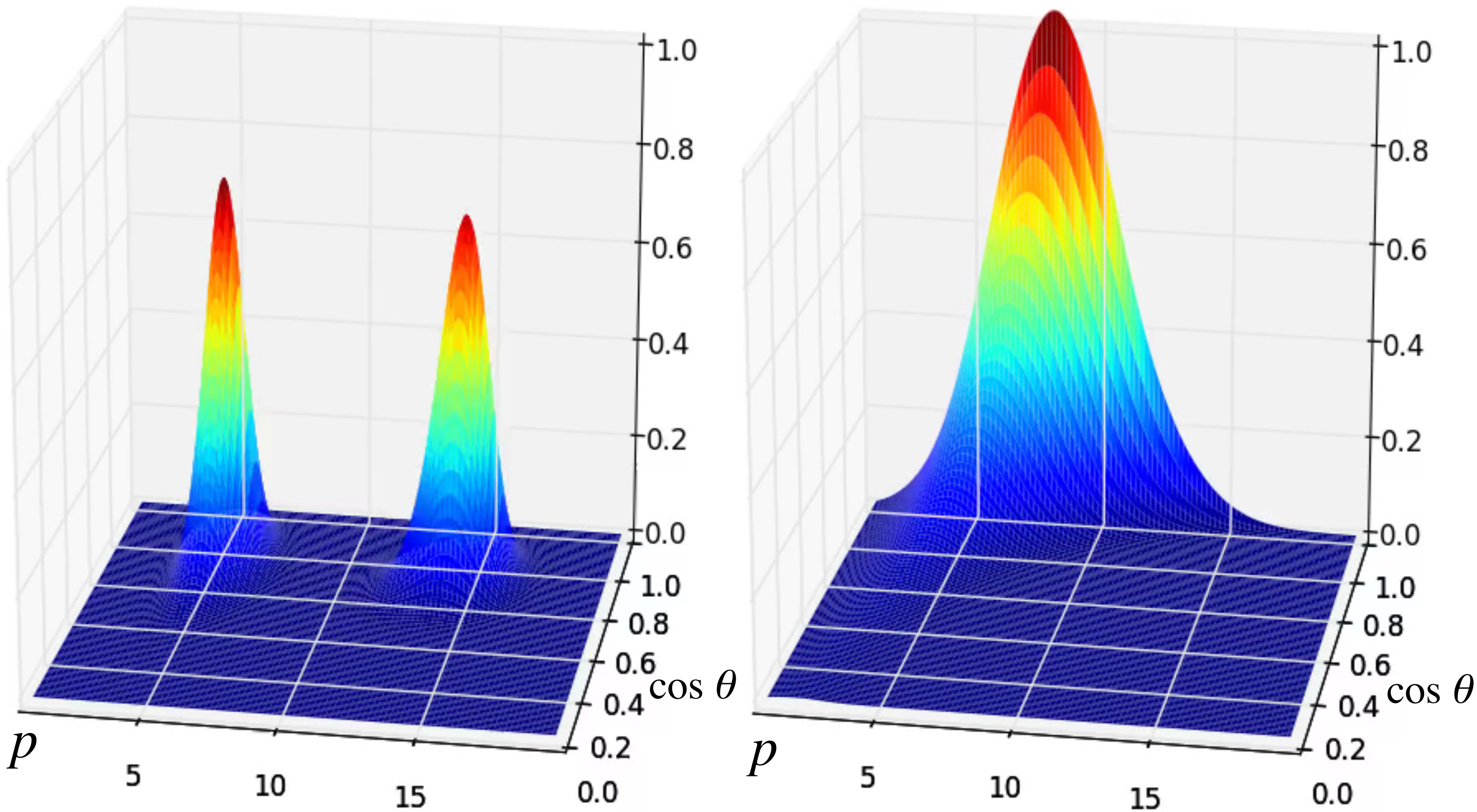
4) Existence of two different threshold fields produces a hysteresis in the runaway evolution

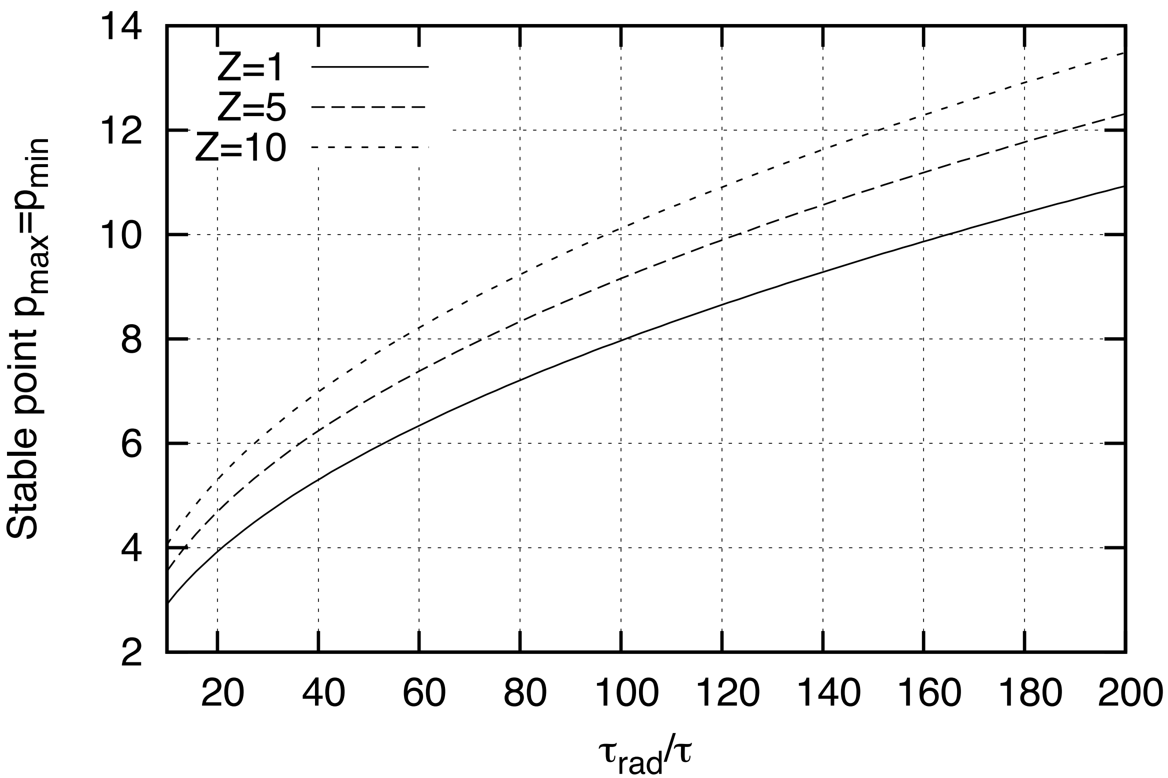
5) Particular features of this regime allows for evaluation of the current decay rate



Benchmark against numerical solution

- Numerical solution exhibits peaked distribution function
- Launching two groups of electrons from $p>p_{max}$ and $p_{min}<p<p_{max}$





14
12
10
8
6
4
2
20
40
60
80
100
120
140
160
180
200
Z=1
Z=5
Z=10
Stable point $p_{max}=p_{min}$
τ_{rad}/τ

Questions

Calculations	Rosenbluth & Putvinski
• Not monotonic distribution	The following formula approximates the energy spectrum of runaway electrons produced by avalanche: $$\frac{1}{n_{\mathrm{RA}}}\frac{\mathrm{d}n_{\mathrm{RA}}}{\mathrm{d}E} \cong \exp(-E/T) \quad (26)$$
• Linear RE current decay	$$\frac{1}{j_{re}}\frac{\partial j_{re}}{\partial t} = \frac{1}{\tau \ln\Lambda}\sqrt{\frac{\pi}{3(Z+5)}}(E-1)$$

- Plateau on the average energy evolution