AVALANCHE GROWTH OF THE SECONDARY RUNAWAY ELECTRON GENERATION

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Motivation

- In recent dedicated runaway electron experiments on DIII-D with gas puffing during flat-top, a turning point of the runaway electron HXR signal was observed.
- Critical electric field found to be several times larger than Connor-Hastie $E_c$.
- Mysterious energy loss mechanisms?

Caveats of Rosenbluth-Putvinski

• Rosenbluth-Putvinski’s theory predicts $E_c$ (Connor-Hastie critical field) is threshold of secondary generation, and avalanche growth rate (almost) proportional to $E/E_c - 1$.

• Issues with the theory
  • Calculation of secondary generation is based on simplified source term that ignores energy and pitch angle distribution of seed electrons.
  
  \[ S = \frac{n_r}{4\pi \ln \Lambda} \delta(\xi - \xi_2) \frac{1}{p^2} \frac{\partial}{\partial p} \left( \frac{1}{1 - \sqrt{1 + p^2}} \right) \]

  \[ \xi_2 = \frac{\sqrt{1 + p^2} - 1}{p} \]

  • Radiation effects (synchrotron, bremsstrahlung) ignored in kinetic model.
  • Other kinetic effects (whistler wave, magnetic fluctuation) are also missing.
Outline

• Kinetic model of runaway electrons
  • Synchrotron radiation reaction force
  • Deriving source term for secondary RE generation
• Calculate runaway probability function
  • PDE solving method
  • Critical electric field for growth
• Avalanche growth simulation
  • Growth rate calculation
  • Simulation of gas-puffing case
• Conclusions
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Kinetic model of runaway electrons

• Collisions, radiation effects, and secondary RE generation included in the kinetic equation.

\[ \frac{\partial f}{\partial t} + E\{f\} + C\{f\} + R\{f\} = S \]

- \(E\): Parallel electric field drive
- \(C\): Collision operator
- \(R\): Synchrotron radiation reaction force
- \(S\): Source term for secondary RE generation

• Collision operator gives correct limits for thermal electrons and relativistic electrons.

• Numerical scheme similar to code CODE.

Synchrotron radiation reaction force

- Synchrotron radiation force is important for high energy electrons (comparable to $E$ field and collisional drag)

\[
F_s = \frac{2}{3}r_e m_e c^2 \beta^2 \gamma \left\{ \frac{\sin^2 \theta}{r_g^2} \left[ (1 + p_\perp^2) p_\perp + p_\perp^2 p_\parallel \hat{b} \right] + \frac{\beta \gamma^3 R_0^2}{R_0^2} \hat{b} \right\}
\]

\[
R\{f\} = \nabla \cdot (F_s f)
\]

- For electrons with $\gamma < 100$ (most), contribution from the magnetic field curvature is negligible compared to Larmor motion ($r_g \ll R_0$).

Deriving source term for secondary generation

- We use Møller scattering cross section to get large angle collision scattering probability for relativistic electrons.
- Scattering angle derived from energy and momentum conservation.

$$\cos \theta = \frac{\gamma_e + 1}{\sqrt{\gamma_e - 1}} \frac{\gamma - 1}{\sqrt{\gamma_e + 1}}.$$ 

- Source term is integrated from scattering probability and electron distribution function

$$S[f] = \frac{1}{2\pi p^2} \int 2\pi p_e^2 dp_e d\xi_e \hat{S}(p, \xi; p_e, \xi_e) f(p_e, \xi_e).$$

C. Møller, Ann. Phys. (Berlin), 406, 531 (1932)
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Calculate runaway probability function

- When $E/E_c > 1$, the electron phase space is separated into the runaway region (electron will run away) and lost region (electron will fall back to the thermal population).

- Two methods to study this phase space structure
  - Test particle method - truncate the kinetic equation to make it deterministic, and locate the singular point in phase space.
  
  \[
  \frac{1}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial f}{\partial \xi} = \frac{\partial}{\partial \xi} (\xi f) + \frac{\partial^2}{(1 - \xi^2) \partial \xi^2} \left( \frac{1 - \xi^2}{2} f \right)
  \]
  
  - Monte-Carlo simulation – Random sampling to obtain runaway probability

- We develop a new method to get runaway probability by solving PDE.


PDE solving method

• Introduce function $P$ representing the runaway probability

\[
P = 1 \quad \text{at high energy boundary}
\]
\[
P = 0 \quad \text{at low energy boundary}
\]

• $P$ is found as a solution to a PDE derived from the kinetic equation. Derivation is similar to first passage problem.

\[
\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}[v(x)f] + \frac{\partial^2}{\partial x^2}[D(x)f]
\]

\[
v(x)\frac{dP(x)}{dx} + D(x)\frac{d^2P(x)}{dx^2} = 0
\]

Adjoint equation of Fokker-Planck equation
Results of runaway probability function

- New method gives smooth probability function rather than separatrix.
- Overcomes caveats of test particle method (truncation & coordinates dependence).
- Agrees well with Monte-Carlo simulation. (Efficiency is better.)
Critical electric field for growth

- In presence of synchrotron radiation force, if $E$ is below a threshold $E_r$, transition solution is missing, with only a (almost) uniform solution left.
- $E_r$ is the critical electric field for runaway electron growth.

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Simulation Result – Avalanche growth

- Time-dependent kinetic equation solved using backward Euler.

\[
\frac{df}{dt} + E\{f\} + C\{f\} + R\{f\} = S\{f\}
\]

- With strong radiation, distribution function is non-monotonic.

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\[Z=1, \quad \tau/\tau_r=0.76 \quad (B=3T, \quad n_e=10^{19}m^3)\]
Avalanche growth rate

• With synchrotron radiation force added, a new threshold $E_r > E_c$ is observed, below which there is no avalanche growth.
Simulation of gas-puffing case

- Three effects after gas puffing: Dreicer loss (loss of low energy electrons), Radiation loss (loss of high energy electrons) and secondary generation.
- HXR signal turning point reflects redistribution of RE energy
- Qualitative agreement with experiment observed. Other loss mechanism not necessary

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- A PDE solving method is developed to calculate the runaway probability function.
- The method can also identify the critical electric field for runaway electron growth.
- In presence of synchrotron radiation reaction and the pitch angle scattering, the threshold electric field for avalanche growth increases from $E_c$ to $E_r$, which depends on $B$ and $Z$.
- Simulation of gas-puffing experiment shows qualitatively agreement with the experimental result.
  - Synchrotron radiation
  - Pitch angle scattering $Z_{\text{eff}}$
  - Redistribution of the runaway electron energy
Thank you!
\[ \tau = \frac{4\pi \epsilon_0^2 m_e^2 c^3}{e^4 n_e \ln \Lambda} \]

\[ \tau_r = \frac{6\pi \epsilon_0 m_e^3 c^3}{e^4 B^2} \]
\[ \int f(x,t)P(x)\,dx = \text{const} \]

\[ 0 = \int \frac{\partial f}{\partial t} P(x)\,dx \]

\[ = \int \left\{ -\frac{\partial}{\partial x} [v(x)f] + \frac{\partial^2}{\partial x^2} [D(x)f] \right\} P(x)\,dx \]

\[ = \int \left\{ v(x) \frac{dP(x)}{dx} + D(x) \frac{d^2 P(x)}{dx^2} \right\} f(x,t)\,dx + \text{Surface term} \]

\[ v(x) \frac{dP(x)}{dx} + D(x) \frac{d^2 P(x)}{dx^2} = 0 \]
Theoretical estimation of the growth rate

- If a distribution function is given, the growth rate can be calculated using the runaway probability function.

\[ \gamma = \frac{1}{n_{re}} \int S(p, \xi)Q(p, \xi)2\pi p^2 d\xi dp \]

- If a growth rate is given, an approximate distribution can be obtained from the kinetic equation.

\[ \Gamma (p) \frac{df}{dt} + E\{ f \} + C\{ f \} + R\{ f \} = S\{ f \} = 0 \]

- The solution is thus the stationary point.
Next steps

• Study other loss mechanisms, including the bremsstrahlung radiation loss, the magnetic field fluctuation and the whistler wave scattering.
• Study the RE generation and decay for sudden cooling of plasma.

Future work

• Couple the kinetic simulation to MHD code.
• Collaborate on the future DIII-D experiments to study the critical electric field for runaway electron growth and the runaway electron energy distribution.
• Develop more complicated synthetic diagnostics simulations and compare the results with the experiments.