Theory and Modeling Needs For Improved Kinetic Treatments of High-Beta Pressure Limits

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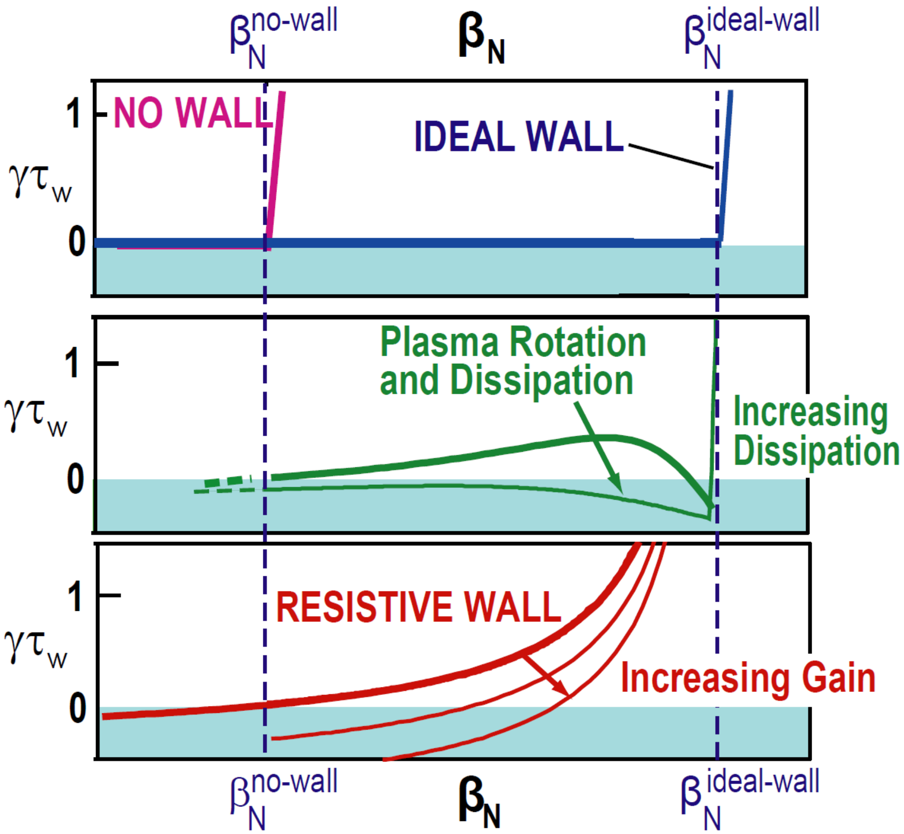
Pressure-driven kink / tearing are strong physics constraints on maximum fusion performance

$$P_{fusion} \propto n^2\langle\sigma v\rangle \propto p^2 \propto \beta_T{}^2 B_T{}^4 \propto \beta_N{}^4 B_T{}^4 (1+\kappa^2)^2 / A\ f_{BS}{}^2$$



M. Chu, et al., Plasma Phys. Control. Fusion 52 (2010) 123001

- Modes grow rapidly above kink limit:
  - $\gamma \sim$ 1-10% of τ_A^{-1} where $\tau_A \sim 1\mu$s
- Superconducting "ideal wall" can increase stable β_N up to factor of 2
- Real wall resistive → slow-growing "resistive wall mode" (RWM)
  - $\gamma\ \tau_{wall} \sim 1$ →
  - ms instead of μs time-scales
- RWM can be stabilized with:
  - kinetic effects (rotation, dissipation)
  - active feedback control

Here we focus on ideal-wall mode (IWM) and TM triggering

NSTX-U

Background

- Characteristic growth rates, frequencies of RWM and IWM
  - RWM: $\gamma\tau_{wall}$ ~ 1 and $\omega\tau_{wall} < 1$
  - IWM: $\gamma\tau_A$ ~ 1-10% ($\gamma\tau_{wall} >> 1$) and $\omega\tau_A \sim \Omega_\phi\tau_A$ (1-30%) ($\omega\tau_{wall} >> 1$)
- Kinetic effects important for RWM
  - Publications: Berkery, et al. PRL 104 (2010) 035003, Sabbagh, et al., NF 50 (2010) 025020, etc…
- Rotation + kinetic effects beginning to be explored for IWM
  - Such effects generally higher-order than fluid terms (∇p, $J_{||}$, $|\delta B|^2$, wall)
- **Calculations for NSTX indicate both rotation and kinetic effects can modify IWM limits and tearing triggering**
  - High toroidal rotation generated by co-injected NBI in NSTX
    - Fast core rotation: Ω_ϕ / ω_{sound} up to ~1, Ω_ϕ / ω_{Alfven} ~ up to 0.1-0.3
  - Fluid/kinetic pressure is dominant instability drive in high-β ST plasmas

MARS-K: self-consistent linear resistive MHD including toroidal rotation and drift-kinetic effects

- Perturbed single-fluid linear MHD:

Y.Q. Liu, et al., Phys. Plasmas 15, 112503 2008

$$(\gamma + in\Omega)\xi = \mathbf{v} + (\xi \cdot \nabla\Omega) R^2 \nabla\phi$$

$$\rho(\gamma + in\Omega)\mathbf{v} = -\nabla \cdot \mathbf{p} + \nabla \times \mathbf{Q} \times \mathbf{B} + \nabla \times \mathbf{B} \times \mathbf{Q}$$

$$\boxed{-\rho\left[2\Omega\nabla Z \times \mathbf{v} + (\mathbf{v} \cdot \nabla\Omega) R^2 \nabla\phi\right] - \nabla \cdot (\rho\xi) R^2 \Omega^2 \nabla Z \times \nabla\phi}$$

$$(\gamma + in\Omega)\mathbf{Q} = \nabla \times (\mathbf{v} \times \mathbf{B}) + (\mathbf{Q} \cdot \nabla\Omega) R^2 \nabla\phi - \nabla \times (\eta \mathbf{j})$$

$$(\gamma + in\Omega) p = -\mathbf{v} \cdot \nabla P - \Gamma P \nabla \cdot \mathbf{v} \qquad \mathbf{j} = \nabla \times \mathbf{Q}$$

- Drift-kinetic effects in perturbed anisotropic pressure p:

$$\mathbf{p} = p_{||}\hat{\mathbf{b}}\hat{\mathbf{b}} + p_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})$$

$$p_{\|} e^{-i\omega t + in\phi} = \sum_{e,i} \int d\Gamma M v_{\|}^2 f_L^1$$

$$p_{\perp} e^{-i\omega t + in\phi} = \sum_{e,i} \int d\Gamma \frac{1}{2} M v_{\perp}^2 f_L^1$$

$$f_L^1 = -f_\epsilon^0 \epsilon_k e^{-i\omega t + in\phi} \sum_{m,l,u} X_m^u H_{ml}^u \lambda_{ml} e^{-in\tilde{\phi}(t) + im\langle\dot{\chi}\rangle t + il\omega_b t}$$

$$H_L = \frac{1}{\epsilon_k}[M v_{\|}^2 \vec{\kappa} \cdot \boldsymbol{\xi}_{\perp} + \mu(Q_{L\|} + \nabla B \cdot \boldsymbol{\xi}_{\perp})]$$

- Rotation and rotation shear effects:
- Mode-particle resonance operator:

Diamagnetic

$$\lambda_{ml} = \frac{n[\omega_{*N} + (\hat{\epsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n(\langle\omega_d\rangle + \omega_E) + [\alpha(m + nq) + l]\omega_b - i\nu_{\text{eff}} - \omega}$$

Precession **ExB** **Transit and bounce** **Collisions**

- Fast ions: analytic slowing-down f(v) model – isotropic or anisotropic

This work

- Include toroidal flow only: $v_\phi = R\Omega_\phi(\psi)$ and $\omega_E = \omega_E(\psi)$

Real part of complex energy functional provides equation for growth-rate useful for understanding instability sources

Dispersion relation

$$\delta K + \delta W = 0$$

Kinetic energy

$$\delta K = \frac{1}{2}\int d^3x \rho (\gamma + in\Omega)^2 \left|\vec{\xi}_\perp\right|^2$$

Potential energy

$$\delta W = -\frac{1}{2}\int d^3x\, \mathbf{F}\cdot\xi_\perp^*$$

$$\delta K_1 = -\frac{1}{2}\int d^3x \rho \left|\vec{\xi}_\perp\right|^2$$

Growth rate equation: mode growth for $\delta W^{re} < 0$

$$\left(\gamma^{re}\right)^2 = \left(\delta W_K^{re} + \delta W_F^{re} + \delta W_{vb} + \delta W_{rot}^{re}\right) / \delta K_1$$

$$\delta W_K = -\frac{1}{2}\int d^3x \mathbf{F}^K \cdot \xi_\perp^* \qquad \mathbf{F}^K = -\nabla\cdot\mathbf{p}^{\text{kinetic}}$$

$$\delta W_{rot} = \delta W_\Omega + \delta W_{d\Omega} + \delta W_{cf} + \delta K_2$$

$$\delta W_F^p = -\frac{1}{2}\int d^3x \mathbf{F}^p \cdot \xi_\perp^*$$

$$= \frac{1}{2}\int d^3x \left[(\xi_\perp\cdot\nabla P)\nabla\cdot\xi_\perp^* + \Gamma P|\nabla\cdot\xi|^2 - \Gamma P(\nabla\cdot\xi)(\nabla\cdot\xi_\parallel^*)\right] + S_F^p$$

$$\delta W_F^j = -\frac{1}{2}\int d^3x \mathbf{F}^j \cdot \xi_\perp^* \;=\; \frac{1}{2}\int d^3x |Q|^2 + S_F^j$$

$$\delta W_F^Q = -\frac{1}{2}\int d^3x \mathbf{F}^Q \cdot \xi_\perp^* \;=\; \frac{1}{2}\int d^3x \left[J_\parallel \hat{\mathbf{b}}\cdot\xi_\perp^*\times\mathbf{Q}_\perp - \frac{Q_\parallel}{B}(\xi_\perp^*\cdot\nabla P)\right]$$

$$S_F^p = -\frac{1}{2}\int [(\xi_\perp\cdot\nabla P) + \Gamma P\nabla\cdot\xi]\,\xi_\perp^*\cdot d\mathbf{s}$$

$$S_F^j = \frac{1}{2}\int BQ_\parallel \xi_\perp^* \cdot d\mathbf{s}$$

Coriolis - Ω

$$\delta W_\Omega = \frac{1}{2}\int d^3x \left[-2\rho\Omega(\gamma + in\Omega)\mathbf{Z}\times\vec{\xi}_1\cdot\vec{\xi}_\perp^*\right]$$

Coriolis - dΩ/dρ

$$\delta W_{d\Omega} = \frac{1}{2}\int d^3x R\left(2\rho\Omega\left(\vec{\xi}_1\cdot\nabla\Omega\right)\vec{\xi}_{\perp\mathbf{R}}^*\right)$$

Centrifugal

$$\delta W_{cf} = \frac{1}{2}\int d^3x R\Omega^2\nabla\bullet\left(\rho\vec{\xi}_1\right)\vec{\xi}_{\perp\mathbf{R}}^*$$

Differential kinetic

$$\delta K_2 = -\frac{1}{2}\int d^3x \rho(\omega + n\Omega)^2\left|\vec{\xi}_\perp\right|^2$$

Note: n = -1 in MARS

Equilibrium force balance model including toroidal rotation

Force balance for species s: $\vec{J}_s \times \vec{B} = \nabla p_s + \rho_s \vec{v}_s \cdot \nabla \vec{v}_s + Z_s e n_s \nabla \Phi$

Assume: $T_s = T_s(\psi)$ $\quad v_{\phi s} = R\Omega_{\phi s}$ $\quad \Omega_{\phi s} = \Omega_{\phi s}(\psi)$

B• above → $n_s(\psi, R) = N_s(\psi) \exp\left(\frac{m_s \Omega_{\phi s}^2 (R^2 - R_{\text{axis}}^2)}{2k_{\text{B}} T_s} - \frac{Z_s e \Phi(\psi, \theta)}{k_{\text{B}} T_s}\right)$

Exact multi-species solution requires iteration to enforce quasi-neutrality → simplify → intrinsically quasi-neutral if all n_s have same exponential dependence.
This approximate solution assumes main ions dominate centrifugal potential.

$$\vec{J} \times \vec{B} = \sum_s \nabla(n_s T_s(\psi)) + \sum_s m_s n_s {\Omega_{\phi s}}^2 \nabla\left(\frac{R^2}{2}\right) \qquad 0 = \sum_s N_s(\psi) Z_s$$

$$n_s(\psi, R) = N_s(\psi) \exp\left(U(\psi)\left(\frac{R^2}{R_{\text{axis}}^2} - 1\right)\right) \qquad U(\psi) = \frac{P_\Omega(\psi)}{P_{\text{K}}(\psi)}$$

$$P_\Omega(\psi) = \frac{\sum_s N_s(\psi) m_s {\Omega_{\phi s}}^2 R_{\text{axis}}^2}{2} \qquad P_{\text{K}}(\psi) = \sum_s N_s(\psi) T_s(\psi)$$

Grad-Shafranov Equation (GSE) including toroidal rotation

Total force balance:

$$\rho\vec{v}\cdot\nabla\vec{v} \approx -\rho\Omega_\phi^2\nabla R^2/2 = \left(\frac{J_\phi}{R} - \frac{FF'}{\mu_0 R^2}\right)\nabla\psi - \nabla p$$

$$\vec{B} = \nabla\psi \times \nabla\phi + F\nabla\phi$$

Rotation-modified GSE:

$$\frac{J_\phi}{R} = \frac{FF'}{\mu_0 R^2} + \left.\frac{\partial p}{\partial\psi}\right|_R \qquad \rho\Omega_\phi^2 R = \left.\frac{\partial p}{\partial R}\right|_\psi$$

$$p(\psi, R) = P_{\mathrm{K}}(\psi)\exp\left(U(\psi)\left(\frac{R^2}{R_{\mathrm{axis}}^2} - 1\right)\right)$$

LRDFIT reconstructions with rotation determine 3 flux functions:

- U(ψ) - based on fitting electron density profile asymmetry (not C^{6+} rotation data)
- P_K(ψ) and FF'(ψ) – full kinetic reconstruction → fit to magnetics, iso-T_e, MSE with E_r corrections, thermal pressure between r/a = 0.6-1.

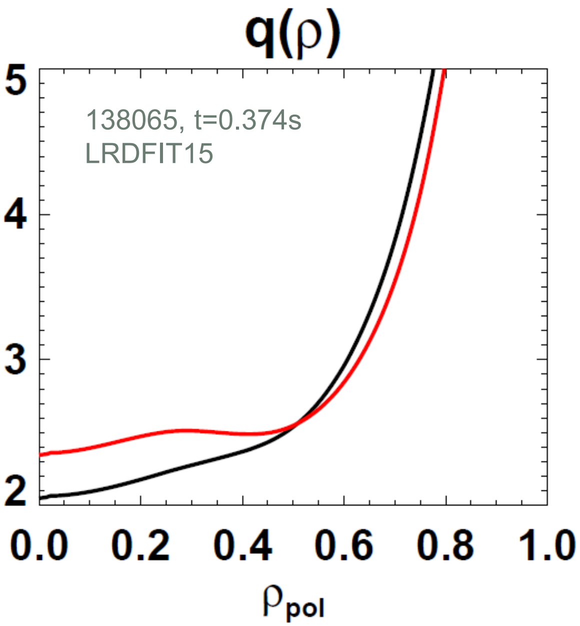
Study 2 classes of IWM-unstable plasmas spanning low to high β_N

- Low β_N limit ~3.5, often saturated/long-lived mode
  - q_{min} ~ 2-3
  - Common in early phase of current flat-top
  - Higher fraction of beam pressure, momentum (lower n_e)

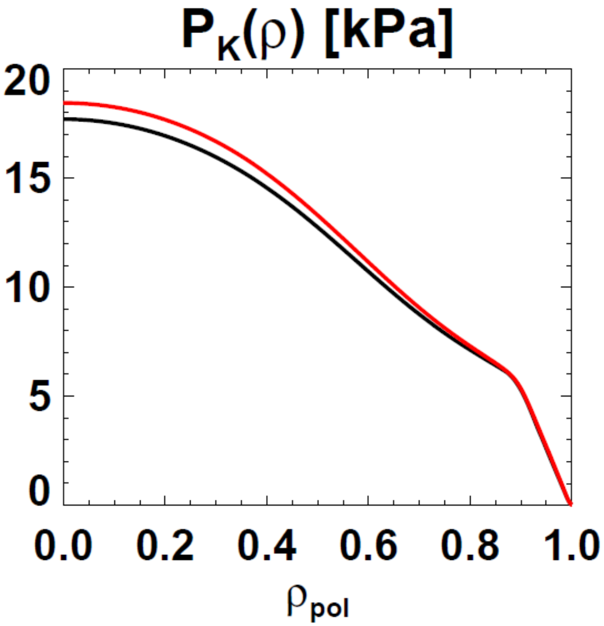
- Intermediate β_N limit ~ 5
  - q_{min} ~1.2-1.5
  - Typical good-performance H-mode, H_{98} ~ 0.8-1.2

Impact of including rotation on q, P_K, P_Ω

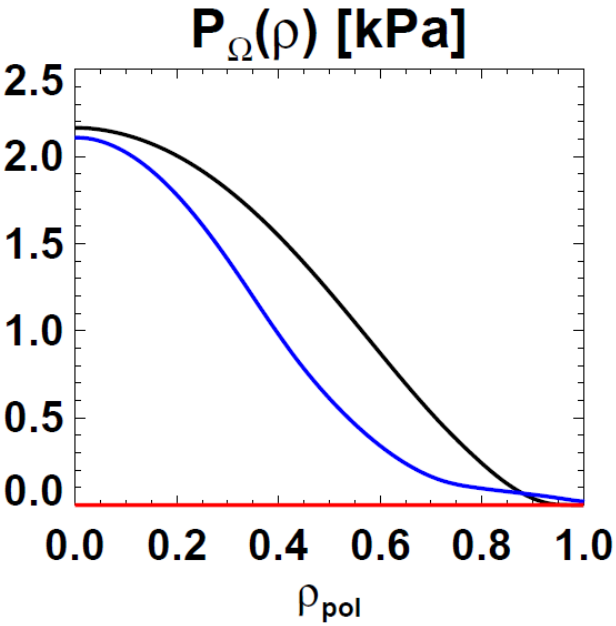
- Black – rotation included (from fit to n_e profile asymmetry)
- Red – rotation set to 0 in reconstruction



Reconstructed core q(ρ) lower with rotation included



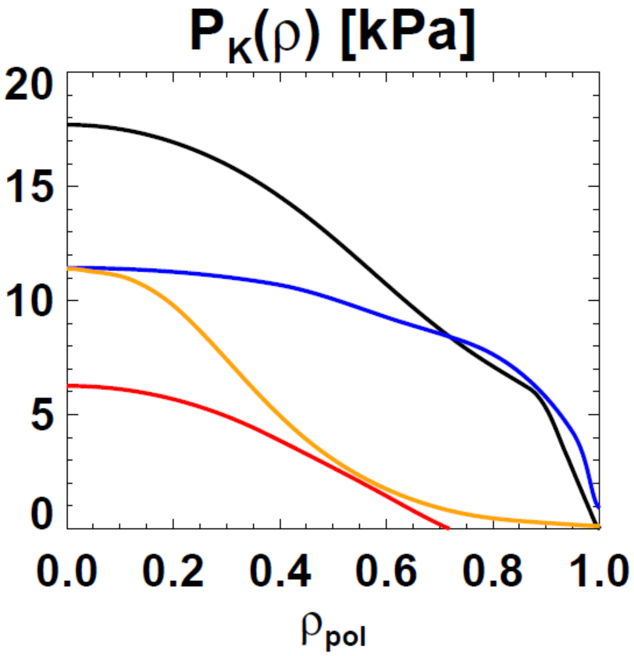
Reconstructed core $P_K(\rho)$ slightly lower w/ rotation included



Reconstructed core $P_\Omega(\rho)$ comparable to measured thermal ion rotation pressure (they should be similar)

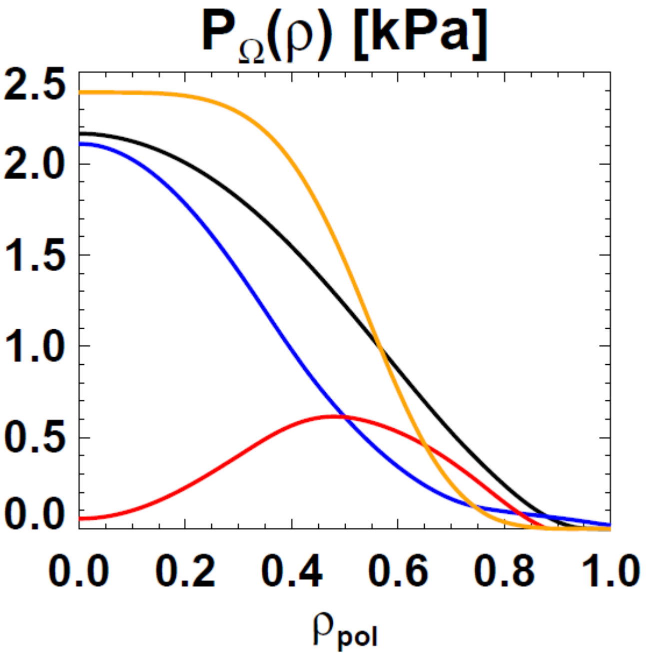
Reconstructions imply significant fast-ion profile broadening

- Black: reconstruction with rotation included (n_e asymmetry)
- Blue: measured thermal
- Red: recons. minus thermal, Orange: TRANSP (no FI diffusion)



$P_K(\rho)$ [kPa]

Reconstructed core fast ion $P_K(\rho)$ significantly lower than TRANSP calculation



$P_\Omega(\rho)$ [kPa]

Reconstructed fast ion $P_\Omega(\rho)$ significantly broader, lower than TRANSP calculation

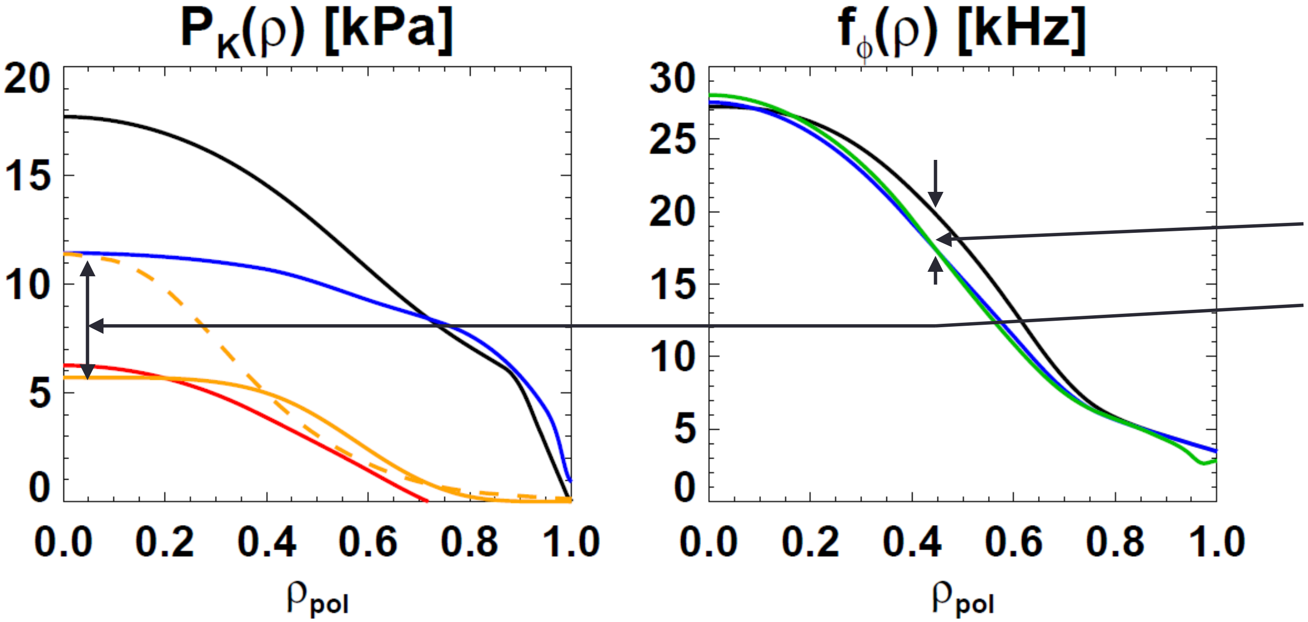
$$U(\psi) = \frac{P_\Omega(\psi)}{P_K(\psi)}$$

$$\exp\left(U(\psi)\left(\frac{R^2}{R_{\text{axis}}^2} - 1\right)\right)$$

NOTE: there is substantial uncertainty in P_Ω near the magnetic axis since U is indeterminate there, i.e. U could be much larger or smaller w/o impacting the density asymmetry fit

Profiles after fast-ion density profile broadening

- Black: reconstruction with rotation included (n_e asymmetry)
- Blue: measured thermal, Red: recons. minus thermal
- Orange: TRANSP with FI density profile broadening (post-facto)



Because fast ion density is low, the impact of fast-ions on total toroidal rotation f_ϕ is weaker than impact on P_Ω

Implication: fast-ion redistribution or loss likely more important for pressure than rotation

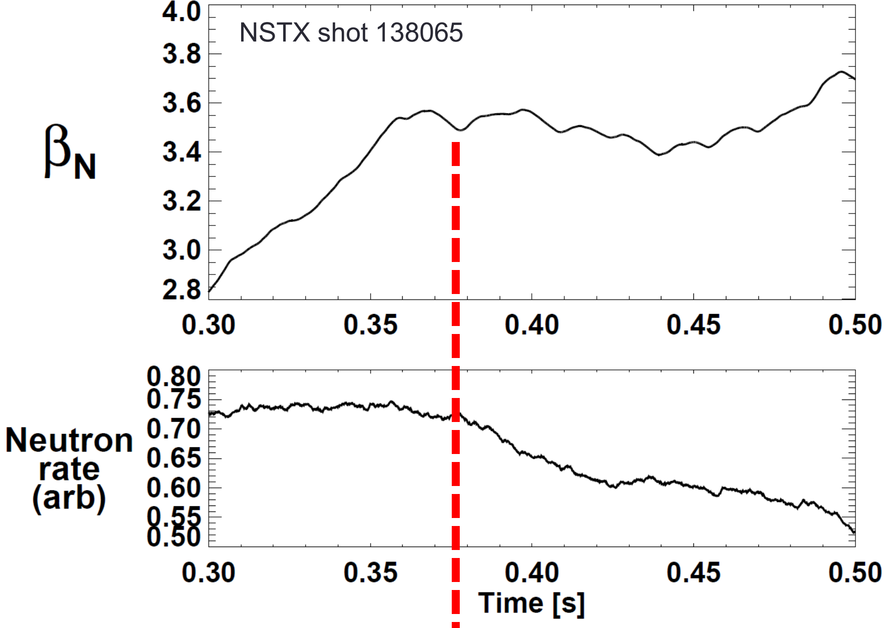
Dash → Solid = fast-ion density broadening: conserve FI stored energy, number, also T_{fast} (ρ)

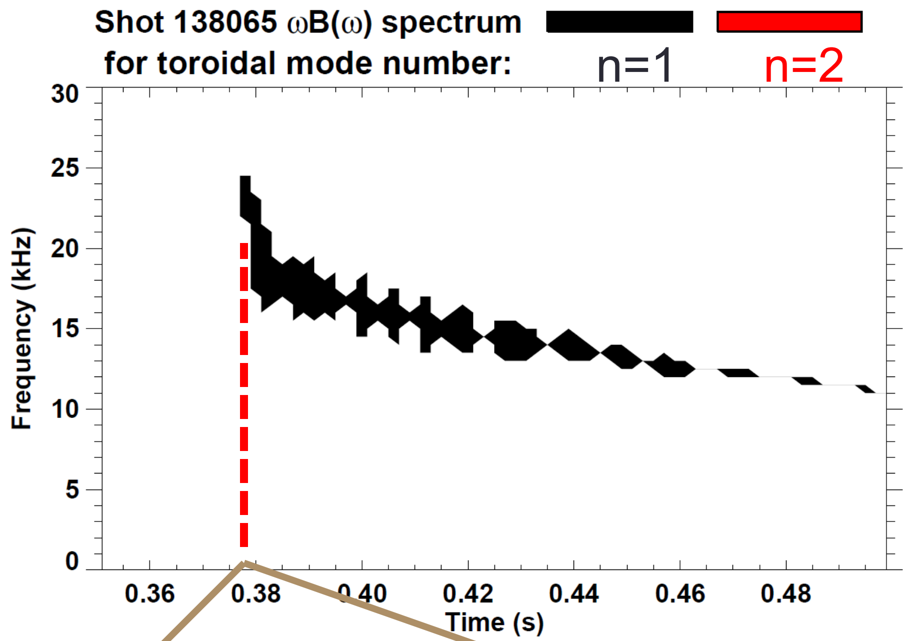
$f_\phi(r)$ including fast ions is broader than $f_{\phi\text{-Carbon}}(\rho)$ & f_{ExB}

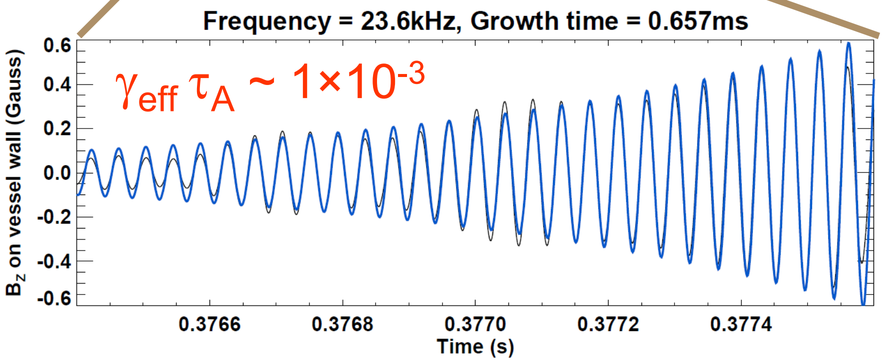
MARS-K uses P_K, f_ϕ , f_{ExB}

Low β_N limit ~ 3.5: Saturated f=15-30kHz n=1 mode common during early I_P flat-top phase

Fixed P_{NBI} = 3MW, I_P = 800kA, β_T=10-15%





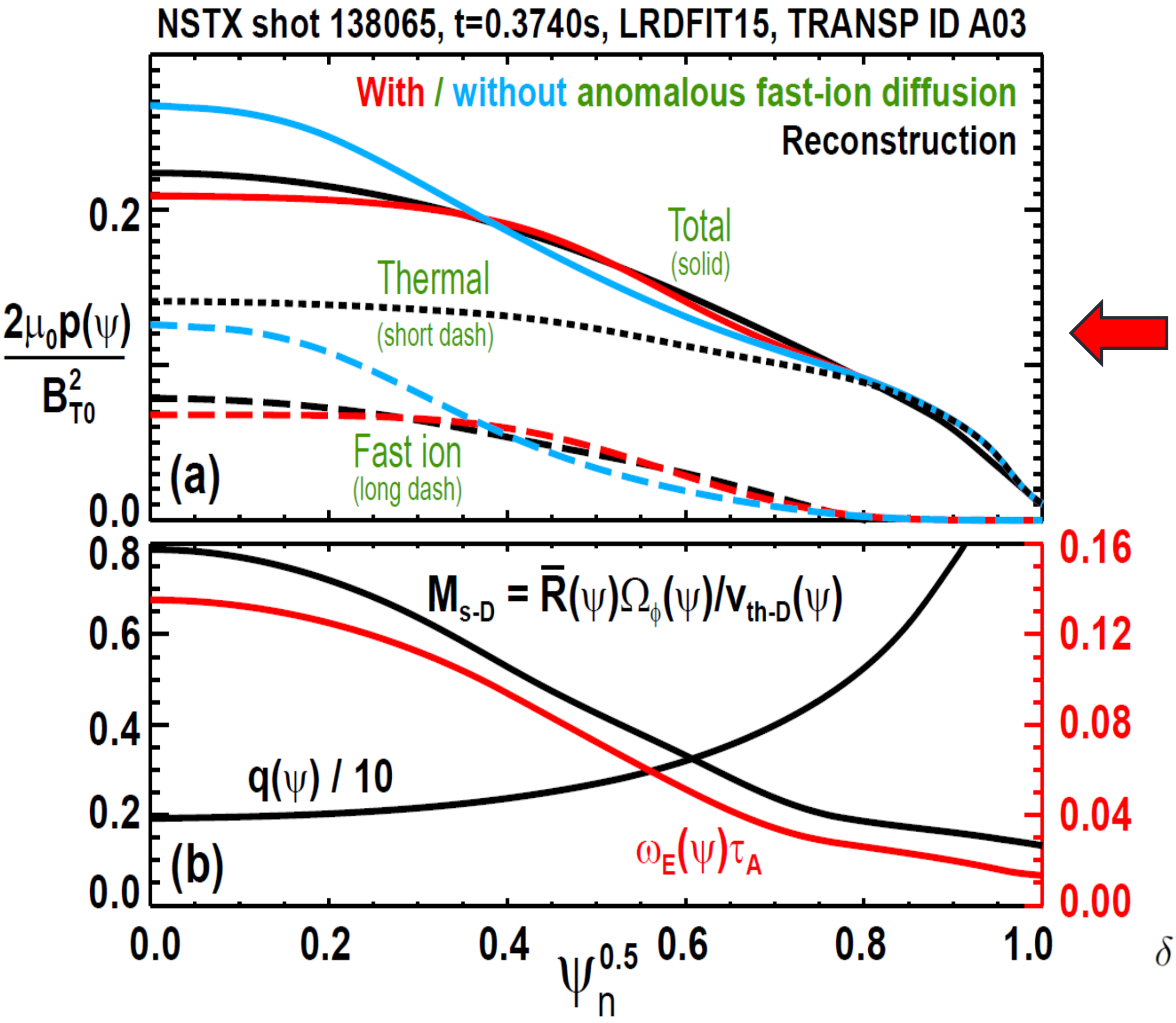


$\gamma_{eff}\,\tau_A$ ~ 1×10^{-3}

Mode clamps β_N to ~3.5, reduces neutron rate ~20%

sometimes slows → locks → disrupts

Kinetic profiles used in analysis



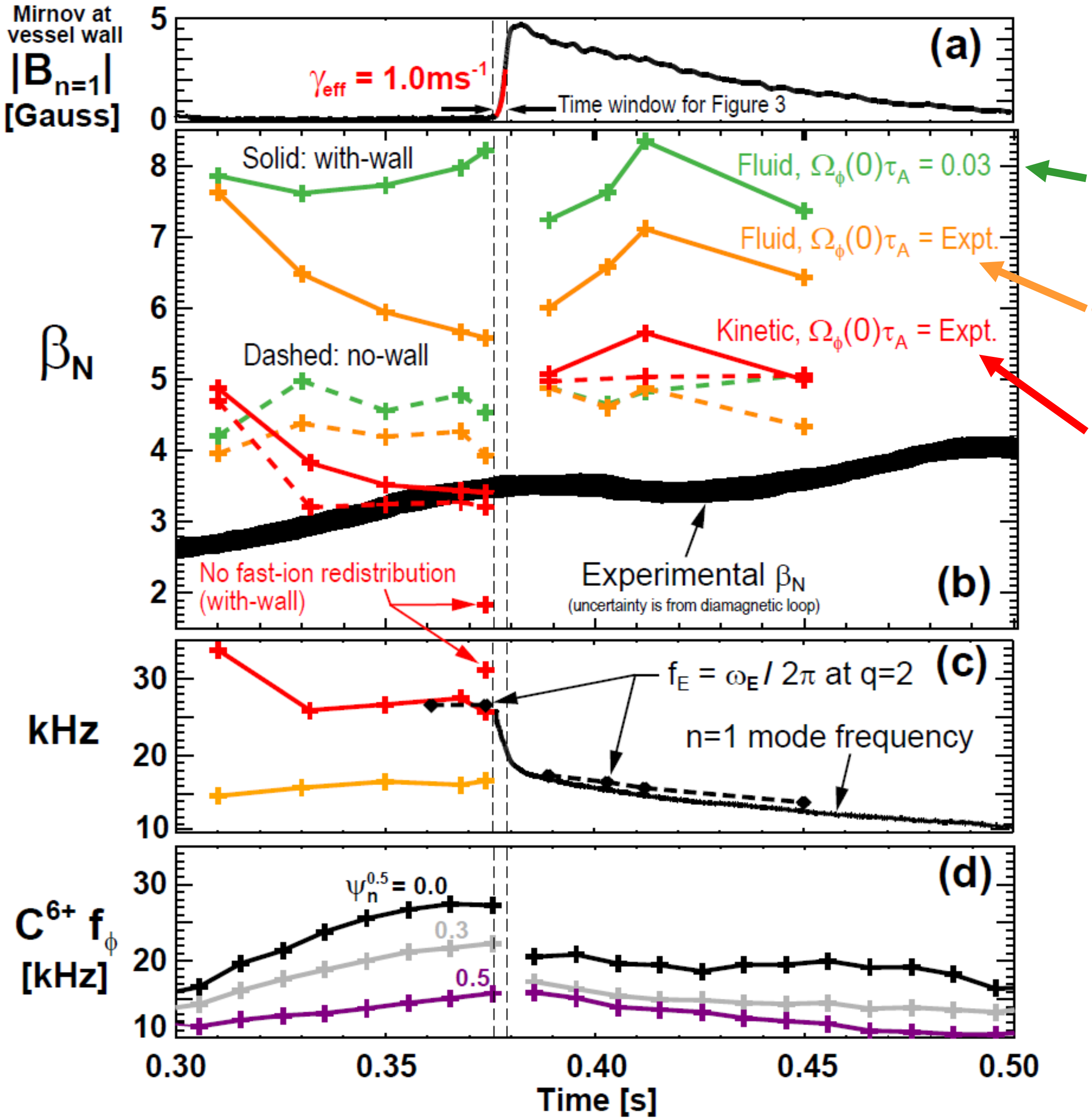
Use broadened fast-ion n, p profiles (red curves) (consistent w/ reconstruction)

- q ≈ 2 in core
- D sound Mach number $M_{s\text{-}D}$ → 0.8 on-axis → significant drive for rotational instability

$$\delta\hat{W}_{rot} \sim \delta W_{\nabla p} \Rightarrow v_\phi \sim v_{th-ion}/\sqrt{q}$$
$$\Rightarrow \Omega_\phi \tau_A \sim \sqrt{\beta_{thermal}/2q}$$

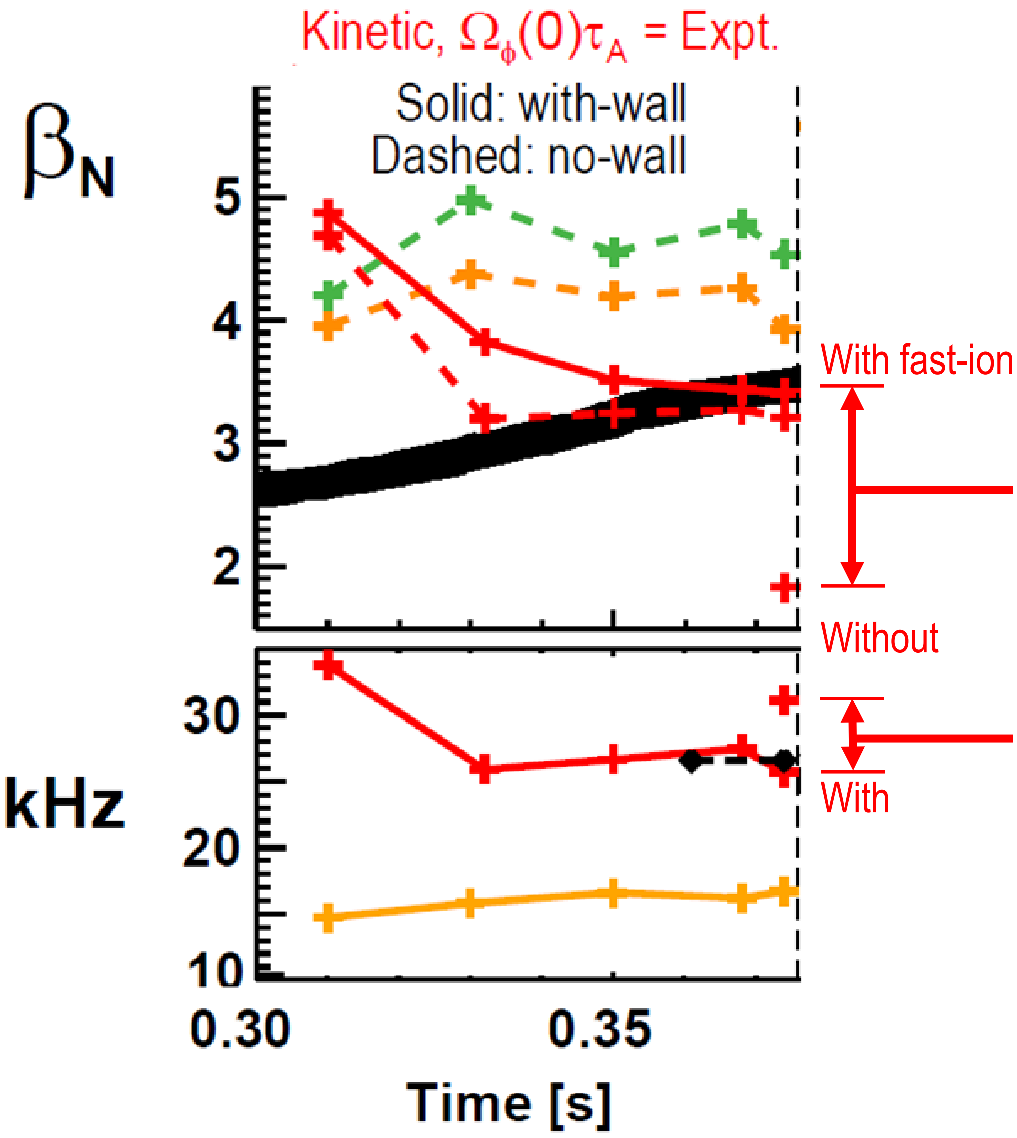
NSTX-U

Predicted stability evolution using MARS-K compared to experiment



- Mode grows quickly, but decays slowly – kink or tearing mode?
- Low-rotation fluid with-wall limit is very high → marginal β_N ~ 7-8
- Increasing rotation lowers max β_N to ~5.5 at mode onset time
- Full kinetic treatment including fast-ions → marginal β_N ~3.5 → most consistent with experiment
- Full kinetic: predicted mode f matches measured f ~ 26kHz
- Fluid model under-predicts f
- Rotation increasing until mode onset, then drops, then remains lower while mode is present.

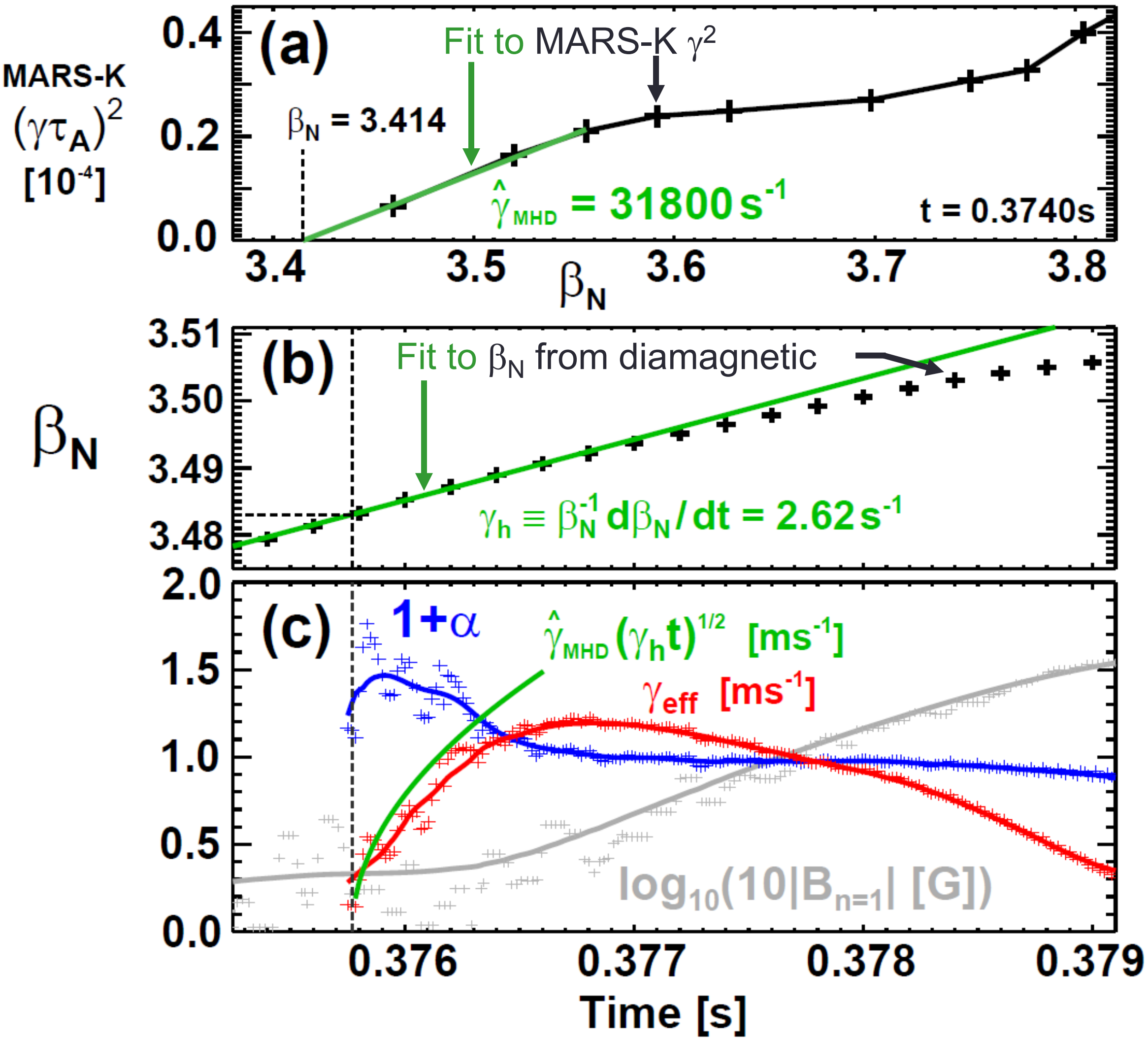
Fast ion broadening has significant impact on predicted stability



Without broadening, predicted marginally stable β_N would be much lower than experiment

And predicted frequency would be higher than observed

Observed mode initial γ consistent with kink/MARS-K



- Use Callen method

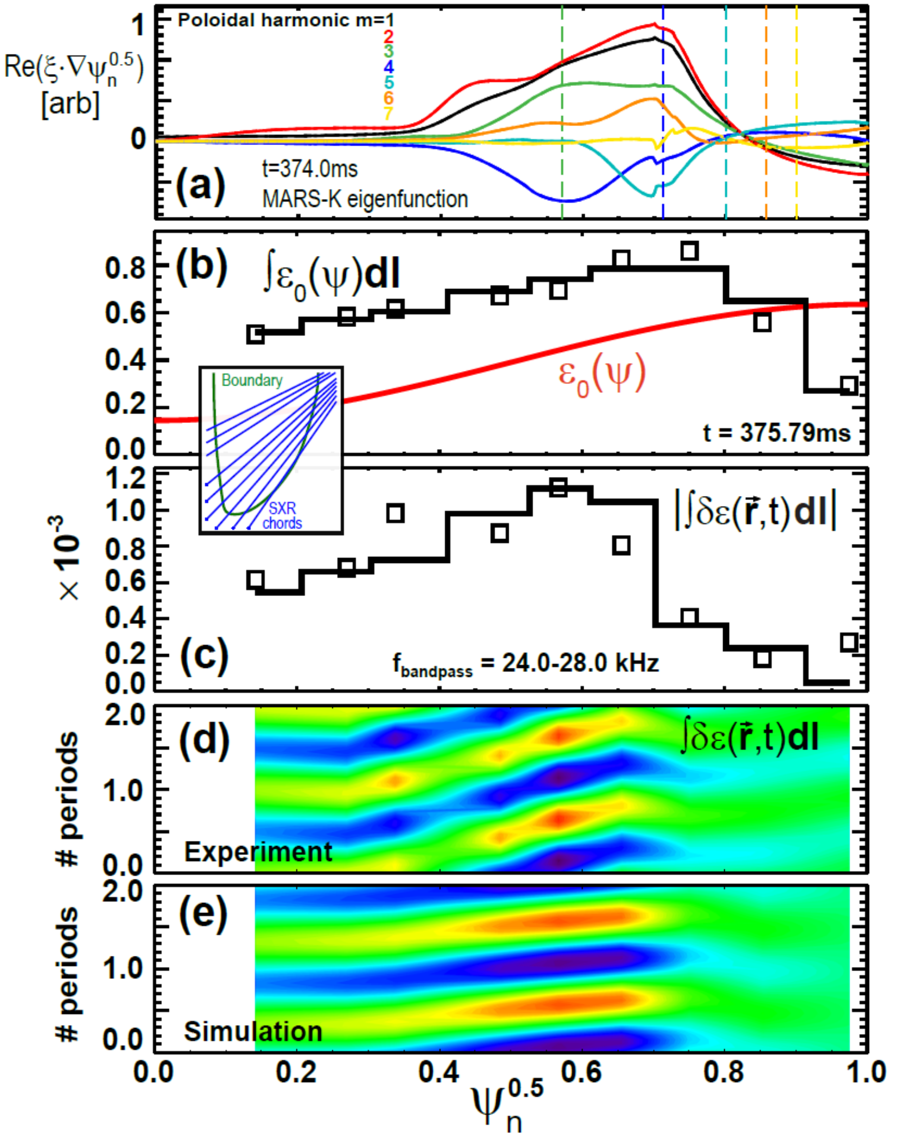
J. D. Callen *et al.*, Physics of Plasmas **6**, 2963 (1999)

- MARS-K γ^2 linear in β_N near marg. stability
- Rate of rise of β_N tracked using diamagnetic loop
- For first 0.5ms, growth is consistent with Callen hybrid γ model for ideal instability:

α = 0.5 for ideal mode

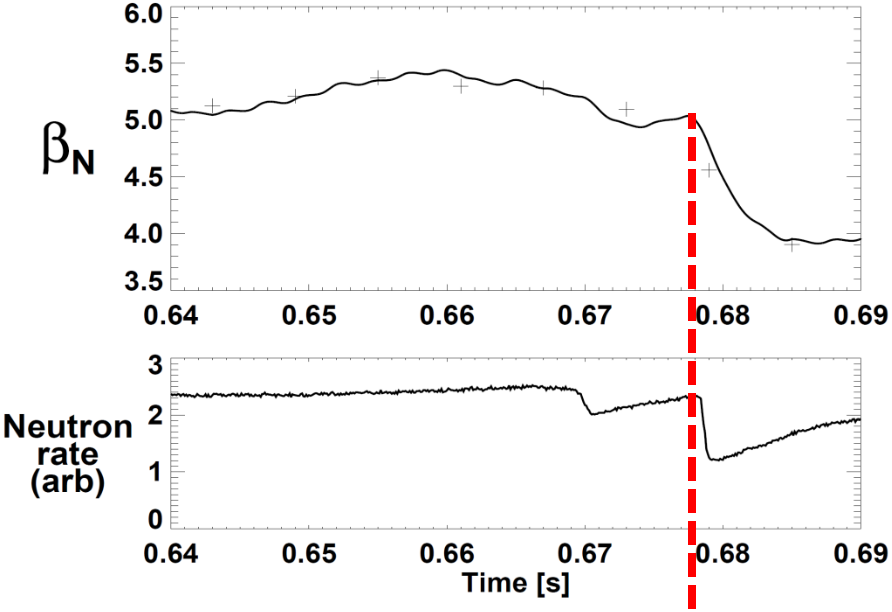
$$\xi = \xi_0 \exp[(\gamma_{eff} t)^{(1+\alpha)}]$$

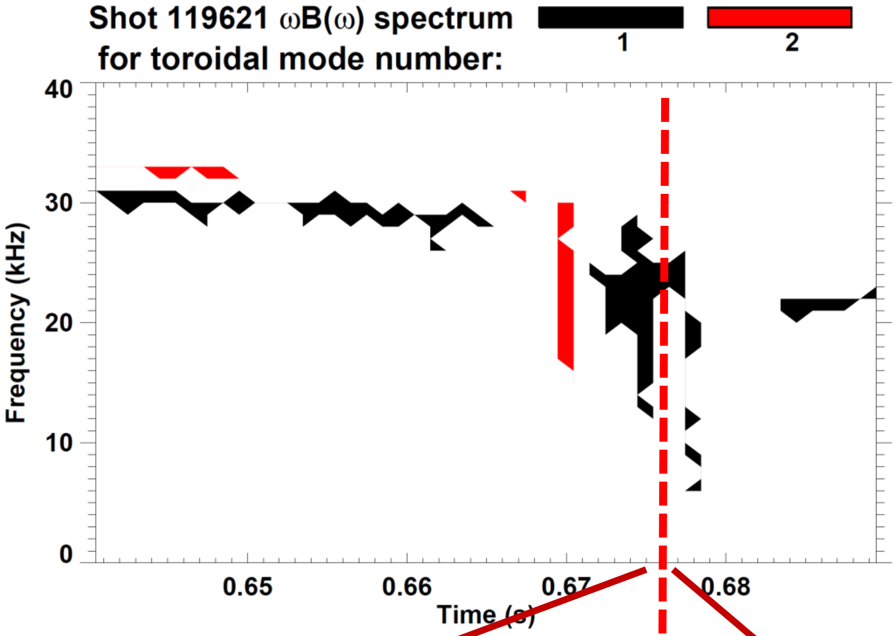
SXR data also consistent with kink/MARS-K at onset

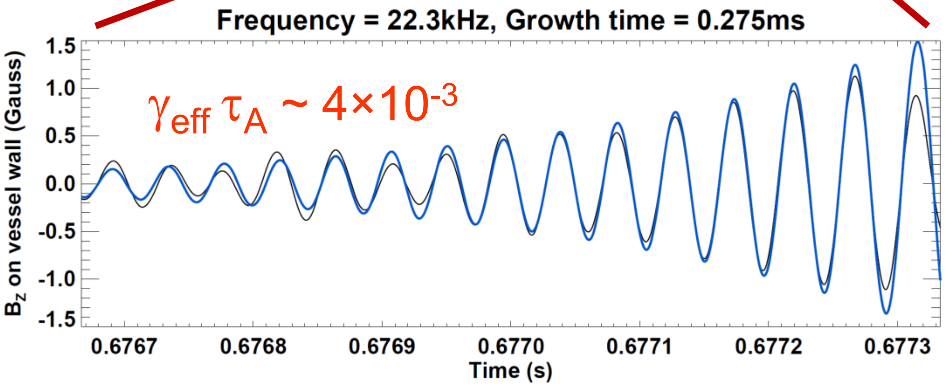


- MARS-K IWM kink eigenfunction largest amplitude for r/a = 0.5-0.8
- Simple/smooth emission profile $\varepsilon_0(\psi)$ can reproduce line-integrated SXR
- …and can reproduce line-integrated SXR fluctuation amplitude profile
- …and has same kink-like structure vs. time and SXR chord position
  - Although the "slope" of the simulated eigenfunction is shallower than measured… rotation or fast-ion effect?
- Later in time, SXR better fit to 2/1 TM (not shown)

Intermediate β_N limit ~ 5: Small f=30kHz continuous n=1 mode precedes larger 20-25kHz n=1 bursts





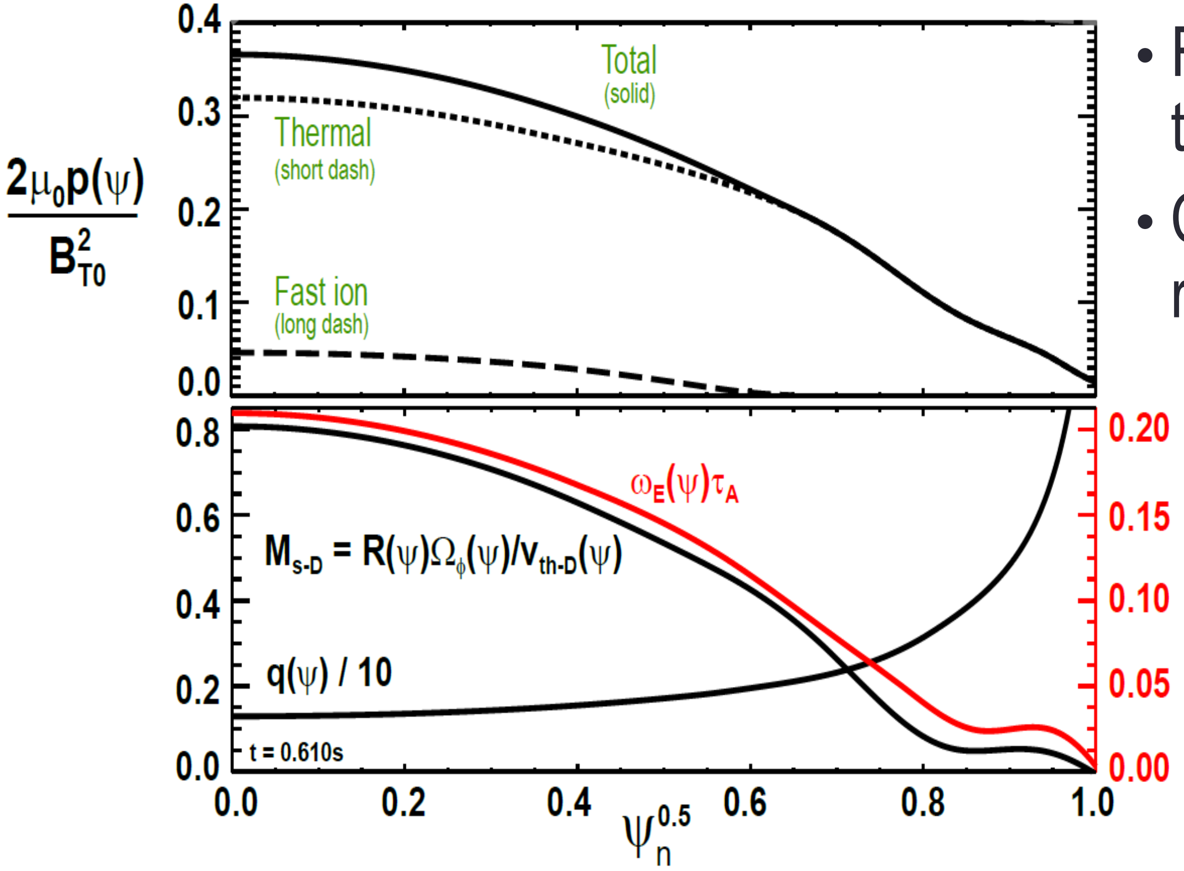


First large n=1 burst → 20% drop in β_N

50% neutron rate drop

Later n=1 modes → full disruption

Kinetic profiles used in analysis



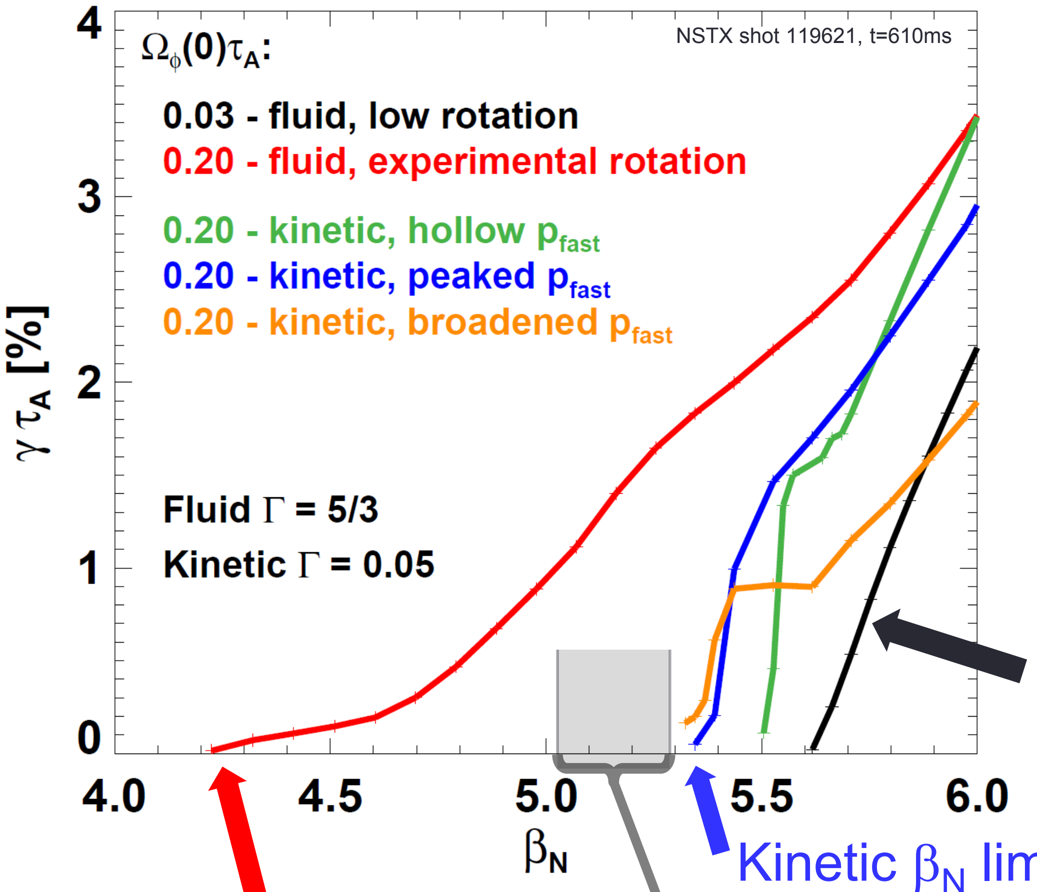
- Fast ion pressure lower in this shot due to higher n_e
- Compute fast-ion from reconstructed total - thermal

- q ≈ 1.3 in core
- D sound Mach number $M_{s\text{-}D}$ → 0.8 on-axis → significant drive for rotational instability

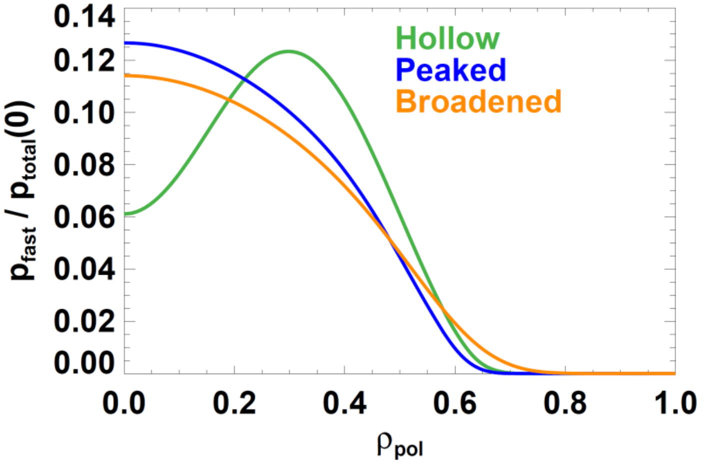
- But, expect weaker rotational destabilization since $M_{s\text{-}D}$ similar, q lower

$$\delta\hat{W}_{rot} \sim \delta W_{\nabla p} \Rightarrow v_\phi \sim v_{th-ion}/\sqrt{q}$$

Kinetic IWM β_N limit consistent with experiment, fluid calculation under-predicts experimental limit



Fast-ion p profiles used in kinetic calculations



Fluid β_N limit for low Ω_ϕ

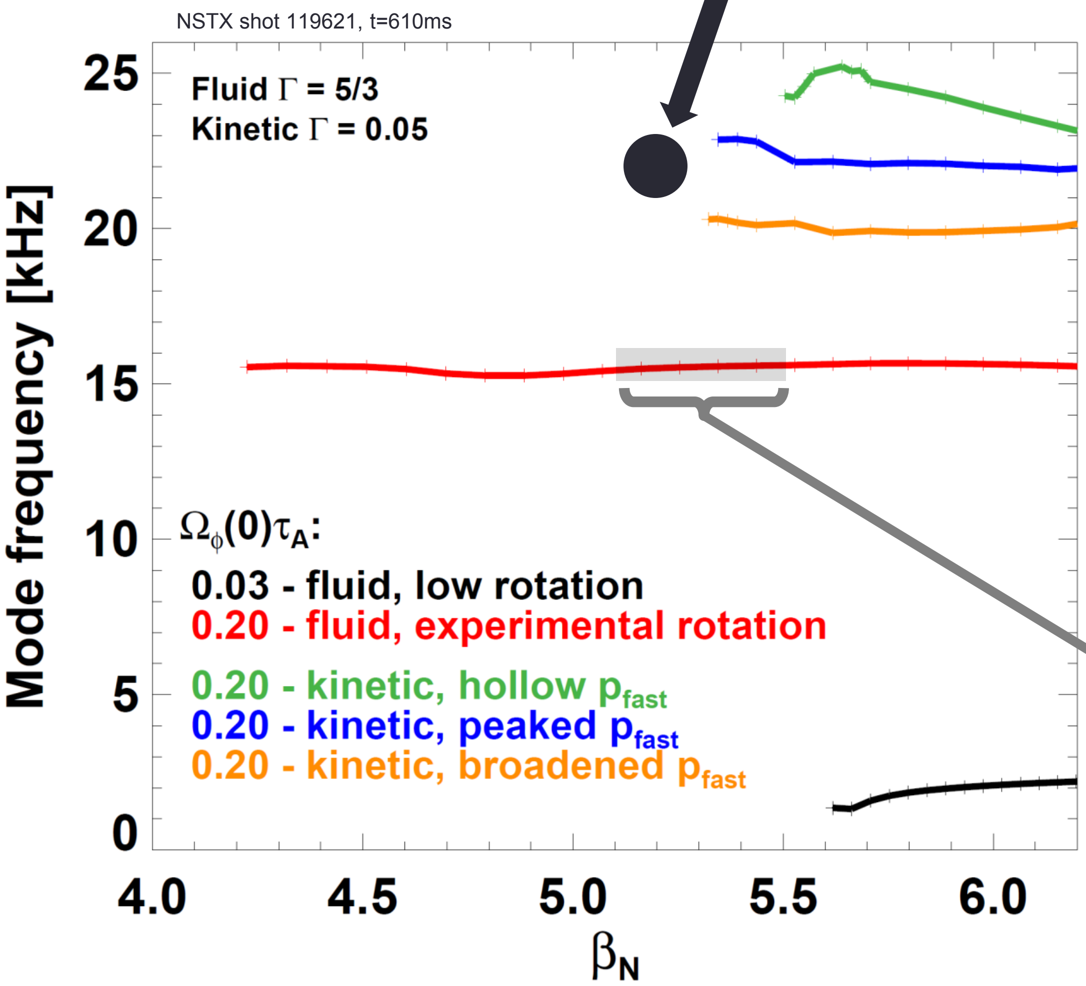
Kinetic β_N limit for experimental Ω_ϕ

(weakly dependent on fast-ion peaking factor)

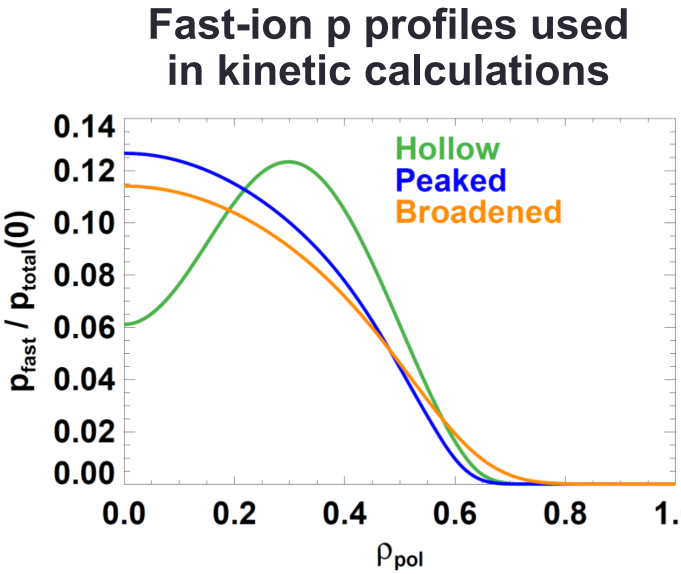
Experimental β_N for n=1 mode onset

Fluid β_N limit for experimental Ω_ϕ ($\Delta\beta_N$ = -1.4 vs. low rotation)

Measured IWM real frequency more consistent with kinetic model than fluid model



Measured mode frequency

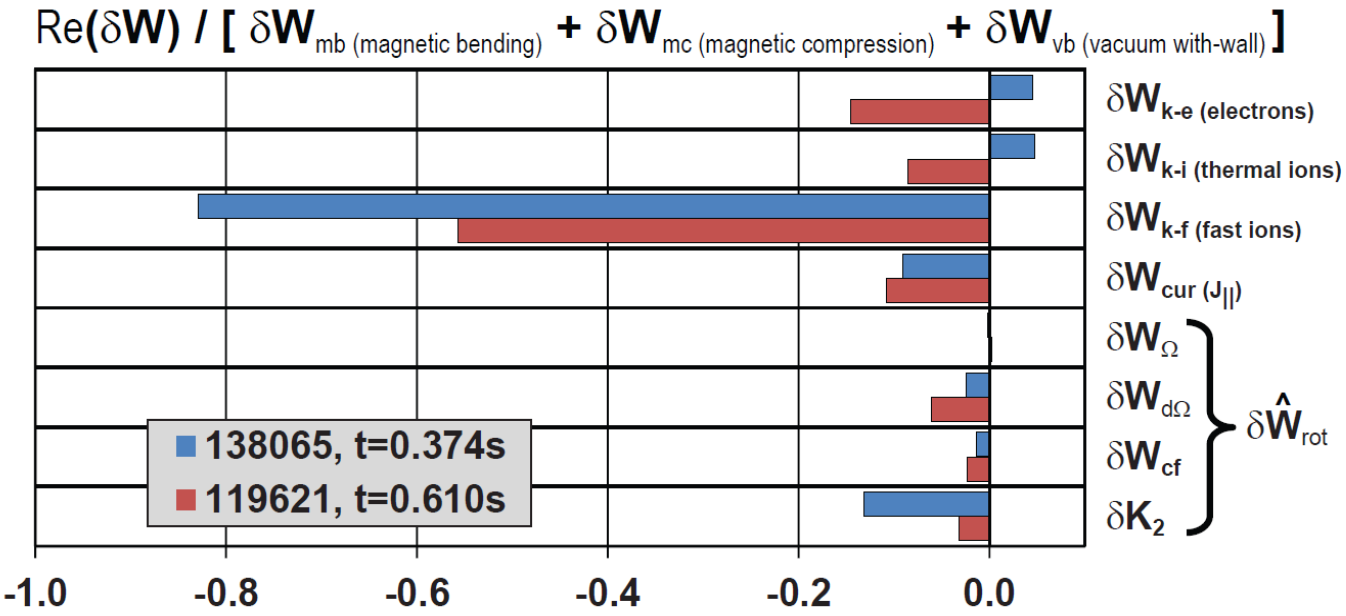

Fast-ion p profiles used in kinetic calculations

$f_E = \omega_E / 2\pi$ at q=2 →

- Not as well matched to measured kink frequency
- Smaller / no subsequent tearing triggered

IWM energy analysis near marginal stability elucidates trends from growth-rate scans

- Fast-ions in Re(δW_k) = dominant destabilization in both shots
  - Balanced against field-line bending + compression + vacuum stabilization
- Shot 138065 has larger destabilization from fast-ions & rotation
  - Consistent w/ larger 55% reduction in β_N = 7-8 → 3.5 (vs. 5.5→ 4.2 or 25%)
- Kelvin-Helmholtz-like $\delta W_{d\Omega}$ and δK_2 are dominant δW_{rot} terms
  - Rotational Coriolis and centrifugal effects weaker



Some theory needs for tearing triggering by IWM, or due to proximity to IWM marginal stability (1)

- Hybrid fluid + drift-kinetic MHD → predicts IWM marginal stability, growth rate, real frequency, eigenfunction
- But when do IWMs lead to disruption, and/or trigger TMs?
- For few cases studied thus far, tearing triggered after kink/IWM onset when $f_{\text{kink-IWM}}$ closer to $f_{\text{ExB at q=2}}$
  - Hypothesis: Require frequency match, sufficient drive + time
  - ω match influenced by rotation shear, thermal / fast-ion ω^*
- Implications for IWM → TM calculations
  - Spontaneous tearing mode calculations likely require modified Δ' and Γ' to include rotation/rotation shear, fast ions
    - Know from Gerhardt's 2/1 NTM work rotation shear likely important for Δ'
  - Forced reconnection triggering of TM likely requires non-linear treatment including differential ω, shielding effects

Some theory needs for tearing triggering by IWM, or due to proximity to IWM marginal stability (2)

- Example non-linear forced reconnection model:

Hegna, Callen, and LaHaye Phys. Plasmas, Vol. 6, No. 1, January 1999

$$L_s = q_0^2 R / q_0' \rho_0$$

$$\Phi = B_\zeta \rho_0^2 / L_s$$

$$\tau_{sp} = \sqrt{\tau_a \tau_r}$$

$$E(t) = E_0 \left(\frac{t}{\tau_G} \right)^\alpha e^{i\omega t} \qquad \frac{\partial \psi_x}{\partial t} = \frac{e^{i\omega t}}{\tau_{sp}} \frac{[2mE(t)]^{3/2}}{\sqrt{\Phi}}$$

$$\psi_x(t) \cong \frac{t}{\tau_{sp}} \frac{[2mE(t)]^{3/2}}{\sqrt{\Phi}} \frac{1}{1+3\alpha/2} \{1 - i\omega t C_0 + \cdots\}$$

$$\omega t \ll 1$$

$$\psi_x(t) \cong \frac{e^{i\omega t}}{i\omega \tau_{sp}} \frac{[2mE(t)]^{3/2}}{\sqrt{\Phi}} \left[1 + O\left(\frac{1}{\omega t} \right) \right]$$

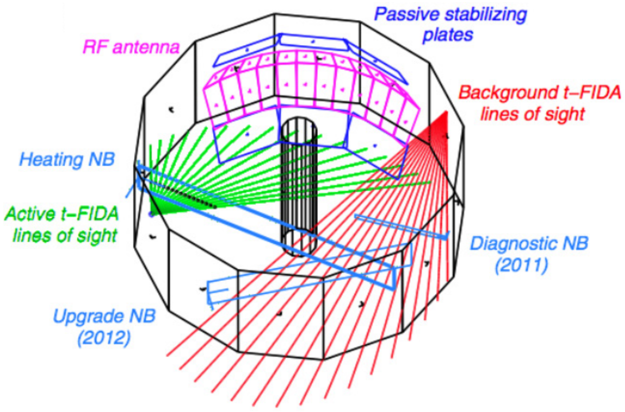
$$\omega t > 1$$

$1/\omega\tau_{sp}$ shielding from differential rotation

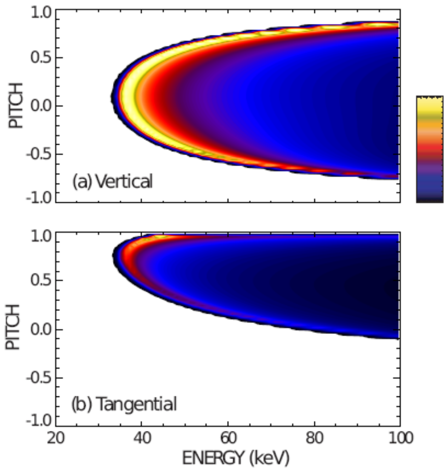
- Can models like this (or more sophisticated) be used:
  - To estimate “fast” evolution, amplitude of driven seed island
  - Link to MRE to understand stabilization, growth/saturation?

Some theory needs related to fast-ion distribution function representation / modeling (1)

- MARS-K has both isotropic and anisotropic (NBI-like) representations for fast-ion distribution function
  - Anisotropic model sometimes leads to singular eigenfunctions and/or different mode characteristics
  - Isotropic model much better behaved… but why?
    - Possible numerical / implementation issues
    - Also likely that fast ions could be more isotropic than TRANSP classical slowing down model due to velocity-space redistribution (?)
  - Need better constraint on pitch angle → FIDA data will help

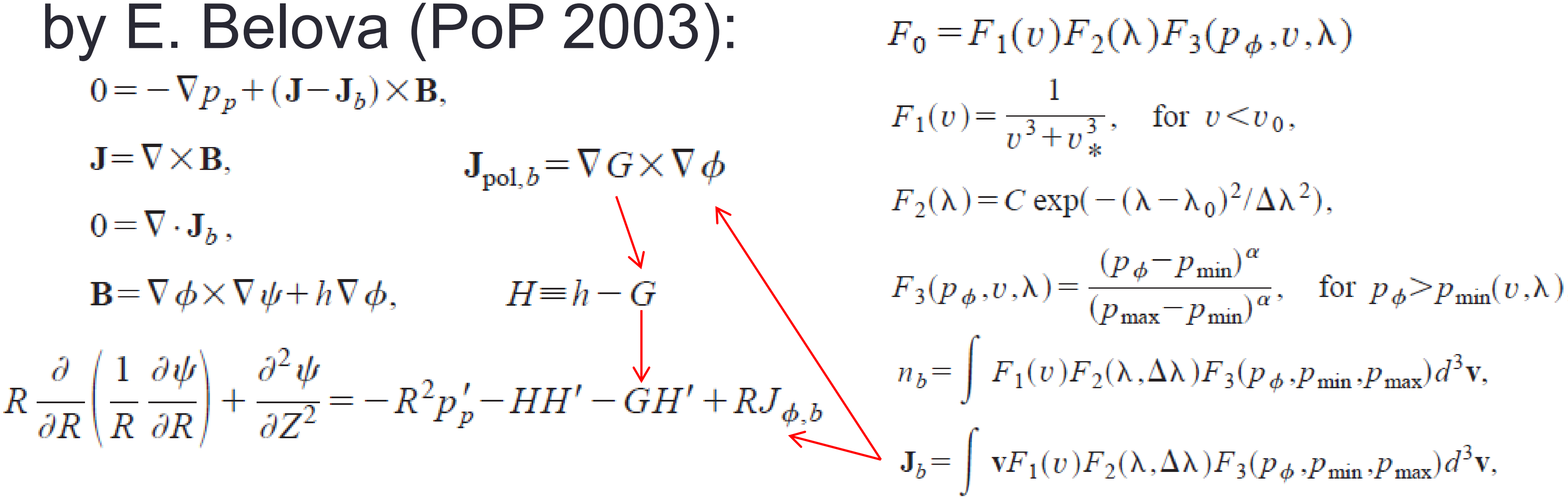


FIDA Views



Some theory needs related to fast-ion distribution function representation / modeling (2)

- Generalized GS equation including beams developed by E. Belova (PoP 2003):



$$0 = -\nabla p_p + (\mathbf{J} - \mathbf{J}_b) \times \mathbf{B},$$

$$\mathbf{J} = \nabla \times \mathbf{B},$$

$$0 = \nabla \cdot \mathbf{J}_b,$$

$$\mathbf{B} = \nabla \phi \times \nabla \psi + h \nabla \phi,$$

$$\mathbf{J}_{\mathrm{pol},b} = \nabla G \times \nabla \phi$$

$$H \equiv h - G$$

$$R \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2} = -R^2 p_p' - HH' - GH' + RJ_{\phi,b}$$

$$F_0 = F_1(v) F_2(\lambda) F_3(p_\phi, v, \lambda)$$

$$F_1(v) = \frac{1}{v^3 + v_*^3}, \quad \text{for } v < v_0,$$

$$F_2(\lambda) = C \exp(-(\lambda - \lambda_0)^2/\Delta\lambda^2),$$

$$F_3(p_\phi, v, \lambda) = \frac{(p_\phi - p_{\min})^\alpha}{(p_{\max} - p_{\min})^\alpha}, \quad \text{for } p_\phi > p_{\min}(v, \lambda)$$

$$n_b = \int F_1(v) F_2(\lambda, \Delta\lambda) F_3(p_\phi, p_{\min}, p_{\max}) d^3\mathbf{v},$$

$$\mathbf{J}_b = \int \mathbf{v} F_1(v) F_2(\lambda, \Delta\lambda) F_3(p_\phi, p_{\min}, p_{\max}) d^3\mathbf{v},$$

- Need to extend the above model to include rotation
- Equilibrium reconstructions may need to include multiple F_0 bases to accurately capture FI effects
  - Investigate constraints by FIDA and other FI diagnostics

Summary

- Hybrid fluid + drift-kinetic MHD model significantly improves predictive capability for linear IWM stability
  - Rotation, fast-ion, kinetic effects can strongly modify IWM
- Accurate reconstructions of fast-ion redistribution by other modes very important for understanding IWM
  - Need improved models of fast-ions in reconstructions and low-n stability codes to understand IWM and related MHD
  - Need to include rotation for self-consistency
- Tearing mode triggering by IWM (or near limits) will need improvements in models:
  - Rotation, rotation shear, fast-ion effects on Δ' and MRE
  - NL / forced reconnection with differential ω and shielding