Error field penetration and locking to the backward wave*

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July 12, 2015

*ArXiv submitted, 1-LANL, 2-Columbia U, 3-Princeton U/PPPL

Static error field interacting with finite frequency modes

- Known: the response to error fields is largest if the tearing mode is *weakly* stable.
- Observation: the response is largest if the Doppler-shifted tearing mode frequency is nearly at rest in the lab frame: $\pm\omega_r + kv \approx 0$

$$\tilde{\psi}(r_t) = -\frac{l_{21}}{\Delta' - \Delta(ikv)}\tilde{\psi}(r_w)$$

- Real frequencies for tearing modes in several regimes: Glasser effect, diamagnetic propagation,...
- Torque (Maxwell stress) applied across the tearing layer:

$$N_m = -\frac{k}{2}|\tilde{\psi}(r_t)|^2 \mathrm{Im}\Delta(ikv)$$

- Although $|\tilde{\psi}(r_t)|^2$ is maximum at $v = \omega_r/k$, the torque is *zero* there. Weak driving torque $\Longrightarrow$ plasma locks to $v \gtrsim \omega_r/k$, *not* to $v \gtrsim 0$.

Simplest layer response function Δ is the Viscoresistive (VR) tearing mode

$$\Delta(\gamma) = \frac{\mu^{1/6}}{\eta^{5/6}|k_{||}'|^{1/3}B^{1/3}}\gamma \;\; (p' = 0) \quad \Delta(\gamma) = \gamma\tau_{vr}$$

Spontaneous modes have $\gamma\tau_{vr} = \Delta'$. Zero real frequency for both signs of Δ'.

$$\Delta' = [\tilde{\psi}']_{r_t}/\tilde{\psi}(r_t) \text{ for } \tilde{\psi}(r_w) = 0.$$

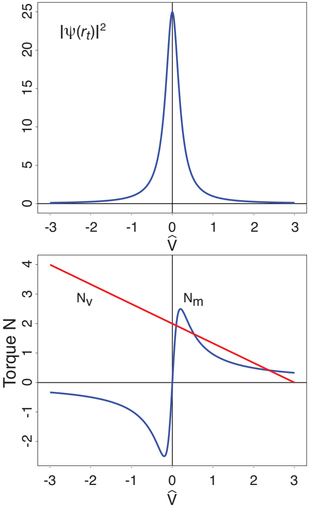
$$\tilde{\psi}(r_t) = -\frac{l_{21}}{\Delta' - \Delta(ikv)}\tilde{\psi}(r_w) = -\frac{l_{21}}{\Delta' - ikv\tau_{vr}}\tilde{\psi}(r_w)$$

Reconnected flux near maximum in locked torque balance state

The field $|\tilde{\psi}(r_t)|^2$ and torque N_m vs. $\hat{v} = kv\tau$ for the VR regime have simple structure near $\hat{v} = 0$.
Equilibrium v_0 determined by flow drive (e.g. beams).

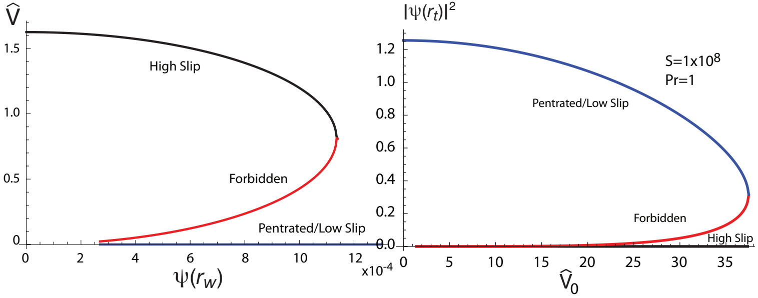
Steady state torque balance $N_v = N_m$ at tearing layer determines roots ($\hat{v}$), stable or unstable.

Viscous torque $N_v \sim \mu(\hat{v}_0 - \hat{v})$ intersects at 1 or 3 equilibria.
Locked state has $\hat{v} \gtrsim 0$.



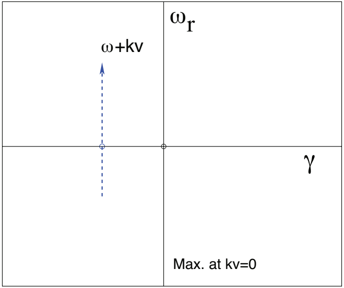
Penetration is a bifurcation to a high reconnected flux, low flow state

Standard bifurcation picture gives penetration threshold in either error field amplitude or equilibrium flow.



Reconnected flux and torque depend on Doppler shift

Denom $= \Delta' - ikv\tau_{vr} = (\gamma - ikv)\tau_{vr}$. γ real $\Longrightarrow$ maximum of $|\tilde{\psi}(r_t)|^2$ is at $\hat{v} \equiv kv\tau_{vr} = 0$



Locus of roots for VR tearing mode, $\omega \rightarrow \omega + kv$

Max. linear response is for $\Delta' \lesssim 0$ and $v = 0$.

$$N_m = -\frac{k^2 l_{21}^2 |\tilde{\psi}(r_w)|^2}{2} \frac{v\tau_{vr}}{\Delta'^2 + k^2 v^2 \tau_{vr}^2} \propto -\frac{v}{c_0^2 + v^2} \quad \text{(Well quoted)}$$

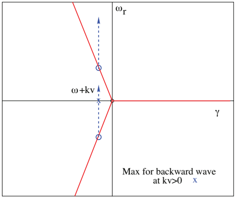
Resistive-inertial (RI) regime, $p' = 0$

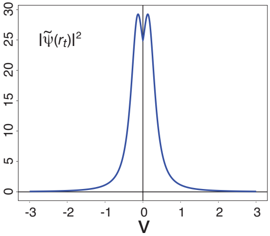
$$\Delta(\gamma) = \frac{\rho^{1/4}}{\eta^{3/4}|k'_{||}|^{1/2}B^{1/2}}\gamma^{5/4} \quad \Delta(\gamma) = \gamma^{5/4}\tau_{ri}^{5/4}$$

Real γ for $\Delta' > 0$; For $\Delta' < 0$, $\gamma\tau_{ri} = |\Delta'|^{4/5}e^{\pm 4\pi i/5}$

$$|\tilde{\psi}(r_t)|^2 = \frac{l_{21}^2|\tilde{\psi}(r_w)|^2}{|\Delta' - (ikv\tau_{ri})^{5/4}|^2} \sim \frac{1}{|(\gamma\tau_{ri})^{5/4} - (ikv\tau_{ri})^{5/4}|^2} \sim \frac{1}{|\gamma - ikv|^2}$$

N. B. denom is $\left(\Delta' - \Delta_r(ikv\tau_{ri})\right)^2 + \Delta_i(ikv\tau_{ri})^2$, not $\Delta'^2 + (\cdots)^2$



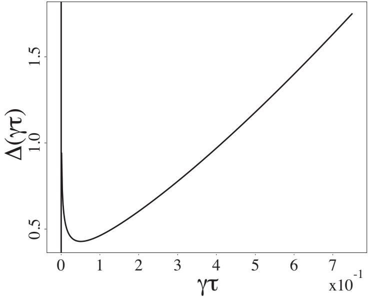


Roots for RI mode, $\omega \to \omega + kv$.

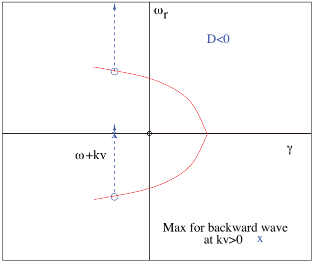
Off-axis peaks but $N_m(v)$ qualitatively the same.

RI with p' and parallel dynamics leads to “Glasser effect” and complex roots

$$\Delta' = \Delta(\gamma\tau) \equiv (\gamma\tau)^{5/4} - \frac{\pi D}{4(\gamma\tau)^{1/4}} \; \ldots \; D \sim -p'(1-q^2) < 0$$



$\Delta(\gamma\tau)$ vs. $\gamma\tau$ showing Δ_{min}



Locus of roots for RI $D < 0$, $\omega \to \omega + kv$.

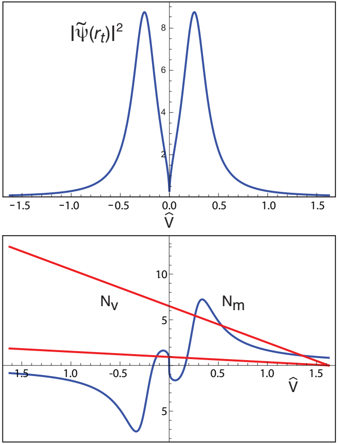
C.c. complex roots if $\Delta' < \Delta_{min} = 2.0|D|^{5/6}$, These roots stabilized if $\Delta' < \Delta_{crit} = 1.13|D|^{5/6}$

RI with Glasser effect has penetrated state near zero torque at significant $\hat{v}$

$$N_m \propto -\frac{\Delta_i(ikv\tau_{ri})}{(\Delta' - \Delta_r(ikv\tau_{ri}))^2 + \Delta_i(ikv\tau_{ri})^2}$$

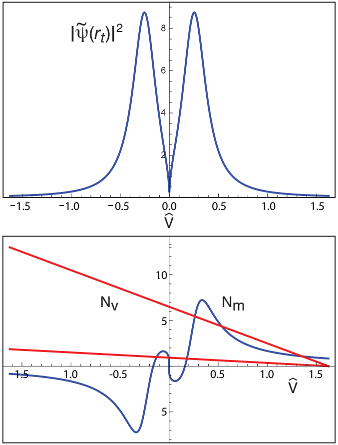
Numerator = 0 near where denominator minimum; $\Delta_i = 0$ at $\omega_r = kv$

- Note pronounced peaks in $|\tilde{\psi}(r_t)|^2$ off axis.
- Viscous torque $N_v(v) \propto \mu(v_0 - v)$ for small μ intersects at $v \gtrsim \omega_r/k$. Fields are locked to the static error field, but the plasma flow is locked to finite value, $v \gtrsim \omega_r/k$ rather than $v \gtrsim 0$.
- For very small μ two other states are possible, L-stable; R-unstable.

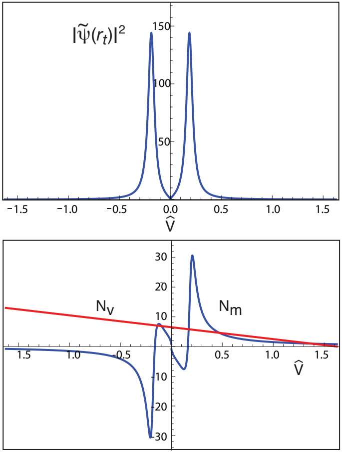


Finite $\hat{v}$ persists in RI with Glasser as $\Delta' \to \Delta_c -$

Numerator = 0 near where denominator minimum; $\Delta_i = 0$ at $\omega_r = kv$



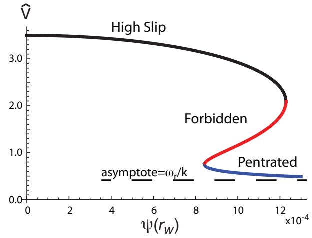
Δ'strongly stable

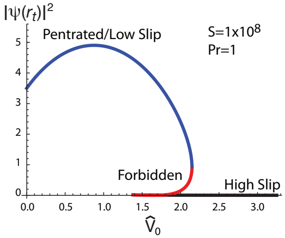


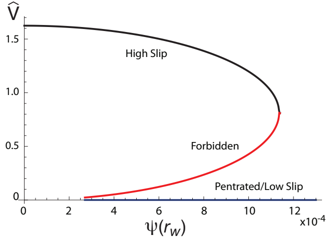
Δ' marginally stable

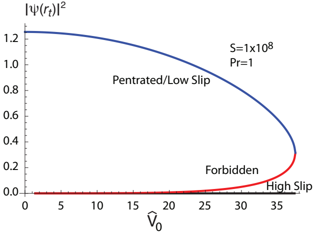
Penetration bifurcation diagram in RI

Plasma velocity locks to $v \gtrsim \omega_r / k$ and penetrated flux has maximum for $\hat{v}_0 > 0$



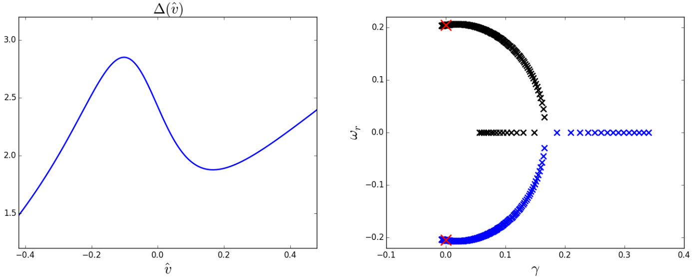






VR regime with p' and parallel dynamics also has complex roots

Numerical computations of the VR dispersion relation, using the constant-ψ approximation

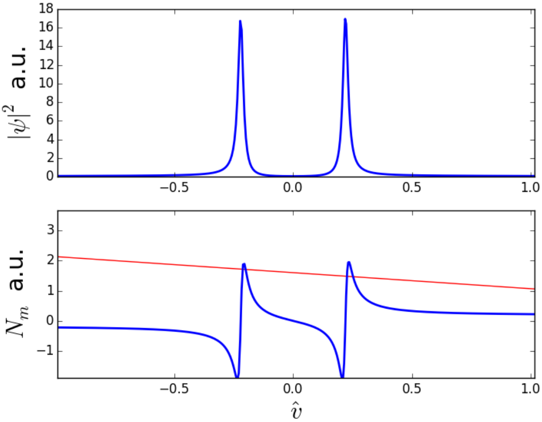


$\Delta(\hat{v})$ is non-monotonic in a range of c_s. $\Delta' \lesssim \Delta_2$ or $\Delta' \gtrsim \Delta_1 \implies$ complex roots. Glasser effect in VR!

Glasser Effect in VR Regime

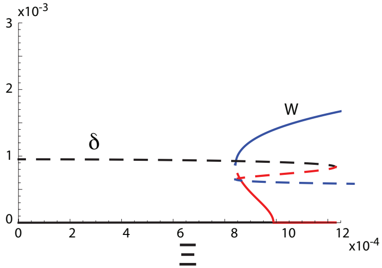
Locus of roots similar to RI. Torque curve similar too.

Two new roots for negative v present in VR too.



Nonlinear effects

- For $|\tilde{\psi}(r_w)|$ large enough and $|\gamma - ikv|$ small and $\Delta' \lesssim 0$ $|\tilde{\psi}(r_t)|$ and hence locked island can become large enough $W \sim \delta$ to enter the Rutherford regime.
- For $|\tilde{\psi}(r_t)|$ larger, the *Scott regime* can be entered, when $k'_{||} W c_s \sim \omega_r$. Island flattening (for $\omega_r = \omega_*$).



Conclusions

- For tearing modes with real frequencies, the peak response $\tilde{\psi}(r_t)$ is for weakly stable modes but also
- The peak response is for $v = \omega_r/k$ for the backward wave.
- Torque N_m is *zero* at $v = \omega_r/k$; for small driving torque, the fields lock to zero frequency, but the *plasma* locks to $v \gtrsim \omega_r/k$ rather than $v \gtrsim 0$. **Source of flow.**
- Effect seen for RI with $D < 0$ and parallel dynamics; Glasser effect and so $v = \omega_r/k$ in VR! There are two new flow roots, L stable, R unstable.
- Who cares? E.g. error fields with (m_1, n_1) and (m_2, n_2) w/ $m_1/n_1 \neq m_2/n_2$; plasma can lock to two different velocities at $q(r) = m_1/n_1$ and $q(r) = m_2/n_2$ and with *NTV* can lead to smooth rotation shear in between.

Conclusions

- In other regimes, there are real frequencies, often $\omega_r \propto \omega_*$, not c.c.
- A point about regimes: the unlocked (high-slip) state can be in one tearing regime and the locked state in another (or even be nonlinear).
- For slow flow, e.g. $v/v_A \sim 10^{-3}$ and very stable, locked state has small $\tilde{\psi}(r_w)$ and small islands, $W \sim \delta$ – Rutherford regime. These calculations with $v \gtrsim \omega_r/k$ are still qualitatively OK.
- For locked state with $\tilde{\psi}(r_w)$ even larger (larger flow or larger Δ'), Scott regime $k'_{||} W c_s \sim \omega_r$ ... pressure gradient flattens due to sound wave and propagation slows.
- For ω_r due to pressure-curvature, does a Scott-effect occur? Probably.