Extended-MHD Modeling of Tokamak Disruptions and RWMs with M3D-C1

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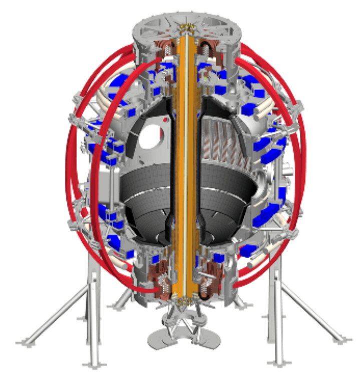


Simulations Seek to Model Onset and Consequences of Disruptive Instabilities

- Stability of disruptive modes often involve plasma / wall interaction
  - Locked Modes
  - Resistive Wall Modes (RWMs)
  - Vertical Displacement Events (VDEs)

- Evolution of disruptions involves large displacement of plasma and penetration of flux through wall

- Both linear stability and nonlinear evolution of disruptive instabilities are naturally modeled with resistive-MHD codes

New Capabilities Have Been Developed in M3D-C1 To Model Disruptions

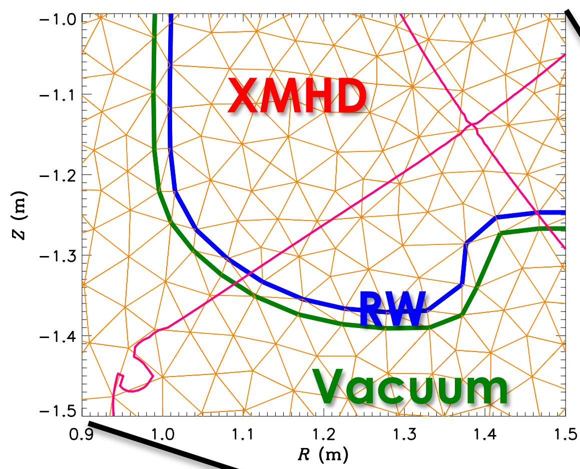
- Resistive wall model
  - Walls of arbitrary thickness
  - Allows current into, out of, and through wall
  - Allows non-axisymmetric resistivity (e.g. ports)

- Capability to switch between 2D Nonlinear / Linear Stability / 3D Nonlinear Calculations

- Improved meshing and modeling of open field-line region

Resistive Wall Model In M3D-C1 Includes Wall Inside Simulation Domain

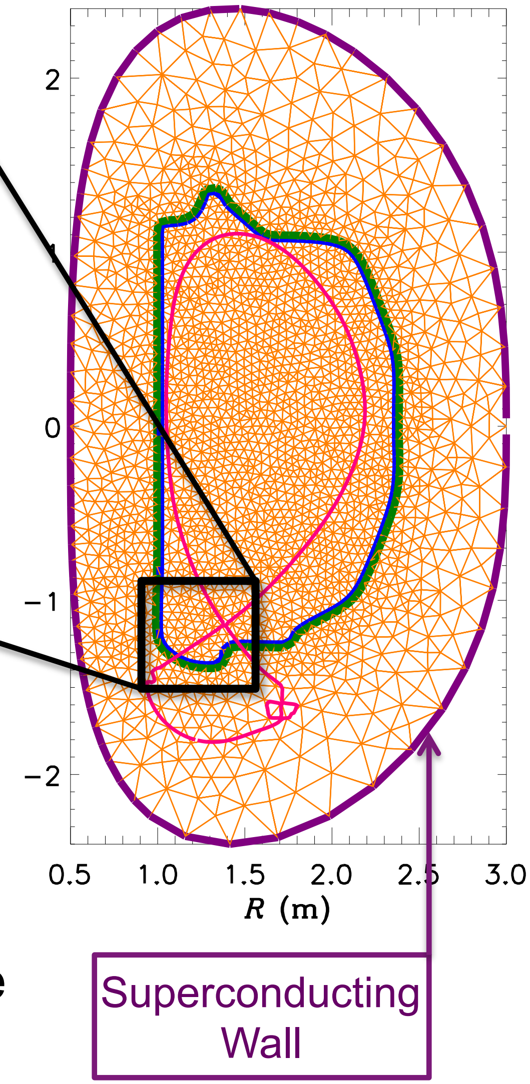
- 3 regions inside domain:
  - XMHD (Extended MHD, includes open field-line region)
  - RW ($\mathbf{E} = \eta_W \mathbf{J}$)
  - Vacuum ($\mathbf{J} = 0$)



- Boundary conditions:
  - $\mathbf{v}$, p, n set at inner wall
  - $\mathbf{B}$ set at outer (superconducting) wall

- *<u>There are no boundary conditions on</u>* B *<u>or</u>* J *<u>at the resistive wall</u>*
  - Current can flow into and through the resistive wall

- All regions advanced simultaneously with implicit time step



Resistive Single-Fluid Model is Considered Here

$$\frac{\partial n}{\partial t} + \nabla \cdot (n_i \mathbf{v}) = 0$$

$$n_i m_i \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_i$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{v} = -(\Gamma - 1) \nabla \cdot \mathbf{q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}$$

$$\Pi_i = -\mu \left[\nabla \mathbf{v} + (\nabla \mathbf{v})^T \right]_Z$$

$$\mathbf{q} = -\kappa \nabla T_i - \kappa_{\|} \mathbf{b}\mathbf{b} \cdot \nabla T_e$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\Gamma = 5/3$$

$$n_e = Z_i n_i$$

- (R, φ, Z) coordinates → no coordinate singularities in plasma
- Three modes of operation:
  - Linear, time-independent (**perturbed equilibrium – not discussed here**)
  - Linear, time-dependent (**linear stability**)
  - Nonlinear, time-dependent (**nonlinear dynamics**)

Resistive Wall Modes

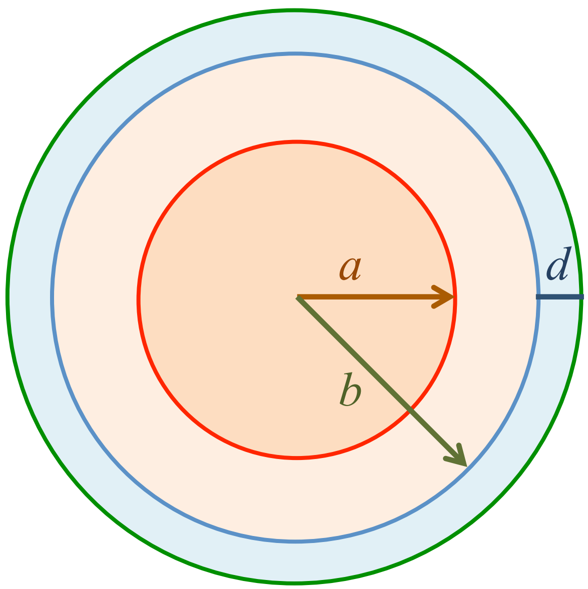
- Code validation for arbitrary wall thickness
- Rotational stabilization and comparison to theory

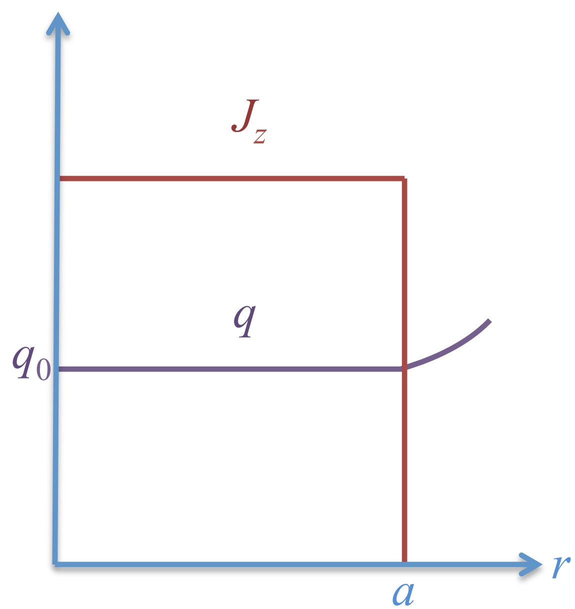
Resistive Model Verified Against Analytic Resistive Wall Mode Result

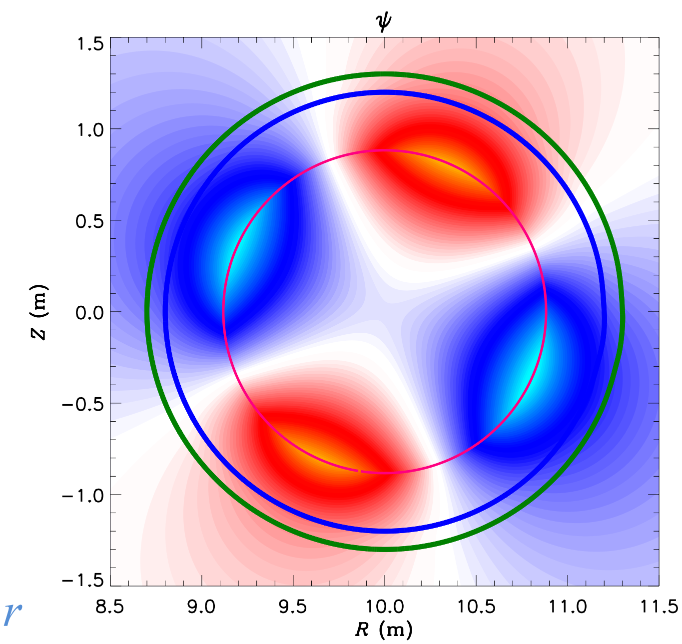
- Circular cross-section, cylindrical plasma with constant q, current density (J_z) and mass density (ρ_0) (Shafranov equilibrium)

- Analytic thin-wall solution provided by Liu *et al.* *Phys. Plasmas* **15**, 072516 (2008)

Wall time: $\tau_W = \mu_0 bd/(2\eta_W)$
Alfven time: $\tau_A = (\mu_0 \varrho_0)^{1/2} R_0/B_0$



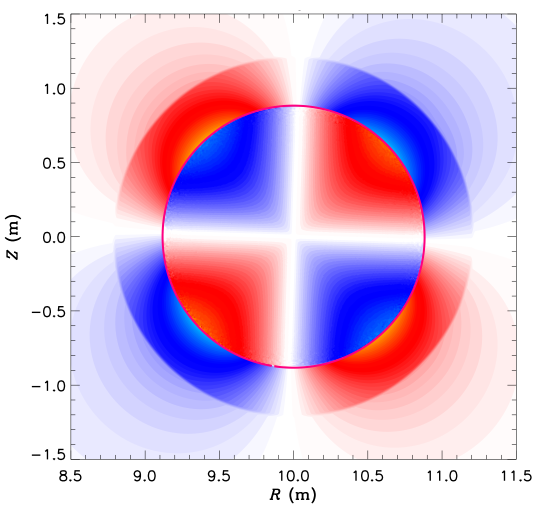


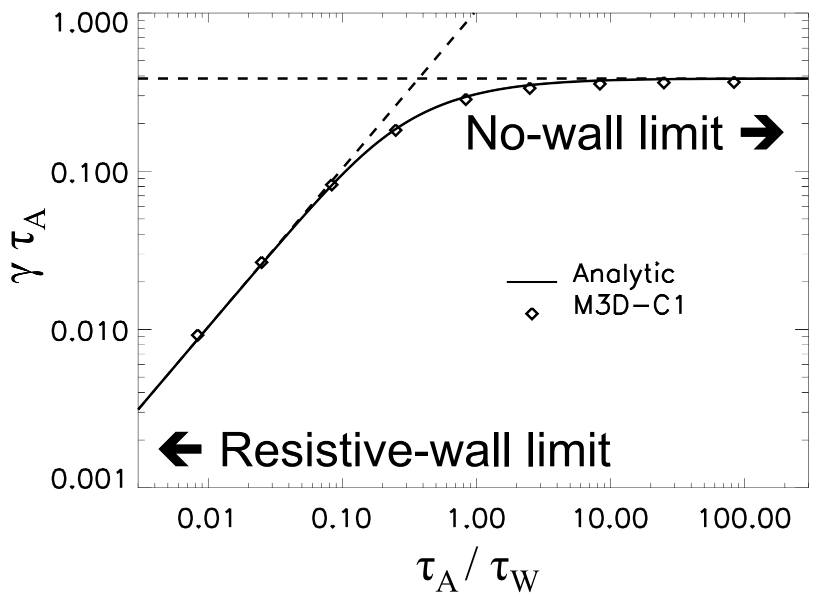


M3D-C1 Reproduces Analytic RWM Result in Thin Wall Limit

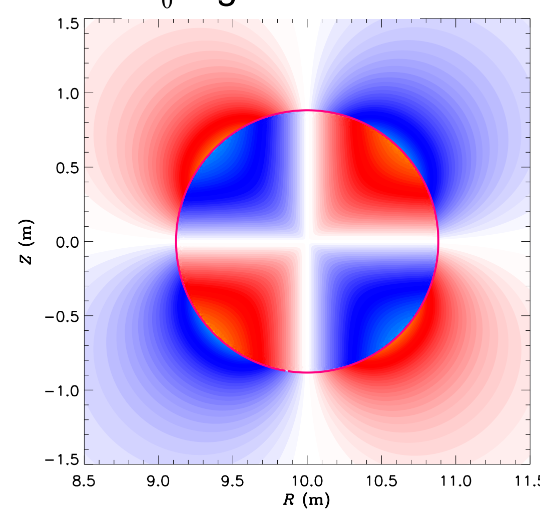
- Growth rate calculated using linear, time-dependent calculation
- M3D-C1 agrees with analytic growth rate in both resistive-wall ($\tau_A \ll \tau_W$) and no-wall ($\tau_W \ll \tau_A$) limits

Resistive-Wall Limit
B_θ Eigenfunction





No-Wall Limit
B_θ Eigenfunction



M3D-C1 Model Verified For Arbitrary Wall Thickness

- Allowing arbitrary wall thickness leads to straightforward modification of Liu *et al.* (thin wall) dispersion relation

$$\frac{\nu}{m-nq_0} - \frac{1}{1-(a/b)^{2\mu}F} = \frac{(\gamma\tau_A)^2}{2}\frac{q_0^2}{(m-nq_0)^2}$$

$$\mu = |m| \qquad \alpha = \sqrt{2\gamma\tau_W b/d}$$

$$\nu = \mathrm{sgn}(m) \qquad \beta = (1+d/b)\alpha$$

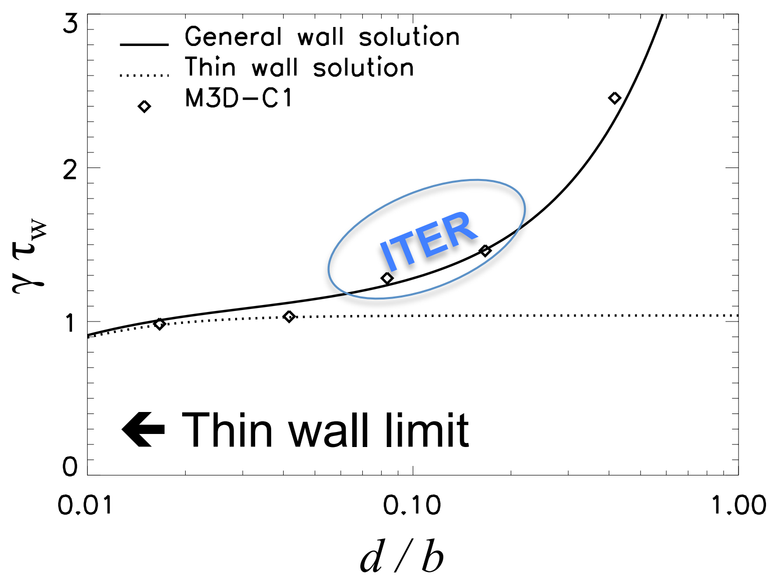
Thin wall ($d \ll b$)

$$F \rightarrow \frac{\gamma\tau_W}{\gamma\tau_W + \mu}$$

General solution

$$F = \frac{I_{\mu-1}(\beta)K_{\mu-1}(\alpha) - I_{\mu-1}(\alpha)K_{\mu-1}(\beta)}{I_{\mu-1}(\beta)K_{\mu+1}(\alpha) - I_{\mu+1}(\alpha)K_{\mu-1}(\beta)}$$

- In thick wall, skin depth limits eddy current depth
  - Weaker eddy currents than in thin wall approximation, which assumes radially uniform current in wall
- M3D-C1 model in good agreement with analytic results for arbitrary wall thickness
- In ITER, $(\gamma\tau_W)(d/b) \sim 0.2$ *
  - Growth rates ~ 20—50% larger than thin wall solution



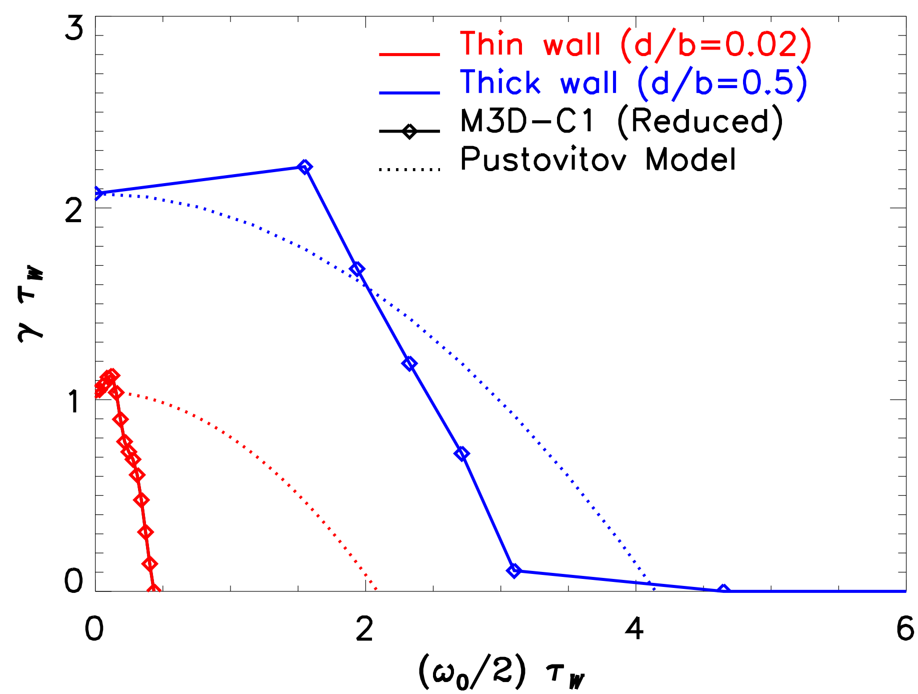
* F. Villone et al. *Nucl. Fusion* **50**, 125011 (2010)

Complete Rotational Stabilization of RWM Observed

- Reduced-model (two-field) calculations show stabilization of RWM by toroidal rotation
  – $\omega = \omega_0 (1 - \psi_N)$

- Qualitative agreement with Pustovitov model*
  – $\gamma = \gamma_0 [1 – (\omega/\omega_c)^2]$ where γ_0 is the growth rate with no rotation and $\omega_c = 2\gamma_0/n$
  – Pustivitov model derived in thick wall limit with uniform rotation

- Work is now ongoing to extend this to full extended-MHD model



*Pustovitov *Nucl. Fusion* **53** (2013) 033001

Vertical Displacement Events

- Axisymmetric simulations
  - Current spike
  - Halo currents
  - Axisymmetric wall forces
  - q-profile evolution

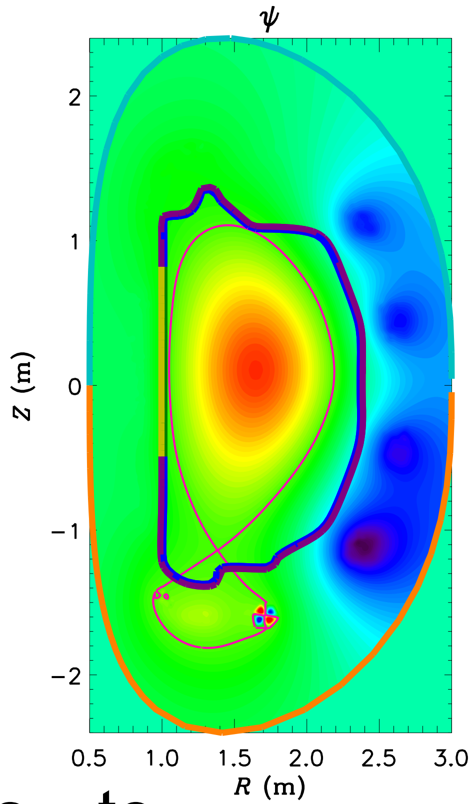
- Non-axisymmetric simulations
  - Linear stability
  - Non-axisymmetric evolution

Disruption Simulations Initialized using Vertically Unstable EFIT Reconstructions

- Nonlinear calculations use fairly realistic plasma parameters
  - Spitzer resistivity: $S_0 \approx 6.8\times10^7$
  - Anisotropic thermal conductivity:

$$\chi_{\parallel}/\chi_{\perp} = 10^6$$

- RW region approximates first wall, not vacuum vessel here



- Cold-VDE calculations have anomalous χ to cause TQ before vertical instability
- Hot-VDE calculations have lower χ and remain hot until after plasma touches wall

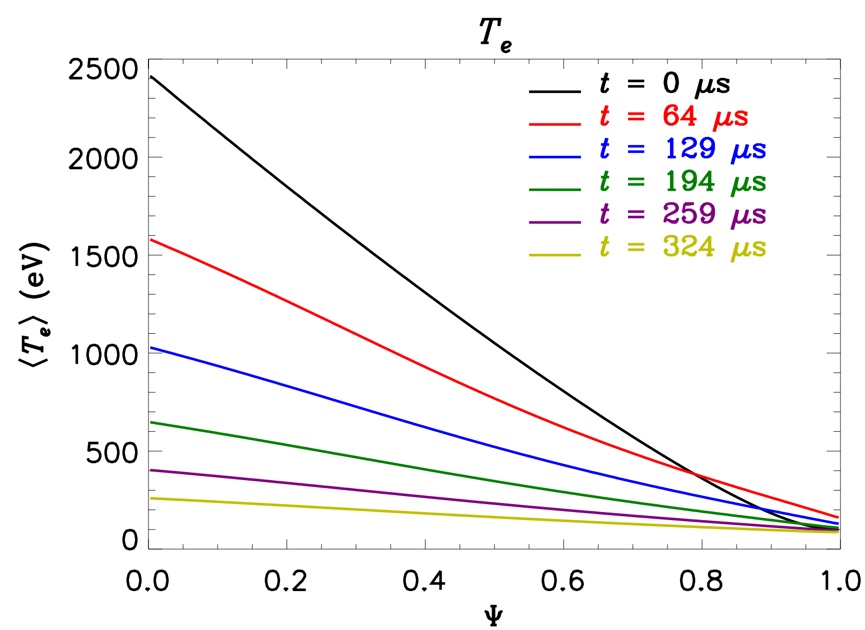
"Cold-VDE" Features Thermal Quench Before Vertical Instability

- Thermal Quench (TQ) is modeled by including anomalous thermal conductivity

$$100 < \chi_{\perp} < 800 \text{ m}^2/\text{s}$$

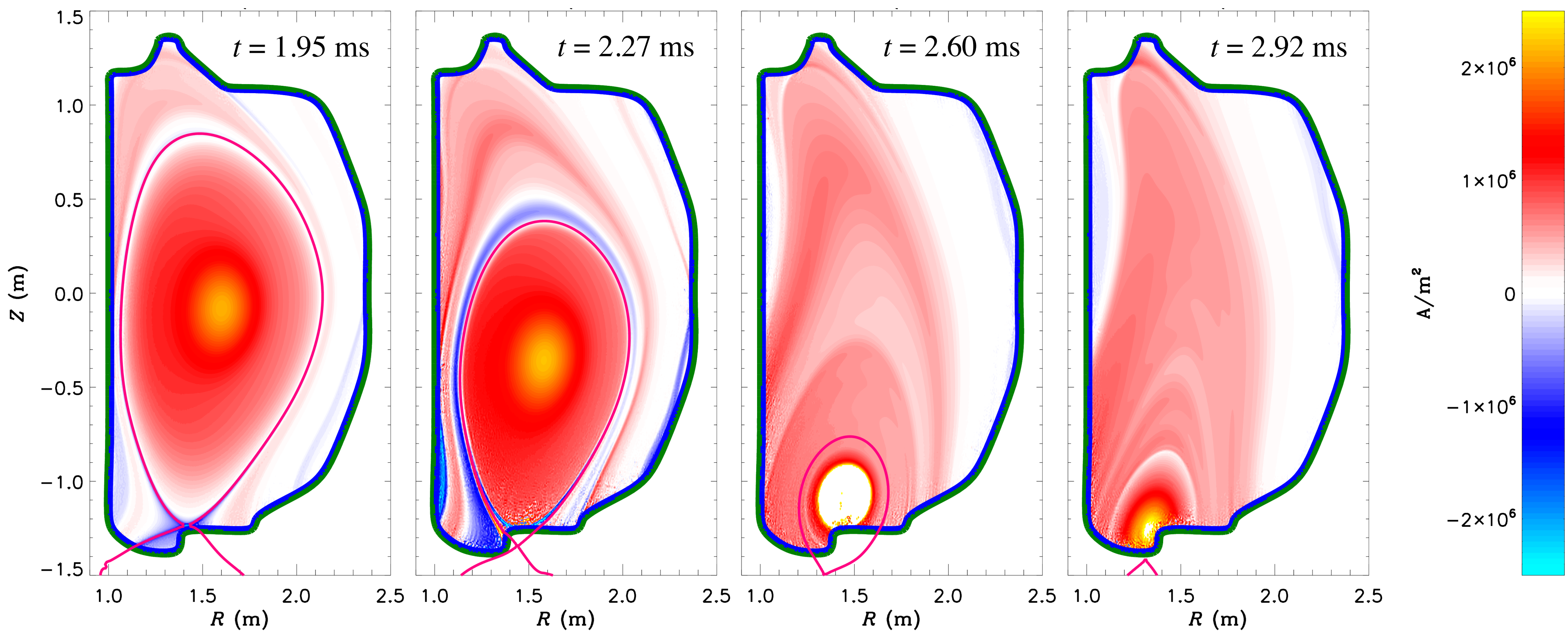
- Thermal quench happens on $\sim 100\ \mu\text{s}$ timescale

- TQ phase not meant to be physically realistic. We are interested in current quench (CQ) phase

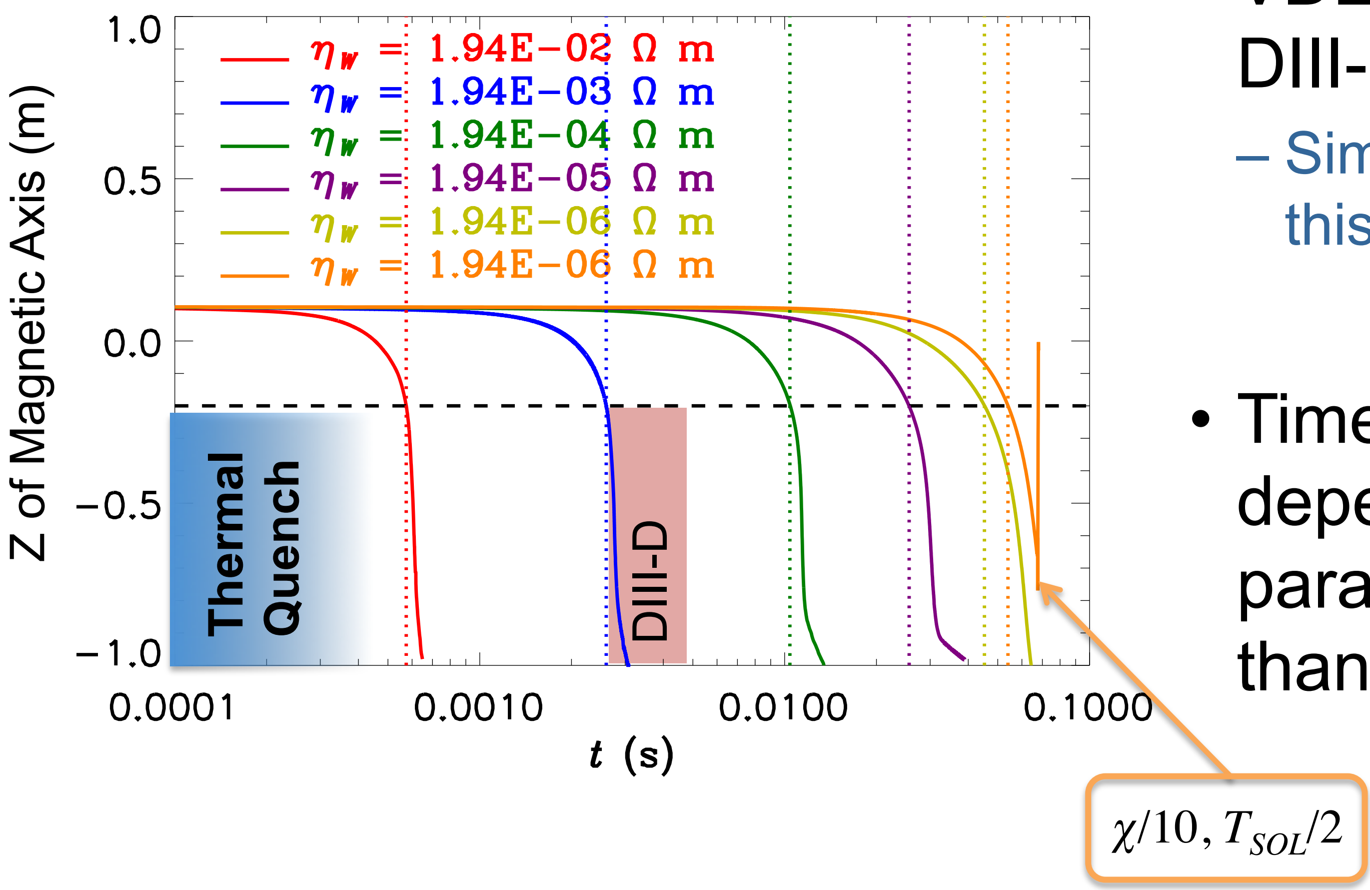


Strong Currents Form in Halo Region; Response Currents form in Wall and SOL

- Both co-I_P and counter-I_P currents are seen in the open field-line region



Timescale of VDE Is Determined by Wall Resistivity (η_W)



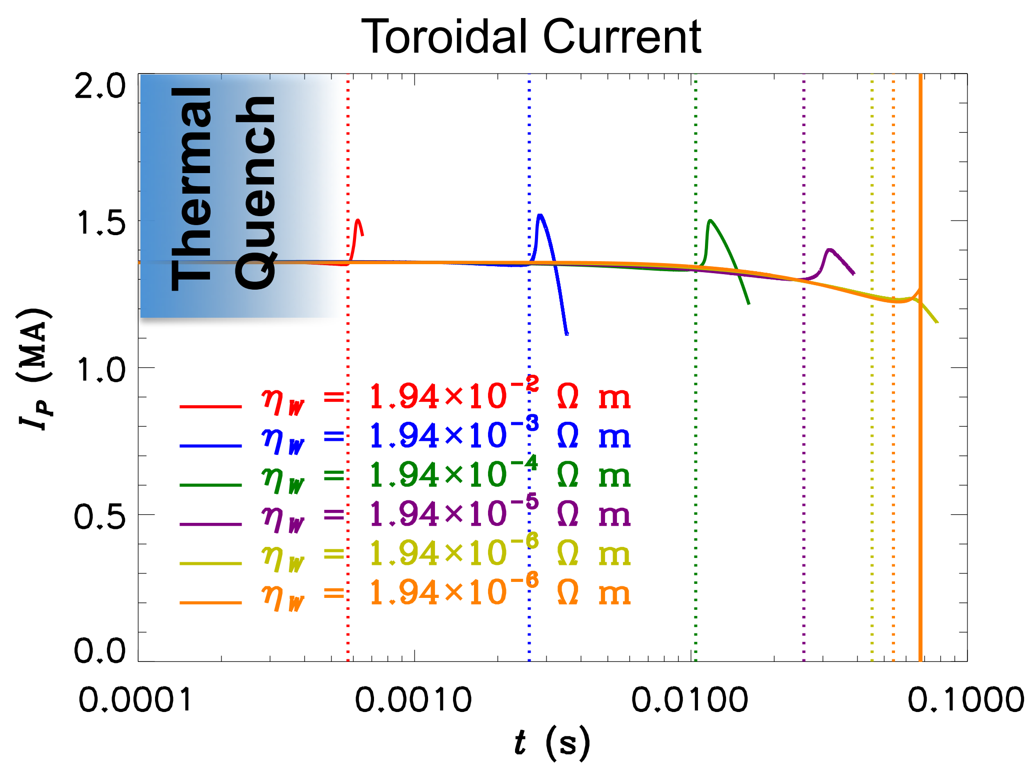
- Physically realistic VDE timescale in DIII-D is a few ms
  - Simulations bracket this regime
- Timescale weakly dependent on parameters other than η_W

Current Spike Observed Just Before Current Quench; Related to Vertical Motion of Plasma

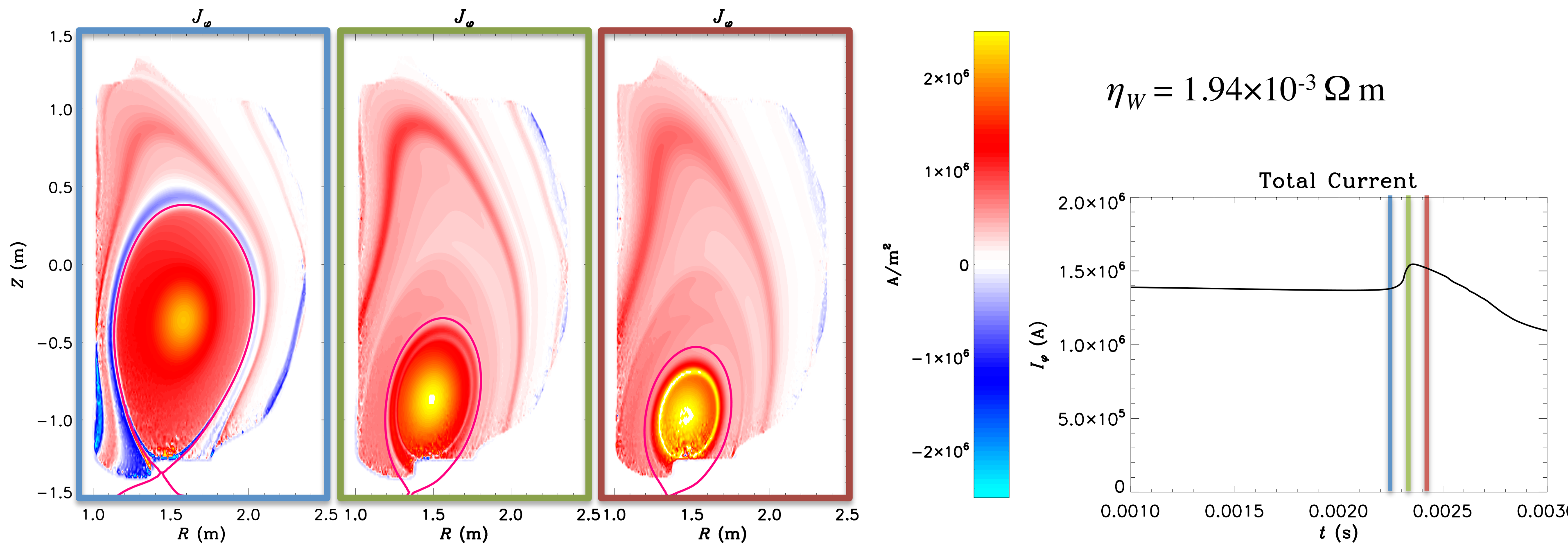
- Current spike occurs soon after plasma makes contact with the wall

- There is no spike associated with the thermal quench

- Spike is smaller when $\eta_W < \eta_{SOL}$



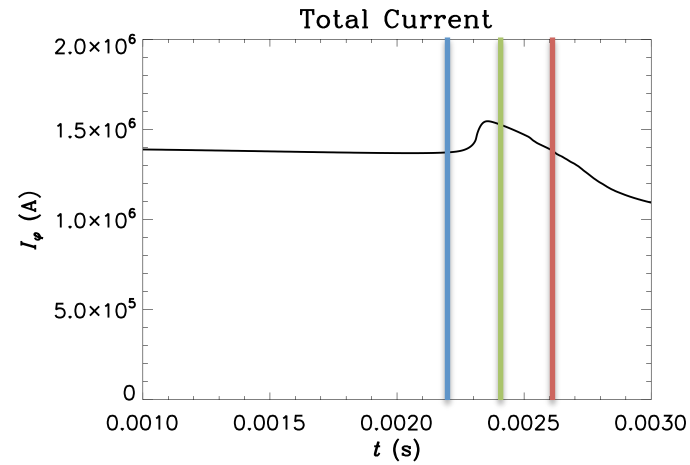
Current Spike Results from Loss of Induced Counter-I_P Currents When Plasma Contacts Wall



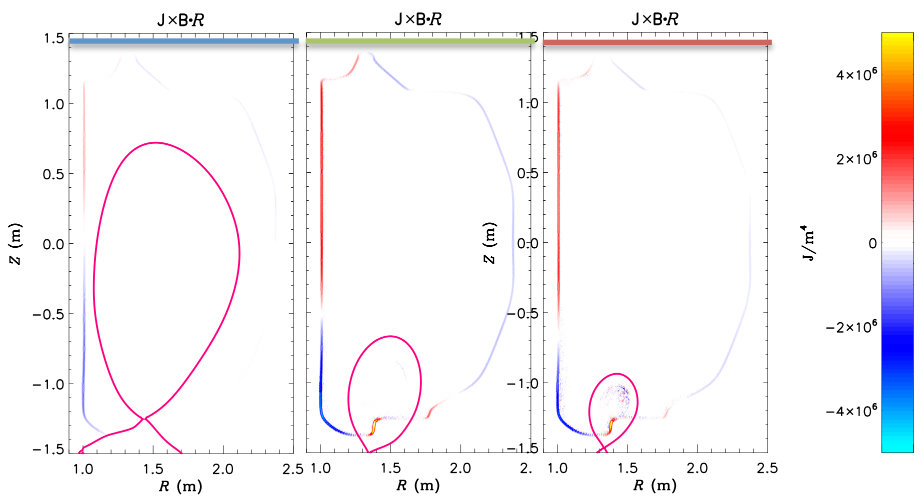
- Counter-I_P response currents are induced by motion of leading edge of plasma
- When plasma contacts wall, these currents quickly dissipate
- Eventually (after spike), toroidal current in wall flips sign to oppose I_P decay

Axisymmetric Forces Reach Maximum Just After Current Spike

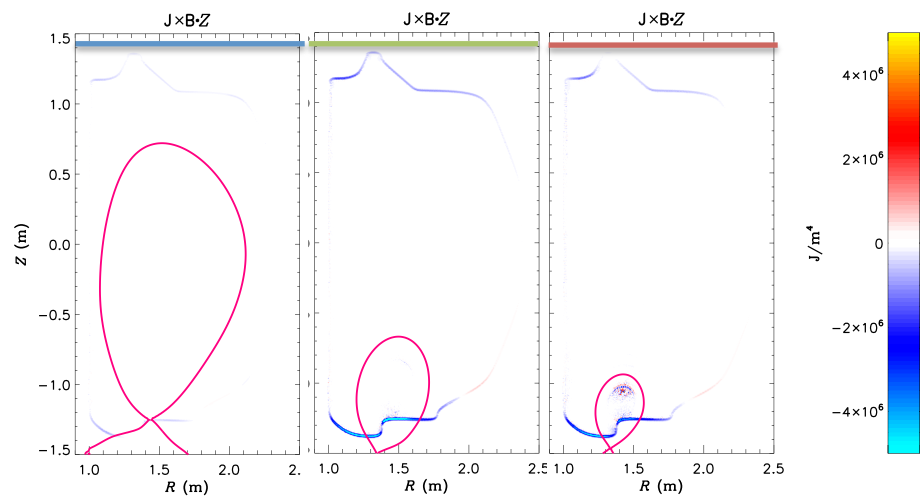
- Axisymmetric forces peak at ~100 kN /m^2
- Force distribution does not evolve significantly
- Currents in plasma are strong, but mostly force-free



Radial J×B Force



Vertical J×B Force



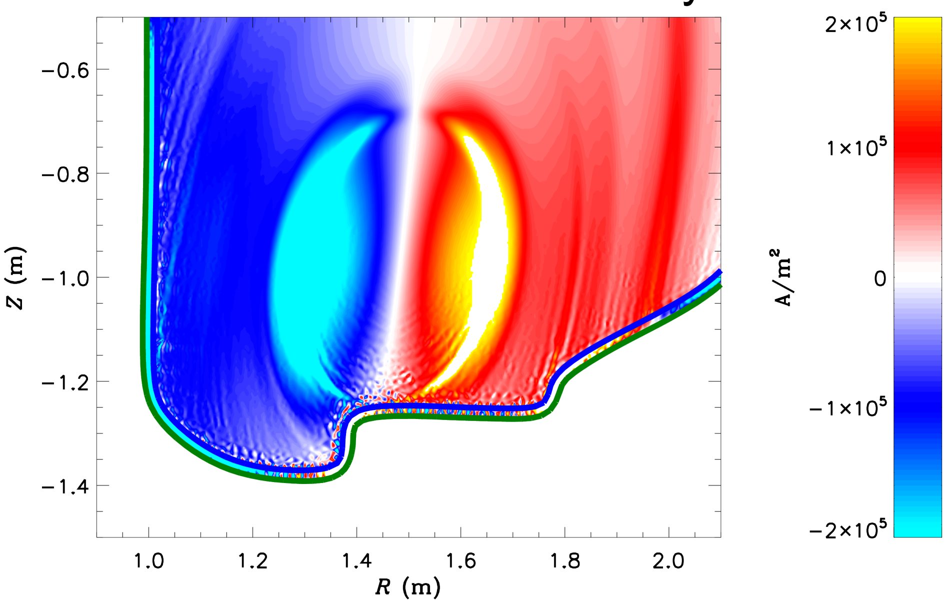
Maximum Axisymmetric Halo Currents and Wall Force Depend Weakly on η_W

- Halo currents can exceed 100 kA/m^2; observed both on divertor floor and center post
  - Distribution likely depends on temperature (resistivity) of open field-line region

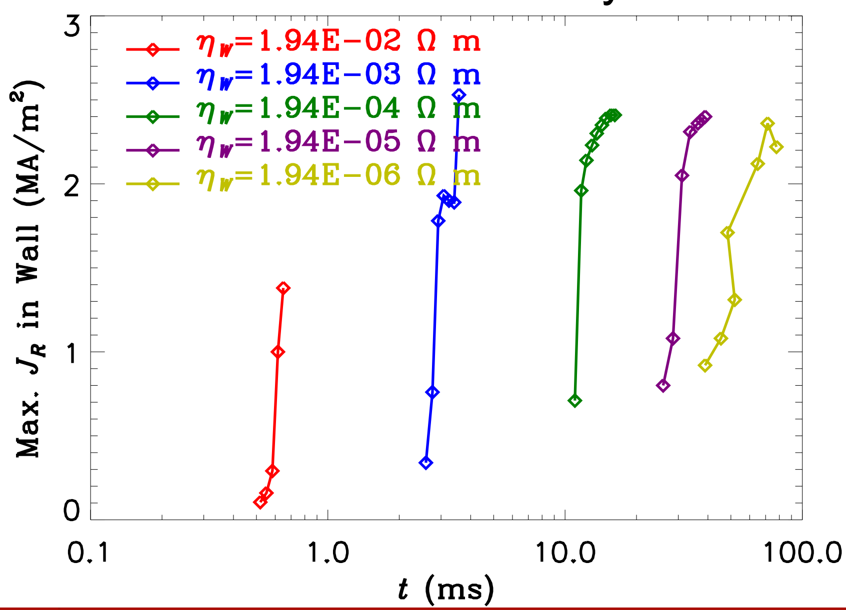
- Maximum Halo currents and force density in the wall is only weakly dependent on wall resistivity

- Impulse to vessel increases with τ_W because force is applied for longer time

Vertical Current Density



Radial Current Density in Wall



3D Evolution Depends on Thermal History of Plasma

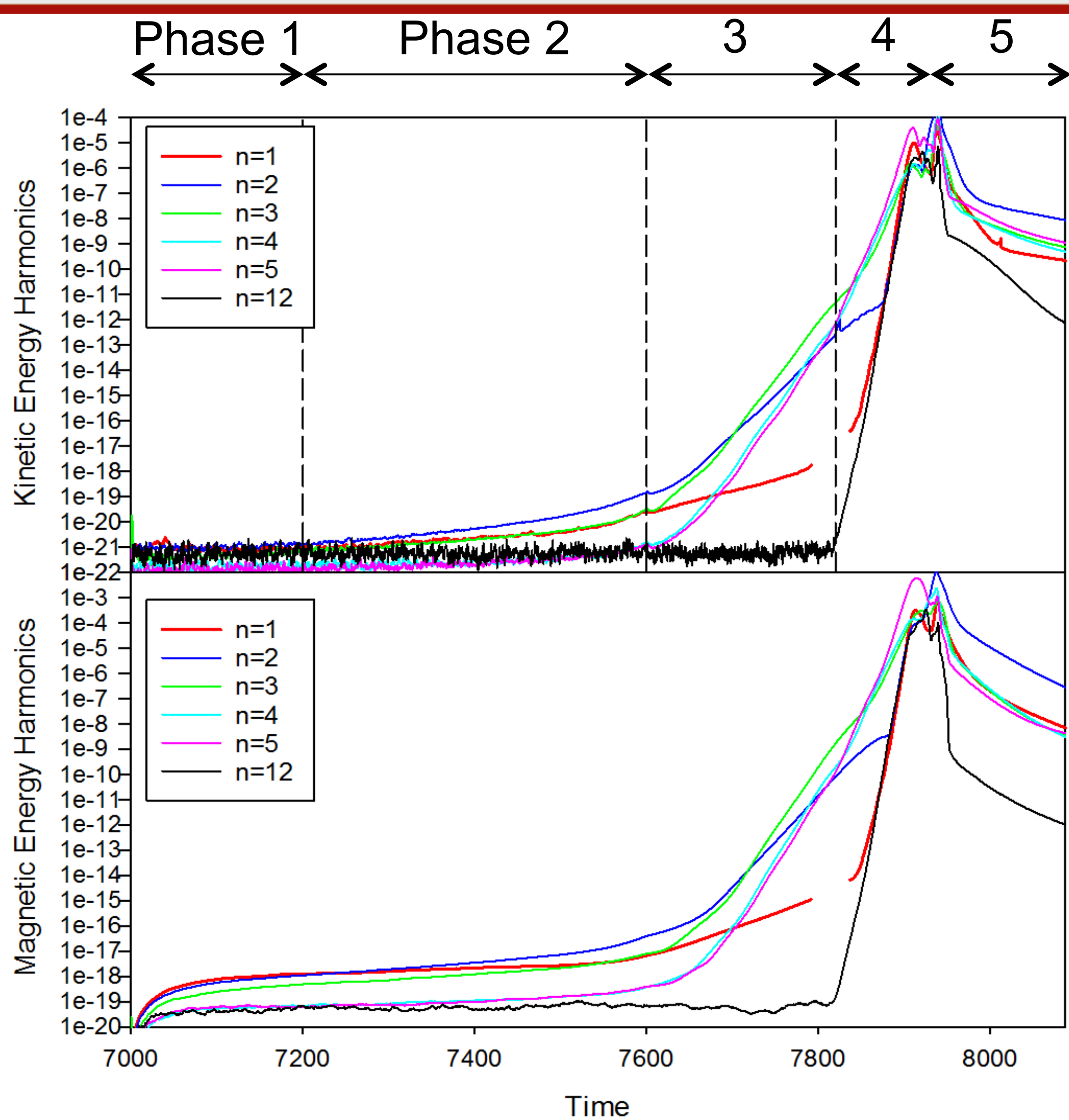
- Two competing effects determine q_{edge} once plasma is limited:
  1. q_{edge} drops as plasma shrinks and is scraped off by limiter
  2. q_{edge} rises because of resistive decay of I_P

- In cold-VDE (TQ happens before VDE), resistive decay is fast and q_{edge} rises
  - Plasma remains stable to $n > 0$ MHD

- In hot-VDE (no TQ before VDE), resistive decay is slow and q_{edge} drops
  - Plasma eventually becomes unstable to $n > 0$ MHD
  - $n > 0$ instability potentially causes strong Halo currents, wall forces, and TQ

- 3D simulations are expedited by testing linear stability of 2D simulations; then turning on 3D model when instability is found

3D Nonlinear Hot-VDE Calculation Shows Development and Saturation of 3D Modes



<u>Phase 1:</u>
Axisymmetric

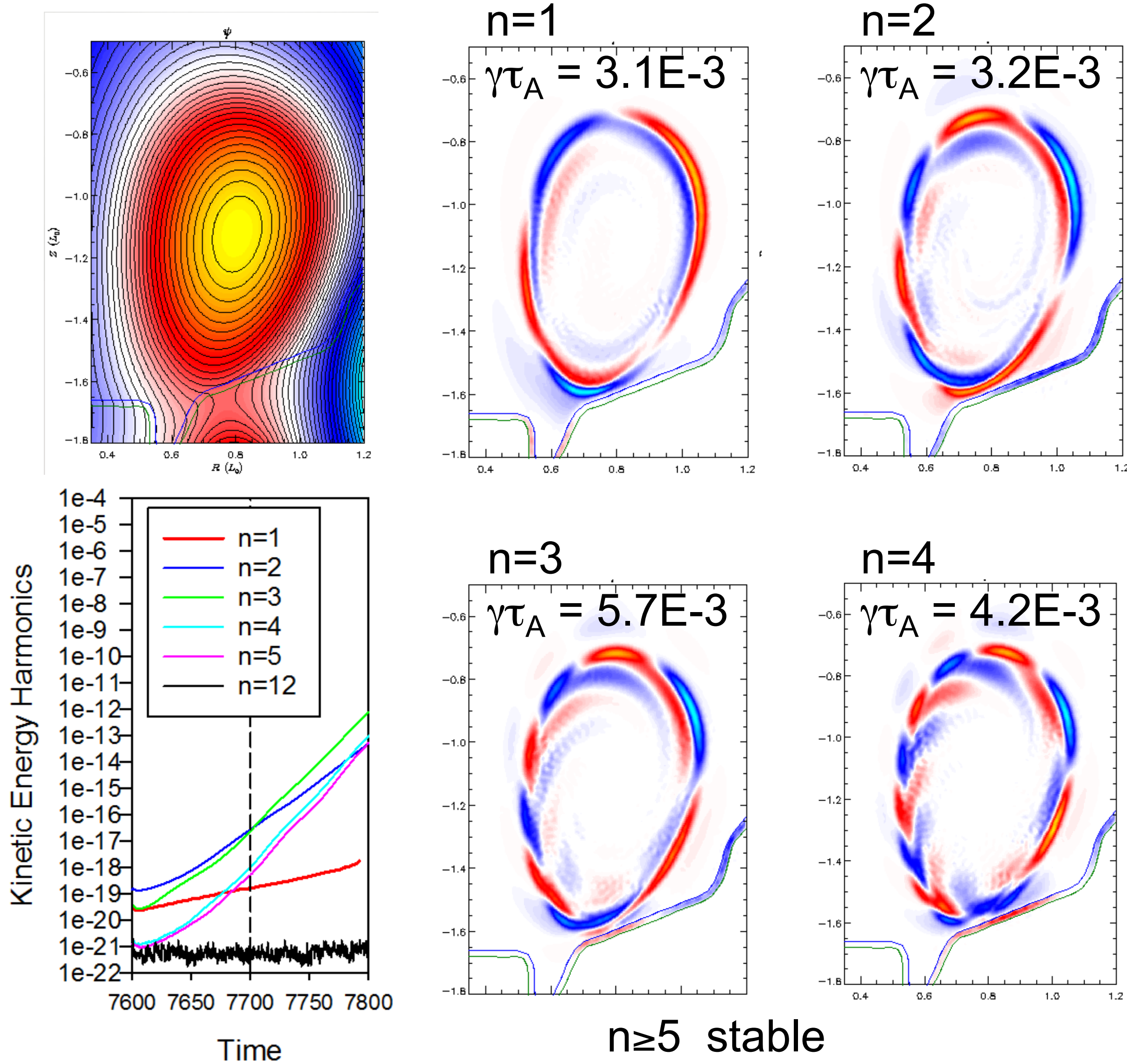
<u>Phase 2:</u>
n=2 tearing? mode dominates

<u>Phase 3:</u>
n=3 tearing? mode begins to dominate

<u>Phase 4:</u>
n=1 and higher-n modes begin to grow

<u>Phase 5:</u>
Plasma gets scraped off and strongly wall stabilized

Linear Stability Analysis Finds Agreement With Nonlinear Calculation



- Linear stability of axisymmetric solution is calculated at $t = 7700\ \tau_A$
  - Evolution of q profile in 2D and 3D cases is nearly identical
- Linear stability finds unstable low-n modes before nonlinear calculation does
- Growth rates are relatively small

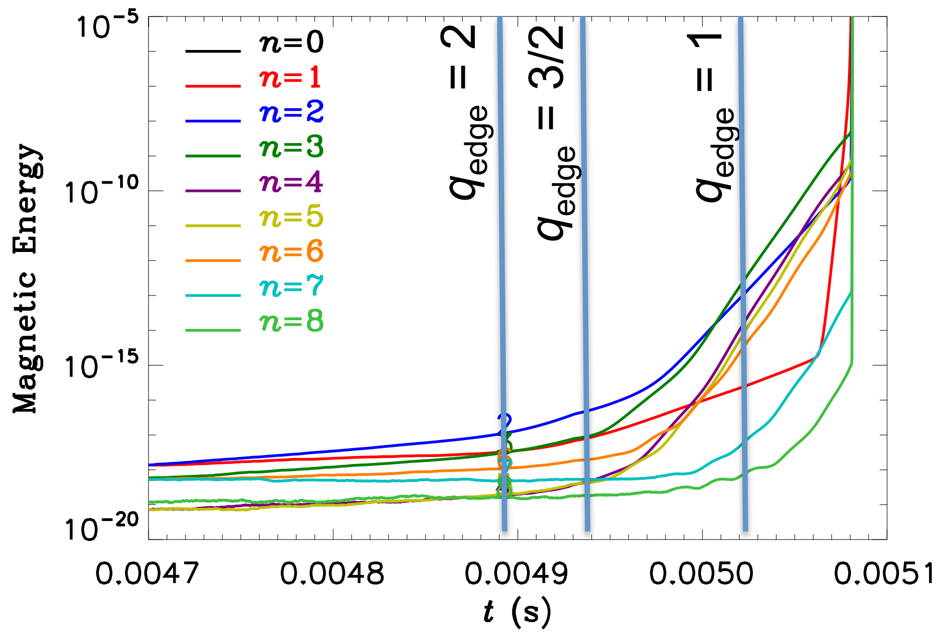
In Hot-VDE Simulations, $q_{\mathrm{edge}} < 1$ Before Non-Axisymmetry Becomes Significant

- Non-axisymmetric modes start growing when $q_{\mathrm{edge}}=2$, but are still at small amplitude when $q_{\mathrm{edge}}=1$

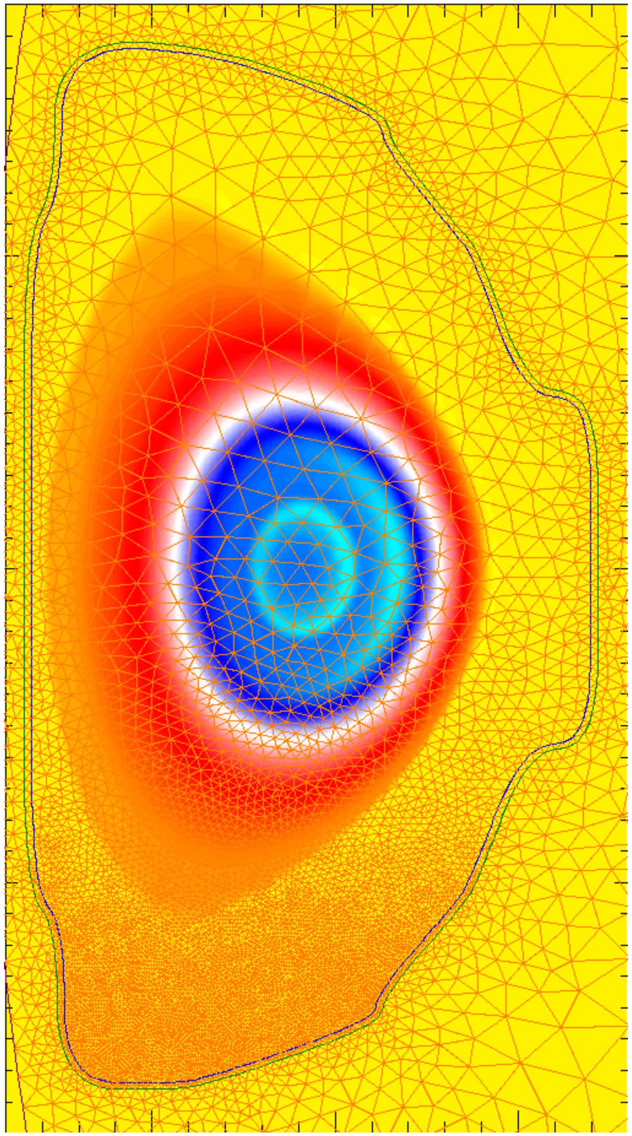
- q_0 is still > 1, so shear is reversed when $q_{\mathrm{edge}}=1$

- Plasma seems unexpectedly stable
  - No strong instability when $q_{\mathrm{edge}} = 2$
  - 1/1 instability onsets after $q_{\mathrm{edge}} = 1$

- Linear calculations confirm that high SOL temperature is responsible for stability



Higher Resolution Meshing will Improve Treatment of SOL



- Practical limit on SOL temperature is set by Te gradient between wall and plasma
- Higher resolution will better resolve this gradient and allow lower temperature in open field-line region
- Artificially increasing the resistivity in the open field-line region has similar effect, but is less self-consistent

Summary

- M3D-C1 provides powerful capability to model disruptive instabilities
  - Linear & nonlinear plasma evolution with resistive walls of arbitrary thickness
- Complete stabilization of RWM is found above critical rotation frequency
  - Reduced model was used here; study with full model is underway
- Current spike always seen in VDE simulations; associated with plasma hitting wall and not with thermal quench
- Axisymmetric forces and halo currents quantified
  - Calculations make no assumptions about Halo region width
  - Axisymmetric forces depend weakly on wall resistivity
- Non-axisymmetric stability of VDE depends on temperature: “cold-VDEs” remain kink stable, while “hot-VDEs” develop kink instability
  - Kink instability sensitive to SOL temperature (resistivity)
- Extensive validation with halo current measurements is planned!

Extra Slides

Linear Growth Rates are Sensitive to SOL Temperature

- The resistivity η_{SOL} in the open field-line region was varied artificially to be consistent with a range of T_{SOL}
- Growth rates are higher at lower T_{SOL} (i.e. higher η_{SOL})

T_{SOL} (eV)	$\gamma_{n=1}\tau_A$	$\gamma_{n=2}\tau_A$	$\gamma_{n=3}\tau_A$	$\gamma_{n=4}\tau_A$	$\gamma_{n=5}\tau_A$
25	.00307	.00321	0.00571	0.00419	0
20	.00352	.00396	0.00689	0.00476	0
15	.00404	.00539	0.00899	0.00593	0
10	.00488	.00787	0.01222	0.00772	0
5	.00650	.01295	0.0181	0.0108	0