Electron Cyclotron Emission (ECE) Radiation of Runaway Electrons

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Outline

- Motivation: Non-thermal ECE signal observed in runaway electron experiments in tokamak
- Introduction to Electron Cyclotron Emission (ECE)
  - Optical thickness
  - Reciprocity theorem
- Runaway electron (RE) growth and distribution in momentum space
- Synthetic diagnostic calculation of RE ECE radiation
  - Absorption and emission of ECE by RE
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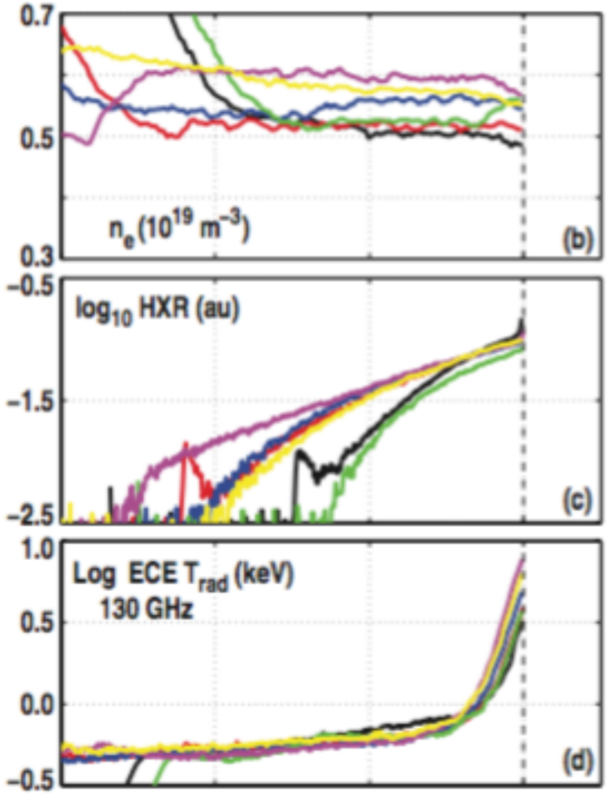
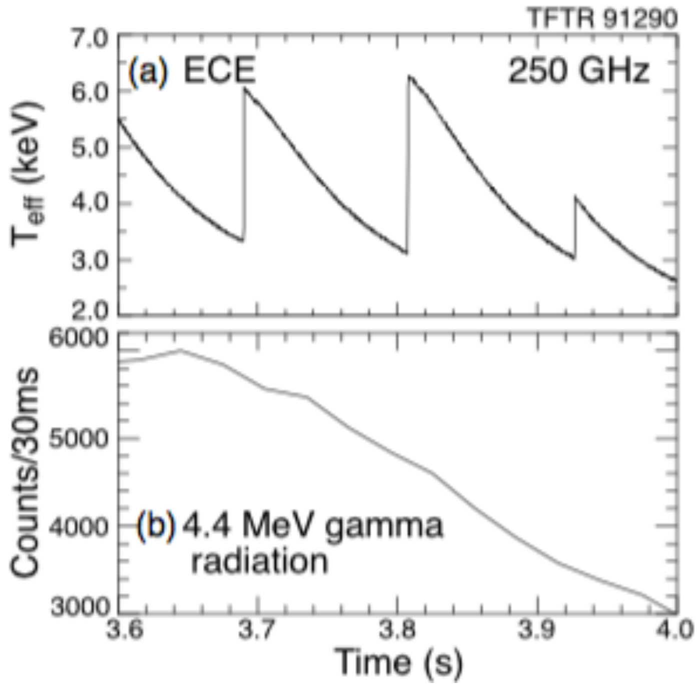
Motivation: Non-thermal ECE signal observed in runaway electron experiments in tokamak

- In both quiescent runaway electron experiments (QRE) in DIII-D, and in disruption experiments in TFTR, fast growth of high harmonics ECE signal has been observed.

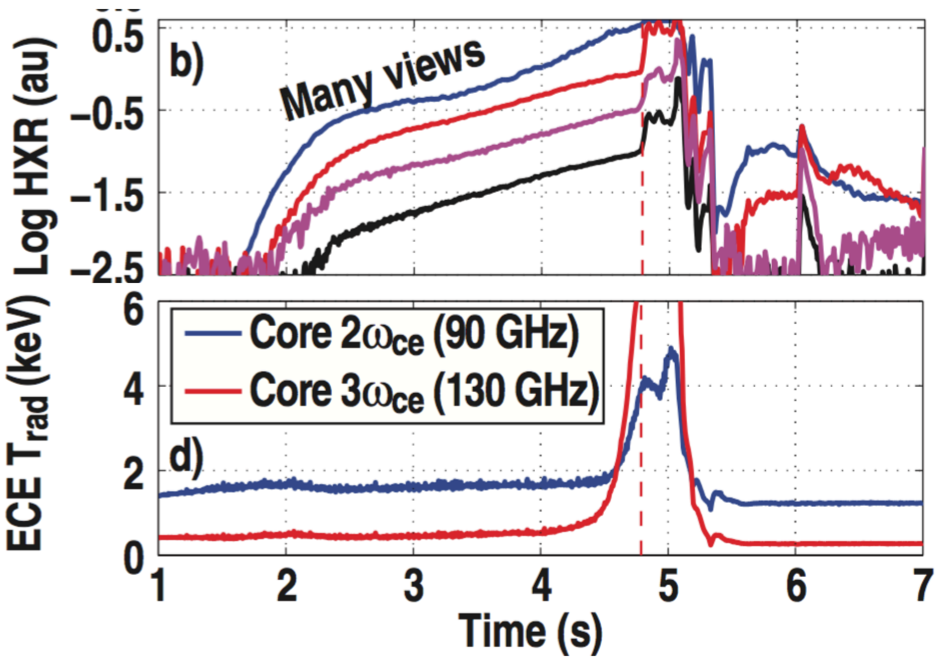


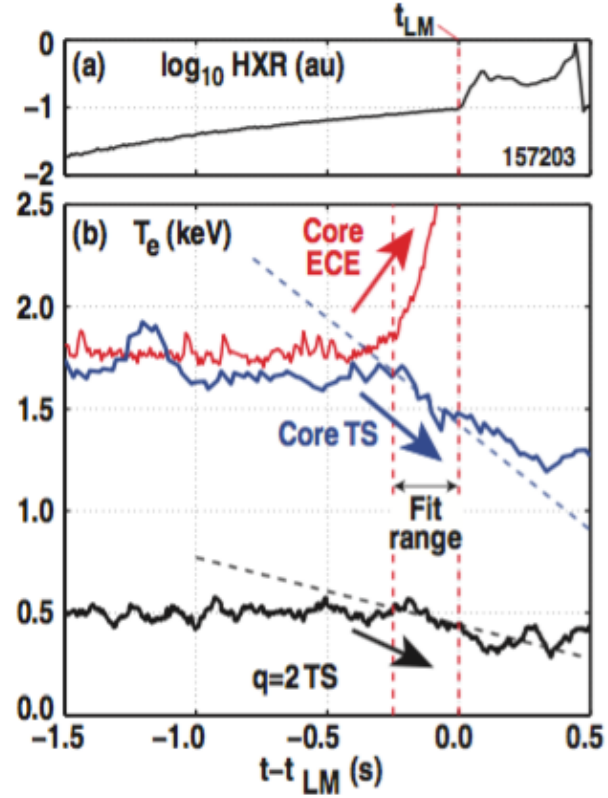


C. Paz-Soldan et al., Nucl. Fusion 56, 056010 (2016).
E.D. Fredrickson, M.G. Bell, G. Taylor, and S.S. Medley, Nucl. Fusion 55, 013006 (2015).

Motivation: Non-thermal ECE signal observed with runaway electron experiments in tokamak (cont'd)

- The non-thermal ECE signal comes from runaway electron in plasma.





- REs can give remarkable contribution to ECE, given their small population (~10^{-4} n_e).
- The contribution to high harmonics is more prompt than low harmonics.
- The growth rate of ECE signal is much higher than HXR growth rate.

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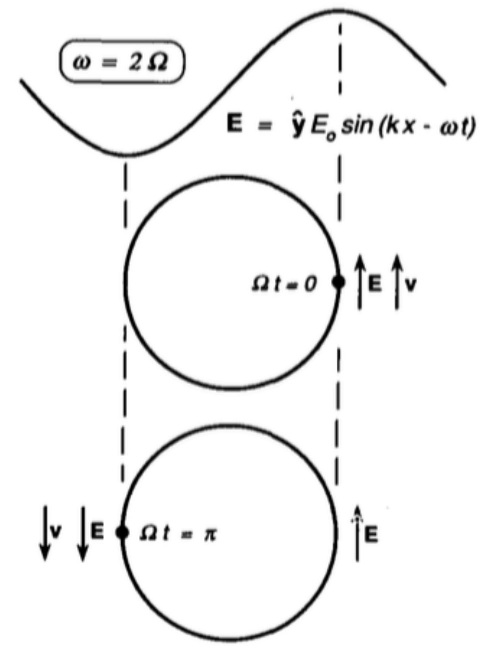
Absorption and Emission of Electron Cyclotron Wave

ECE Resonance condition: $\omega - k_{\|} v_{\|} - \dfrac{n\omega_{ce}}{\gamma} = 0$



- The absorption of the ECE can be represented by ϵ^{A} (anti-Hermitian part of permittivity tensor)
- The emission of ECE can be represented by source current correlation tensor K

$$\underline{\underline{K}}_{\omega,k} 4\pi\delta(\omega - \omega')\delta(k - k') = < \vec{j}_{\omega,k} \vec{j}_{\omega',k'} >$$

  - Both terms come from the electrons satisfying the resonance condition.

- For thermal equilibrium (Maxwellian distribution), the ratio of K and ϵ^{A} is proportional to T_{e}, a result of Fluctuation-Dissipation theorem (or Kirchhoff's law),

$$\underline{\underline{K}}_{\omega,k} = \omega T_e \underline{\underline{\epsilon}}^{A}_{\omega,k} / \pi^2$$

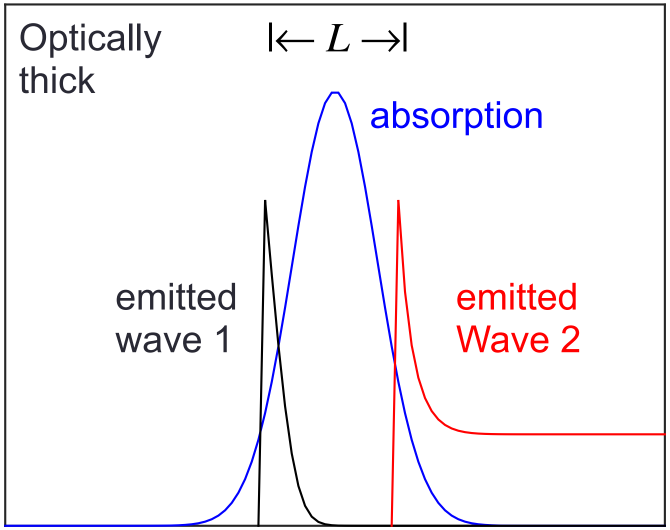
Waves in Plasmas, T. H. Stix

Optically thick vs. optically thin

- For ECE, plasma can be considered as optically thick if absorption length is much smaller than scale length of resonance region.

$$1/k_i \ll L$$

  - For optically thick case, the wave power radiated out is emission/absorption, which is only related to the local T_e at resonance point.
  - For optically thin case, the wave power is related to n_e, T_e, and the resonance region length L.



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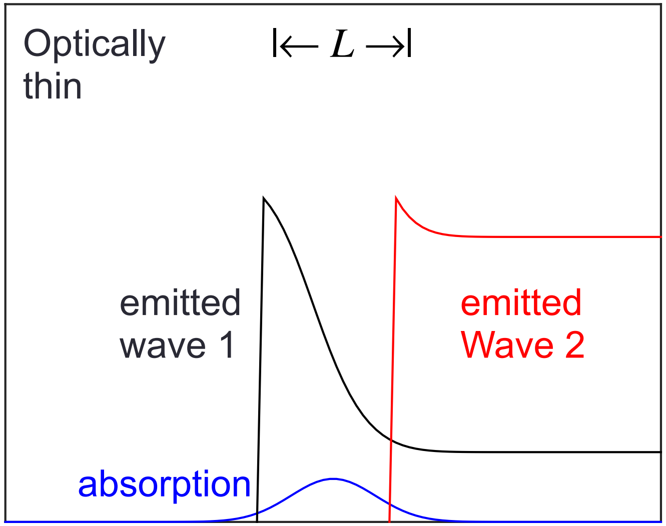

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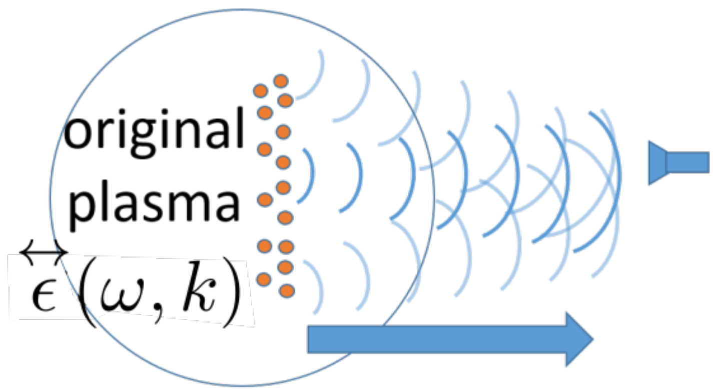
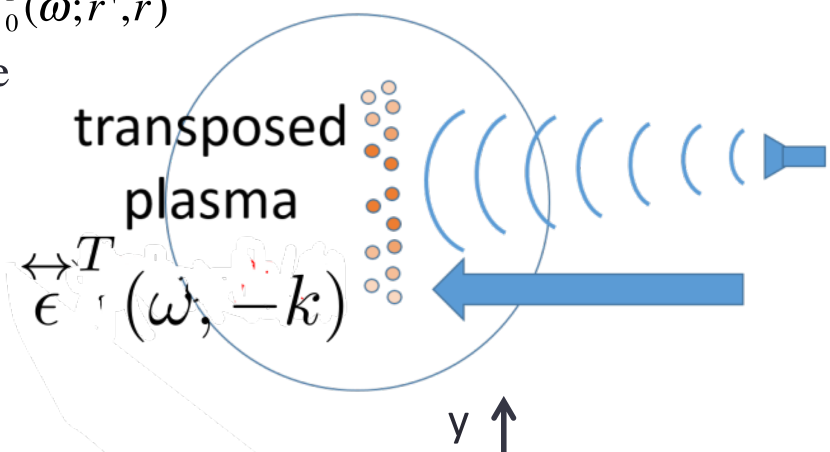
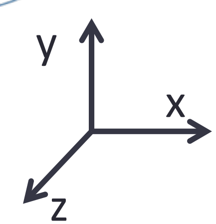
Reciprocity theorem

- Instead of solving the radiated power P by collecting waves from all the sources, we solve the reciprocal problem, and calculate P using reciprocity theorem.



- Reciprocal Problem:
    - Reversed in time
    - Study the transposed plasma with $\underline{\underline{\epsilon}}(\omega; r, r') = \underline{\underline{\epsilon}}_0^T(\omega; r', r)$
    - Calculate E^+ of the wave that launched from the receiver, propagates into plasma, absorbed by resonant electrons.
    - Much faster: calculate one wave propagation instead of many





- The measured radiation power

$$P(\omega) = \frac{1}{8\pi}\int d\vec{x}\,\vec{E}^+_{\omega,k}\cdot\underline{\underline{K}}\cdot\vec{E}^{+*}_{\omega,k}$$

A.D. Piliya and A.Y. Popov, Plasma Phys. Control. Fusion **44**, 467 (2002).

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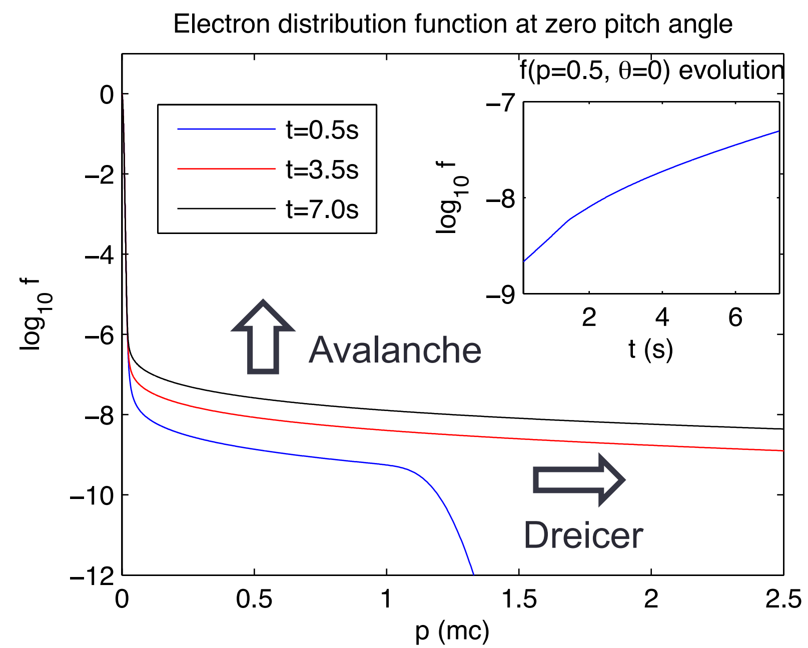
Runaway electron distribution in momentum space evolution

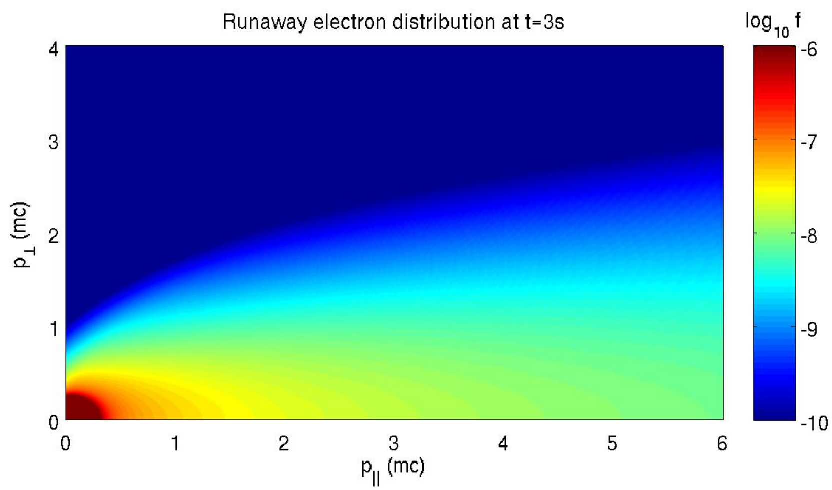
- RE distribution evolution in momentum space is calculated using CODE
  - Include the synchrotron radiation energy damping
  - Include the secondary RE generation (avalanche)

$$n_e = 0.5\times 10^{19} m^{-3}, T_e = 1keV$$
$$E / E_{CH} = 12, B = 1.5T$$
$$n_{RE}(t = 7s) / n_e \approx 10^{-3}$$

- The calculated RE tail is flat in p, but strongly anisotropic in pitch angle especially for highly relativistic electrons.

A.H. Boozer, Phys. Plasmas 22, 032504 (2015).





Major contributions to ECE emission comes from REs in the intermediate energy regime

- The source current correlation tensor can be calculated from RE distribution

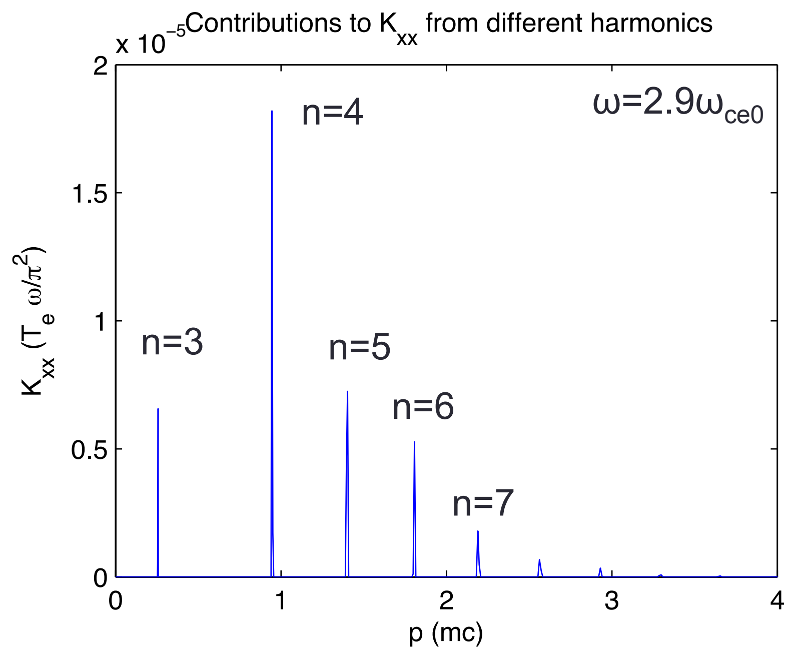
$$K_{xx} = 4e^2 n_e \sum_{n=-\infty}^{\infty} \int d^3p\, f_e\, \delta(\omega - k_{\|} v_{\|} - \frac{n\omega_{ce}}{\gamma}) \left[v_{\perp}^2 \frac{n^2}{k_{\perp}^2 \rho_e^2} J_n^2(k_{\perp}\rho_e) \right]$$

- J_n is sensitive to $k_{\perp}\rho_e$ ($k_{\perp}\rho_e = k_{\perp}p_{\perp}/(Be) = k_{\perp}\gamma m_e v_{\perp}/(Be)$) especially for large n.

$$J_n^2(z) \sim \frac{1}{n!^2}\left(\frac{z}{2}\right)^{2n}$$

- The value of $k_{\perp}\rho_e$ depends on both electron energy (γ) and pitch angle ($v_{\perp}$).

- Results show that major contributions to K_{xx} comes from REs in the intermediate energy regime ($0.5<p/m_e c<2$).



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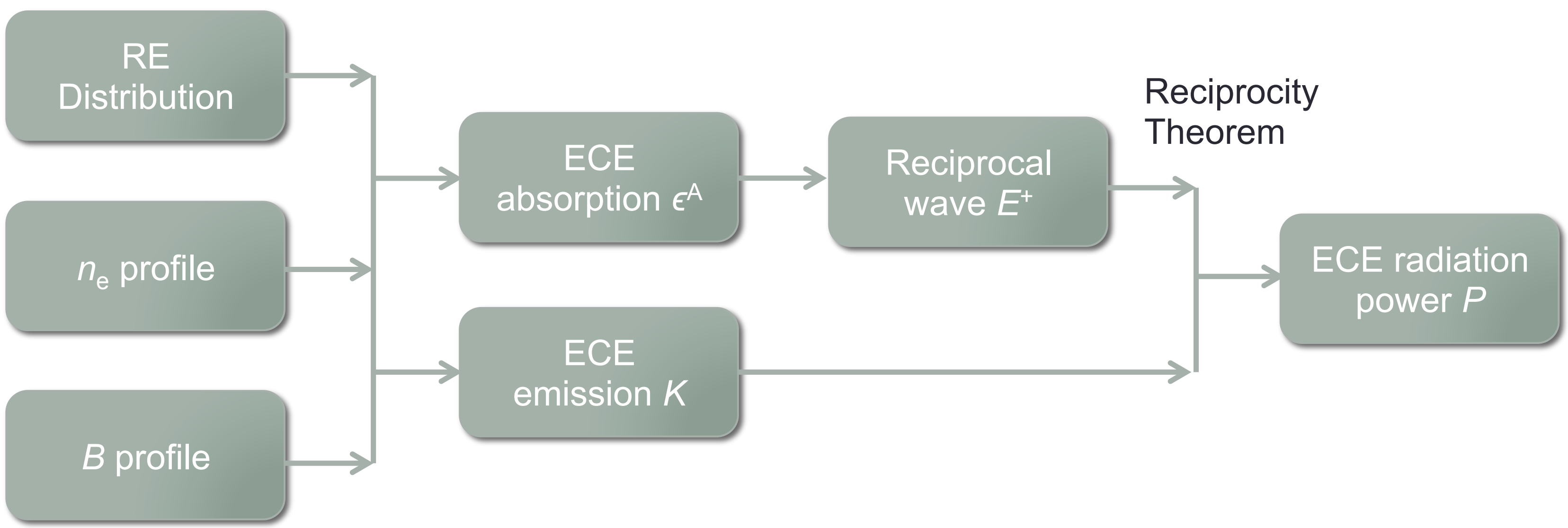
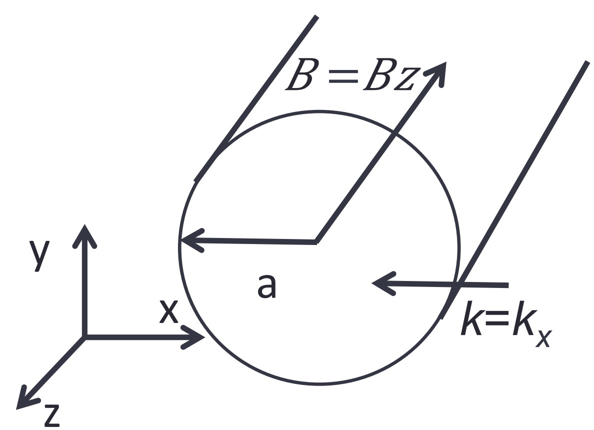
The process of RE ECE synthetic diagnostic calculation



- For current calculation we used simplified model
  - n_e and T_e are treated as a constant, $B \sim 1/R$.
  - $k_y = k_z = 0$, k is perpendicular to B.
  - Only include X-mode, ignoring mode conversion

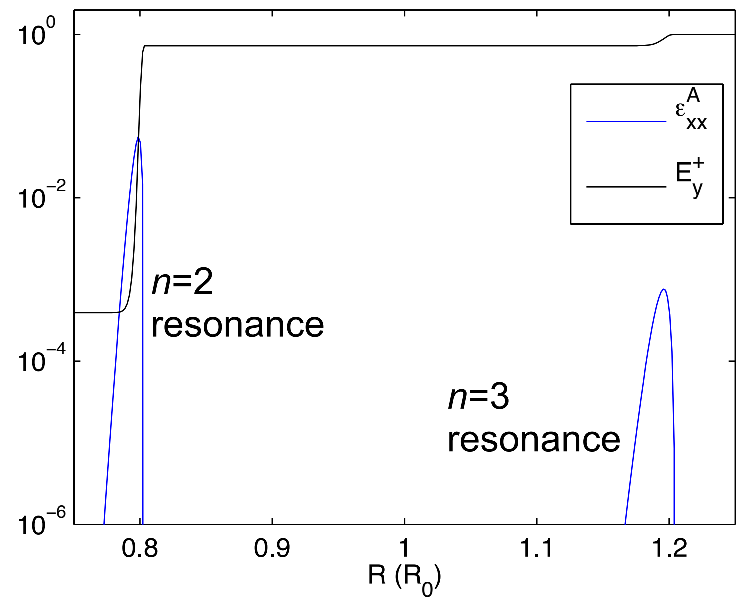


ECE absorption from thermal electrons is strongly localized

Here we show one example of calculation of reciprocal wave E^{+} and current correlation K_{xx} for ω=2.5$\omega_{ce}(R_0)$.

- High harmonic absorption is significantly lower than low harmonic.
- The absorption is localized near the resonance region.
- Plasma is optically thick for n=2 resonance, whereas optically thin for n=3 resonance.

The reciprocal wave calculation for thermal electrons

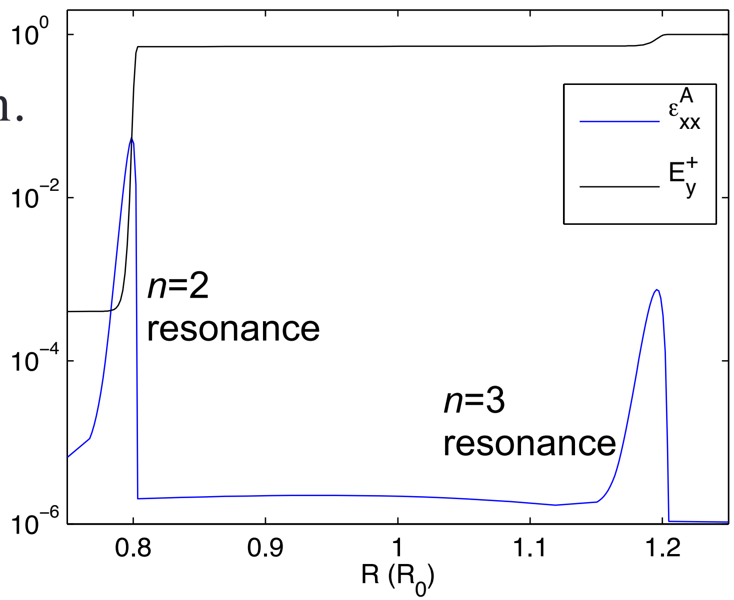


ECE absorption from RE is nonlocal, but very small

Here we show one example of calculation of reciprocal wave E^{+} and current correlation K_{xx} for $\omega=2.5\omega_{ce}(R_0)$.

- RE contribution to the absorption is nonlocal and not limited to resonance region.
- The runaway electrons have little impact on the ECE wave absorption.

The reciprocal wave calculation for RE (t=7s)

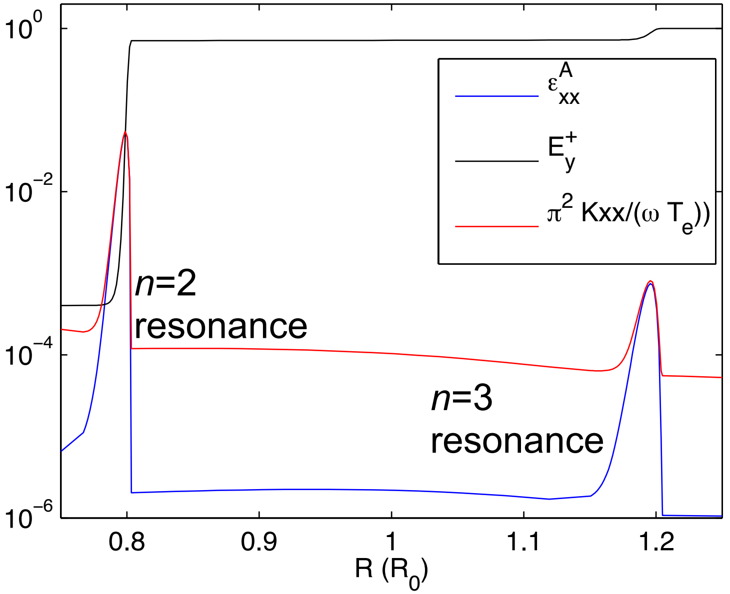


ECE emission from RE is important and accessible

Here we show one example of calculation of reciprocal wave E^{+} and current correlation K_{xx} for $\omega=2.5\omega_{ce}(R_0)$.

- The emission-absorption ratio of ECE from RE is much larger than thermal electrons.
  - RE distribution is not in thermal equilibrium.
- Given plasma is optically thin for high harmonics ($n\geq3$) ECE wave, the radiation from RE can propagate outside and get collected by receiver.

Emission and absorption of ECE from RE



Effective Temperature of RE distribution

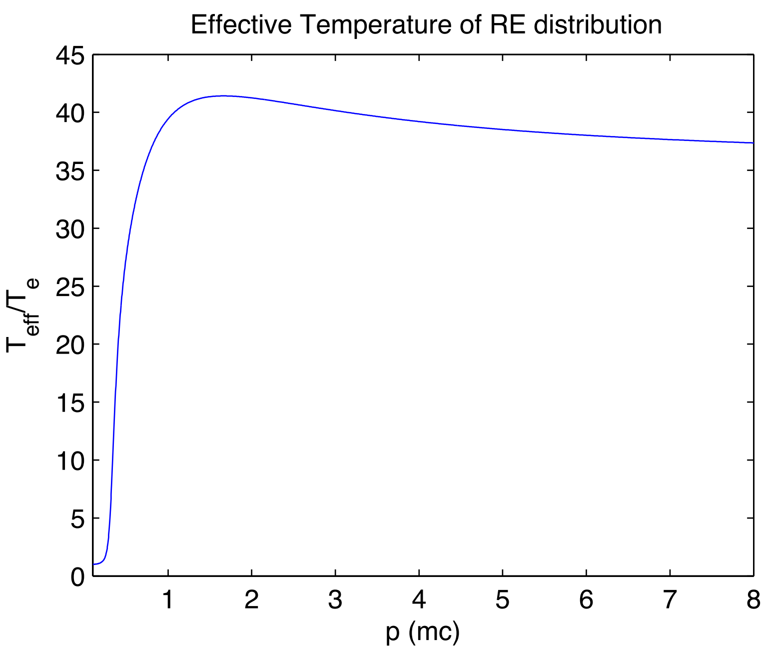
$$K_{xx} = 4e^2 n_e \sum_{n=-\infty}^{\infty} \int d^3p\, \delta(\omega - k_\parallel v_\parallel - \frac{n\omega_{ce}}{\gamma}) \left[v_\perp^2 \frac{n^2}{k_\perp^2 \rho_e^2} J_n^2(k_\perp \rho_e) \right] f_e \qquad \xi = p_\parallel / p$$

$$\epsilon_{xx}^A = \frac{4e^2 n_e}{\omega} \sum_{n=-\infty}^{\infty} \int d^3p\, \delta(\omega - k_\parallel v_\parallel - \frac{n\omega_{ce}}{\gamma}) \left[v_\perp^2 \frac{n^2}{k_\perp^2 \rho_e^2} J_n^2(k_\perp \rho_e) \right] \left[\frac{1}{v}\frac{\partial f_e}{\partial p} - \frac{\omega\xi - k_\parallel v}{\omega p v}\frac{\partial f_e}{\partial \xi} \right]$$

- To show the reason of high emission-absorption ratio, we define "effective temperature" at every momentum p for RE distribution

$$T_{eff}(p) = \frac{\int d\xi\, f(p,\xi)}{-\int d\xi \frac{1}{v}\frac{\partial f(p,\xi)}{\partial p} - \frac{1}{pv}\frac{\partial f(p,\xi)}{\partial \xi}}$$

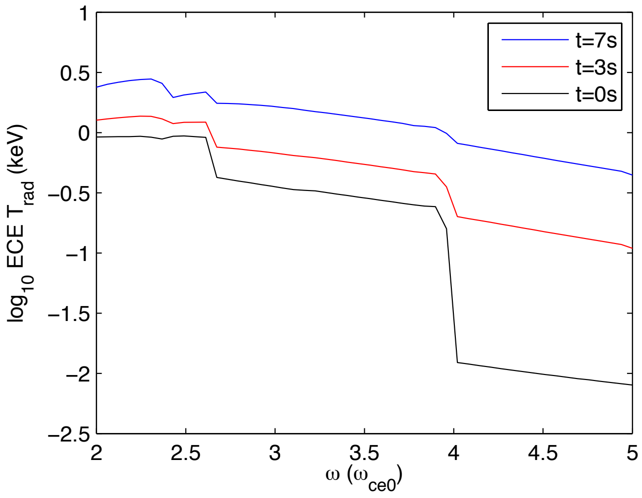
- Results show $T_{\text{eff}} \gg T_{\text{e}}$ for large p, consistent with the small absorption of ECE by RE tail.



Spectrum of ECE signal from RE is smoother than thermal electrons

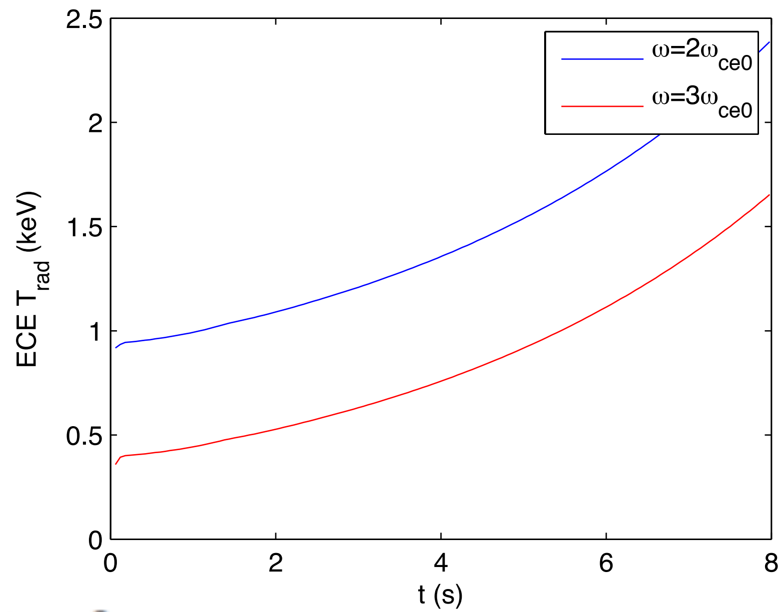
- For thermal electrons, the radiation power behaves like step-function as frequency increases.

- Growth of RE makes the spectrum flatter, which is consistent with the experiments.

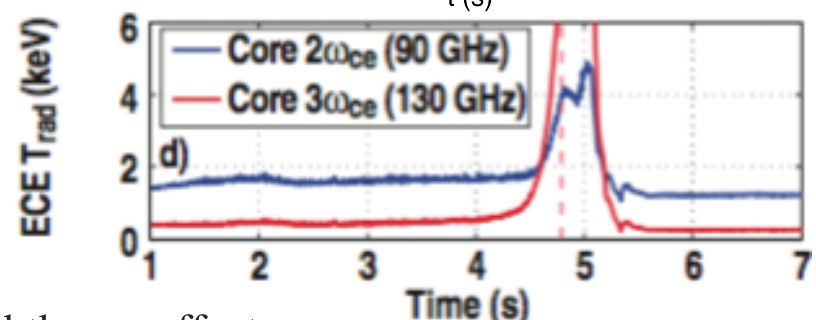


Time evolution of RE ECE signal

- ECE signals shows exponential growth, with the same growth rate as RE avalanche growth.
- The growth rate is still much smaller compared to experiments.
- The catch-up of higher harmonic signal is not observed.



- Conjectures
  - Kinetic modes that cause fast pitch angle scattering effect (fan instability) for RE distribution.
  - RE forming moving filaments when confined in magnetic islands, which can give fast and instantaneous signal through lighthouse effect.



V.V. Parail and O.P. Pogutse, Nucl. Fusion 18, 303 (1978).
G. Pokol, T. Fülöp, and M. Lisak, Plasma Phys. Control. Fusion 50, 045003 (2008).

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Summary

- Using newly-developed RE ECE synthetic diagnostic tool, we successfully calculate ECE signals from RE.
  - Due to the small absorption and large emission, and nonlocal spatial distribution of both, RE can give significant contributions to ECE radiation.
  - Flat RE ECE spectrum is observed.
  - The fast growth of ECE signal observed in experiments is not explained.

- Most ECE radiation comes from REs in intermediate energy regime ($0.5<p/m_e c<2$).

- Conjectures for fast ECE growth
  - Kinetic modes causing fast pitch angle
  - Non-uniform RE spatial distribution and lighthouse effect.

- Future plans: Using realistic plasma parameters and integrate the module to SDP (Synthetic Diagnostic Platform) developed at PPPL.

Thanks!