Runaway electrons in disruptions: sliding and screening

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1 Starting remarks

2 Sliding

3 Screening
Runaway team

- Ola: Close collisions, Bremsstrahlung
- Linnea: Partial screening effects
- Mathias: Synthetic synchrotron diagnostics
- George: Self-consistent electric field
Tools available for runaway studies at Chalmers

- 0D2P relativistic Fokker-Planck solvers
  - **CODE** – runaway electrons, linearized collision operator
    - synchrotron radiation
    - Bremsstrahlung
    - effect of partial screening **NEW**!
    - Rosenbluth-Putvinskii, Chiu-Harvey, Boltzmann avalanche operator
  - **NORSE** – nonlinear collision operator **NEW**!
  - **CODION** – runaway ions
- Radiation
  - **SOFT** – synthetic synchrotron diagnostics **NEW**!
  - **SYRUP** – synchrotron spectra
Outline

1. Starting remarks
2. Sliding
3. Screening
Motivation

- The more runaways, the bigger the problem
- Existing tools break down when more than a few % runaways
- Such RE densities obtainable in experiments
NORSE: NOOn-linear Relativistic Solver for Electrons

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Features

• 2D in momentum space, no spatial dependence
• Full Braams & Karney collision operator
• Arbitrary electric field strengths
• Radiation reaction
• Time-dependent plasma parameters
Benchmark: relativistic weak-field conductivity

- **Braams & Karney** list conductivities
  - weak-field
  - large $T$ range
  - same collision operator

\[ \bar{\sigma}/Z_{\text{eff}} \]

\( Z = 1 \)
\( Z = 2 \)
\( Z = 5 \)
\( Z = 10 \)

B&K

\( \bar{\sigma} \): normalized conductivity

\( \Theta = T/m_e c^2 \)
Benchmark: relativistic weak-field conductivity

- Braams & Karney list conductivities
  - weak-field
  - large $T$ range
  - same collision operator
- NORSE reproduces these perfectly

\[ \bar{\sigma}/Z_{\text{eff}} \]

$\bar{\sigma}$: normalized conductivity

\[ \Theta = T/m_e c^2 \]
Benchmark: conductivity in strong fields

- Comparison to Weng et al. [PRL 100, 185001 (2008)]
- They calculate modified Spitzer conductivity in strong $E$ field
- Non-relativistic
- Nice agreement!

(Numerical heating in Weng’s data for $E/E_D = 0.01$)

\[ \frac{j}{\hat{E}} \] normalized conductivity
\[ \frac{\hat{E}_\tau}{\sqrt{\Theta}} \] normalized time

$E/E_D = 0.01$
$E/E_D = 0.1$
$E/E_D = 1$
Distribution evolution

$\hat{E}_\tau / \sqrt{\Theta}$

$E/E_D = 0.01$

$F$

$-15$

-10

-5

0

Maxwellian

$E/E_D = 0.1$

$E/E_D = 1$

$p_{||}$

$-0.1$ $0$ $0.1$ $0.15$

$-0.1$ $-0.05$ $0$ $0.05$ $0.1$ $0.15$
Distribution evolution

$E/E_D = 0.01$

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Distribution evolution

\( \frac{E}{E_D} = 0.01 \)
Bulk heating

- \( E \) field is a source of heat!
  - Must be removed in a linear treatment
  - Automatically accounted for in NORSE
- In practice bulk keeps temperature or even cools – a heat sink is useful
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Current evolution and transition to slide-away is highly sensitive to the details of the sink!

Slide-away: Net parallel force experienced by electrons is positive in the entire momentum space.

- $E/E_D = 0.035$
- $E/E_c = 3.5$
- $T = 5.11$ keV
- $\tau_{th} = 63$

\[
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- Does the details of the heat sink influence the RE generation?

![Graph showing $E/E_D = 0.035$, $E/E_c = 3.5$, $T = 5.11$ keV, and $\tau_{th} = 250$ for different $p_\parallel$ values with and without the sink and Maxwellian distribution.](image-url)

$E/E_D = 0.035$

$E/E_c = 3.5$

$T = 5.11$ keV

$\tau_{th} = 250$
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Slide-away: Net parallel force experienced by electrons is positive in the entire momentum space.
An ITER-like scenario calculated by GO [Smith et al (2006)]

- GO: generation of runaway electrons coupled to a diffusion equation for the electric field.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E}{\partial r} \right) = \mu_0 \frac{\partial}{\partial t} \left( \sigma \parallel E + n_r e c \right)
\]

and

\[
\frac{\partial n_r}{\partial t} = \left( \frac{\partial n_r}{\partial t} \right)^{Dreicer} + \left( \frac{\partial n_r}{\partial t} \right)^{avalanche}
\]

- \( T_e^{final} = 10 \text{ eV}, \ B = 5.3 \text{ T}, \ Z_{eff} = 1, \)
  \( j_0 = 0.62 \text{ MA/m}^2, \) thermal quench time 1 ms.
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Electric field in V/m and normalized to the Dreicer field after the thermal quench.
Transition to slide-away depends on the heat-sink

- **No heat sink**: all energy supplied by the electric field remains.

- **Weak heat sink**: the energy-removal rate of the heat sink is restricted to 0.5 MW/m³

- **Strong heat sink**: keep the bulk temperature at 10 eV; any excess heat in the bulk region is removed
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Normalized current density in the different heat-sink scenarios. Current density becomes half of the original at $t_N$ (no HS), $t_W$ (weak HS) and $t_S$ (strong HS).
Runaway electron population

- Maximum particle energies depend on the heat-sink scenario.
  - No HS and weak HS: particle do not reach relativistic energies
  - Strong HS: particle energies of 22 MeV are reached just before slide-away.
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Tail of the parallel electron distribution. Thin lines $f$ at $t_N$ (no HS), $t_W$ (weak HS) and $t_S$ (strong HS), and thick lines $f$ immediately before the transition to slide-away.
Runaway electron population

- Maximum particle energies depend on the heat-sink scenario.
  - No HS and weak HS: particle do not reach relativistic energies
  - Strong HS: particle energies of 22 MeV are reached just before slide-away.
- In the strong HS case the $n_r/n$ grows more slowly and the runaways have time to reach high energies.
Feedback loop

- Collisional friction is lower in a hotter distribution.
- Dreicer field is $\propto 1/T$.
- For a given field strength $E/E_D$ increases as the bulk heats up.
- Decreasing $n_{\text{bulk}}$ also leads to a positive feedback.
- Eventually the friction becomes low enough that the parallel balance of forces becomes positive everywhere: Slide-away!

Effective temperature of the bulk population
Feedback loop

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Effective normalized E-field strength
Summary

NORSE [Stahl et al CPC (2017)]

- Relativistic, non-linear electron dynamics
- Radiative effects, time-dependent scenarios
- Efficient, freely available

Non-linear effects

- Conductivity different from Spitzer for strong fields
- Large heating of electron bulk by parallel $E$-field
- Slide-away at much weaker electric fields than previously expected.

Heat-sink

- Severity of disruptions can be affected by the properties of heat sink.
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Effect of partial screening

- Disruption mitigation via material injection: typically $n_Z > n_D$.
- In the cold post-disruption plasma, impurities are weakly ionized.
- Collision frequencies for fast electrons are expected to be enhanced.

Previous work

- **Elastic collisions**: Thomas–Fermi theory (limited to intermediate distances from the nucleus, and does not capture the shell structure of the ion): [Kirillov et al, Fizika Plazmy (1975)] and [Zhogolev and Konovalov VANT (2014) in Russian]


- **Inelastic collisions**: Rosenbluth–Putvinski rule of thumb: half of the bound electrons [Rosenbluth and Putvinski, NF (1997)]

- **Stopping-power formula for inelastic collisions** was used in a test-particle approach in [Martin-Solis et al, PoP (2015)].
Modelling of the effect of partial screening

- Generalized collision operator including the effect of partial screening

\[ C_{\text{test}}^e = \nu_D \mathcal{L}(f_e) + \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^3 \left( \nu_S f_e + \frac{1}{2} \nu_{\parallel} p \frac{\partial f_e}{\partial p} \right) \right] \]

- Model elastic collisions quantum-mechanically using density functional theory.

- Using kinetic simulations demonstrate the effect of partial screening on the distribution function, current decay and critical electric field.

- Analytical expression of the enhanced critical electric field.

[Hesslow et al, PRL (2017)]
Effect of partial screening

- **Definitions**
  - **Complete screening:** the electron interacts only with the net ion charge
  - **No screening:** the electron experiences the full nuclear charge
- **Elastic collisions**
  - Interaction strength proportional to the charge squared.
  - No screening enhances the interaction strength by a factor $X^2 = (Z/Z_0)^2$, where $Z_0$ is the ionization state and $Z$ is the charge number of the nucleus.
- **Inelastic collisions** (leading to excitation of the ion)
  - Increase the effective electron density of the plasma, as experienced by the fast electron.
  - The rate of e-e collisions will be an order $X$ larger.
Elastic collisions $\nu_{ei}^D$

Cross section in Born approximation, valid for $v/c \gg Z\alpha$

$$\frac{d\sigma_{ej}}{d\Omega} = \left( \frac{r_0^2}{4p^4} \right) \left( \frac{\cos^2(\theta/2)p^2 + 1}{\sin^4(\theta/2)} \right) |Z_j - F_j(q)|^2$$

Form factor: $F_j(q) = \int \rho_{e,j}(r)e^{-iq \cdot r/a_0} \, dr$

$q = \frac{2p}{\alpha} \sin(\theta/2)$, $p = \gamma \frac{v}{c}$, $Z$: atomic number, $Z_0$: net charge

Limits:
Low energy $|Z - F| \rightarrow Z_0$: complete screening (usual case)
High energy $|Z - F| \rightarrow Z$: no screening (interaction with nucleus)
Elastic collisions: density and form factor

From density functional theory (DFT)

\[ \rho(r) \left[ a_0^{-3} \right] \]

\[ 10^3 \quad 10^1 \quad 10^{-1} \quad 10^{-3} \]

\[ r \left[ a_0 \right] \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ F \]

\[ 0 \quad 5 \quad 10 \quad 15 \]

\[ 2p/\alpha \]

\[ 10^{-2} \quad 10^0 \quad 10^2 \quad 10^4 \]

\[ \text{Ar}^+ \]

\[ \text{Ar}^{2+} \]

\[ \text{Ne}^+ \]
Elastic collisions $\nu_D^{ei}$

\[
\nu_D^{ei} = \nu_D^{ei,cs} \left( 1 + \frac{1}{\sum_j n_j Z_{0,j}^2} \sum_j n_j Z_{0,j}^2 \frac{g_j(p)}{\ln \Lambda} \right)
\]

completely screened collision frequency

DFT simulations
Elastic collisions $\nu^e_D$

$$\nu^e_D = \nu^e_{D,\text{cs}} \left(1 + \frac{1}{\sum_j n_j Z^2_{0,j}} \sum_j n_j Z^2_{0,j} \frac{g_j(p)}{\ln \Lambda}\right)$$

*completely screened collision frequency*

**Full formula**

$$g_j(p) = \int_0^1 \left(\frac{[Z_j - F_j(q)]^2}{Z^2_{0,j}} - 1\right) \frac{dx}{x}$$

**DFT simulations**
Elastic collisions $\nu^e_D$

\[
\nu_D^e = \nu_{D,cs}^e \left( 1 + \frac{1}{\sum_j n_j Z_{0,j}^2} \sum_j n_j Z_{0,j}^2 \frac{g_j(p)}{\ln \Lambda} \right)
\]

- **Full formula**
  \[
  g_j(p) = \int_0^1 \left( \frac{[Z_j - F_j(q)]^2}{Z_{0,j}^2} - 1 \right) \frac{dx}{x}
  \]

- **TF-DFT model**
  \[
  g_j(p) = \frac{2}{3} (X_j^2 - 1) \ln(y_j^{3/2} + 1) - \frac{2}{3} (X_j - 1)^2 y_j^{3/2} - \frac{2}{3} y_j^{3/2} + 1
  \]

- **Effective length** $\alpha_j$
  \[
  y_j = 2 \alpha_j p / \alpha
  \]

- **DFT simulations**

- **completely screened collision frequency**
Enhancement of deflection frequency $\nu_{ei}^D$

- Compare to completely screened
- Excellent agreement between analytical model (TF-DFT) and full DFT
- Significant effect already at $p \sim p_c \sim 0.1$
- $p \gg 1$: $\nu_{ei}^D/\nu_{ei,CS}^D \sim (Z/Z_0)^2 \sim 10^2$
- Parameters: $T = 10$ eV, $n_{Ar^+} = 10^{20}$ m$^{-3}$

$p = \gamma \frac{\nu}{c}, \ E = 10 \text{ MeV} \leftrightarrow p = 20.$
Inelastic collisions $\nu^e_S$

Bethe stopping power formula (matched with low energy asymptote)

\[
\nu^e_S = \nu^e_{S,cs} \left\{ 1 + \sum_j \frac{n_j N_{e,j}}{n_e \ln \Lambda} \left[ \frac{1}{k} \ln \left( 1 + h^k_j \right) - \beta^2 \right] \right\},
\]

\[h_j = p \sqrt{\gamma - 1}/l_j, \quad k = 5, \quad \beta = v/c\]

$l_j$ mean excitation energy [Sauer et al, Advances in Quantum Chemistry 2015]

- Rosenbluth–Putvinski rule of thumb:
  \[
  \nu^e_{S,rp} \approx \nu^e_{S,cs} \left( 1 + \frac{1}{2} \sum_j \frac{n_j N_{e,j}}{n_e} \right),
  \]
  where $N_e$ is the number of bound electrons.

- RP rule of thumb leads to greater enhancement than the full formula up to $p \approx 1$. 
- Enhancement due to elastic collisions kicks in for lower momenta and is larger for high momenta than the corresponding one for inelastic collisions.

Parameters: $T = 10$ eV, $n_{\text{Ar}^+} = 10^{20}$ m$^{-3}$
Effect on distribution function

- Implemented in CODE.
- Collisional deceleration of initial beam-like distribution.
- Contours of $\log_{10}(F)$, $F = (2\pi m_e T)^{3/2} f_e / n_e$
- Parameters: 25 ms collisional deceleration $T = 10$ eV, $Ar^+$, $n_{Ar} = n_D = 10^{20}$ m$^{-3}$
Current decay

- Same initial distribution as previous figure.
- Decay time is proportional to $1/n_{Ar}$ for $n_{Ar} \gtrsim n_D$.
- Bands represent $n_{Ar} \in [0.5 \ n_D, 100 \ n_D]$.
- RP model underestimates the decay rate and shows a different current evolution.
Critical electric field

- Important for generation and decay
- Constant $\ln \Lambda$ and no screening or radiation effects:

$$E_c = \frac{n_e e^3 \ln \Lambda_0}{4\pi \epsilon_0^2 m_e c^2}$$
Critical electric field

- Important for generation and decay

- Constant $\ln \Lambda$ and no screening or radiation effects:

  $$E_c = \frac{n_e e^3 \ln \Lambda_0}{4\pi \epsilon_0^2 m_e c^2}$$

- $E_c$ enhanced by
  - Partially ionized atoms
  - Synchrotron radiation
  - Bremsstrahlung
  - Energy-dependent Coulomb logarithm $\ln \Lambda$
Enhanced critical electric field $E_{\text{eff}}$

- Large enhancement of $E_{\text{eff}}$ due to partial screening
- Significant effect from elastic collisions
- RP model underestimates $E_{\text{eff}}$

\[
\frac{E_{\text{eff}}}{E_c} \approx 1 + \frac{1}{\ln \Lambda_0} \left( 7 - \ln \sqrt{T_{\text{eV}}} + 240 \frac{n_{\text{Ar, tot}}}{n_e} \right)
\]
Derivation of $E_{c}^{\text{eff}}$

- Assume fast pitch-angle dynamics in Fokker–Planck equation:\footnote{Lehtinen et al, JGR (1999), Aleynikov and Breizman, PRL (2015)}

\[
\frac{\partial \bar{f}}{\partial t} = \frac{\partial}{\partial p} \left[(p \nu_{S} - eE\xi)\bar{f}\right] + \frac{\partial}{\partial \xi} \left[(1 - \xi^{2}) \left(\frac{eE}{pmc}\bar{f} + \frac{1}{2\nu_{D}} \frac{\partial \bar{f}}{\partial \xi}\right)\right] = 0
\]

where $\bar{f} = p^{2}f$.

- Averaged force balance: $\langle eE_{c}^{\text{eff}} \rangle = \min_{p} p \nu_{S}$

- Up to triply ionized argon\footnote{Hesslow et al, PRL (2017); Details in Hesslow et al, EPS (2017)} $n_{Ar} \gtrsim 0.1n_{D}$ (synchrotron neglected)

\[
\frac{E_{c}^{\text{eff}}}{E_{c}} \approx 1 + \frac{1}{\ln \Lambda_{0}} \left(7 - \ln \sqrt{T_{eV}} + 240 \frac{n_{Ar,\text{tot}}}{n_{e}}\right)
\]
Simulate dissipation of runaway beam [1/2]

- Linear current decay predicted\(^1\): \(-\frac{\partial j}{\partial t} \propto E \approx E_{\text{eff}}^c\)

- Implemented in Fokker–Planck solver CODE with 0-D inductive electric field\(^2\)

\[ E = -\hat{L} \frac{\partial j}{\partial t}, \quad \hat{L} = \frac{AL}{2\pi R} \sim \frac{\mu_0 A}{2\pi} \]

- Forward-beamed initial distribution obtained by simulation with large E-field, average runaway energy: 17.2 MeV

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\(^1\)Breizman NF (2014)

\(^2\)Wilkie et al in preparation; Stahl et al EPS P2.150
Simulate dissipation of runaway beam [2/2]

- Test $-\hat{L} \frac{\partial j}{\partial t} = \hat{E} \approx E_c^{\text{eff}}$
- Good agreement at high inductance:
  $\rightarrow$ current decay rate is $\propto E_c^{\text{eff}} / \hat{L}$
- Enhanced $E_c^{\text{eff}} \Rightarrow$ faster dissipation
- Parameters: $T = 10 \text{ eV}$, Ar$^+$ with $n_{\text{Ar}} = 4n_D$, $n_D = 10^{20} \text{ m}^{-3}$, initial average runaway energy 17.2 MeV
Summary: partial screening

Enhanced collision frequencies

- Analytical expressions for the deflection and slowing-down frequencies.
- Significant enhancement compared to complete screening, already at sub-relativistic electron energies.

Current decay time is reduced

- Low inductance case: current decay time is approximately half compared to the RP rule of thumb.
- High inductance case: current decay rate is $\propto \frac{E_{\text{eff}}}{L}$

Critical electric field

$$\frac{E_{\text{eff}}}{E_c} \approx 1 + \frac{1}{\ln \Lambda_0} \left( 7 - \ln \sqrt{T_{eV}} + 240 \frac{n_{\text{Ar,tot}}}{n_e} \right)$$
Highlights

- Recent papers
  - NORSE: A solver for the relativistic **non-linear** Fokker-Planck equation for electrons in a homogeneous plasma
    [Stahl, Landreman, Embréus and Fülöp, CPC 212, 269 (2017)]
  - Runaway-electron formation and electron **slide-away** in an ITER post-disruption scenario
    [Stahl, Embréus, Landreman, Papp and Fülöp, JPCS 775 012011 (2016)]
  - Effect of **partially ionized impurities** on fast electron dynamics
    [Hesslow, Embréus, Stahl, DuBois, Papp, Newton and Fülöp, PRL 118, 255001 (2017)]

- In preparation
  - SOFT: a synthetic **synchrotron diagnostic** for runaway electrons
    [M Hoppe et al]
  - On the relativistic **large-angle** electron collision operator for runaway avalanches in plasmas [O Embréus et al]
Outline

4 SOFT
SOFT: Synthetic synchrotron diagnostics

- SOFT – Synchrotron-detecting Orbit Following Toolkit
- Takes spectrum, camera location/size/viewing direction into account
- Uses experimentally obtained magnetic equilibria
- Solves the guiding-center equations of motion to distribute particles poloidally (accounts for geometric effects)
- Momentum-space distribution of runaways (e.g. obtained by CODE) given as input

Experimental image provided by A. Tinguely and R. Granetz
Strange synchrotron image? A case for SOFT!

Spare slides
The total energy change can be written as

$$\frac{dW}{dt} = m_e c^2 \int_{\Omega} d^3 p \left( \gamma - 1 \right) \left( \frac{eE}{m_e c} \cdot \frac{\partial f}{\partial p} + \frac{\partial}{\partial p} \cdot (F_s f) + k_h \frac{\partial}{\partial p} \cdot (S_h f) \right)$$

from which $k_h$ can be determined in each time step by demanding that $dW/dt = 0$.

- $S_h(p)$ is an isotropic function of momentum (a natural choice is a Maxwellian).
- The momentum space need not necessarily encompass the entire population domain.
- In the figures $\Omega$ is the bulk of the distribution, which was defined as all particles with $\nu < 4 \nu_{Th0}$ where $\nu_{Th0}$ is the thermal speed at the initial temperature.