Simulation of runaway electron diffusion in momentum space due to whistler wave instabilities

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Theory and Simulation of Disruptions Workshop
July 18 2017

Highlights

- Due to bump-on-tail and anisotropicity of runaway electron distribution in momentum space, whistler waves can be driven unstable.
- Excited whistler waves cause diffusions and strongly alter runaway electron distribution in momentum space.
- Experiments observations, such as ECE signals and whistler wave observations, can be explained by RE kinetic simulation including whistler wave diffusion

Outline

1. Motivation: observations of runaway electron kinetic instabilities
2. Simulation model
3. Simulation results
4. Explanations of whistler wave excitation
5. Summary

Motivation: observations of runaway electron kinetic instabilities

Runaway electrons (RE) are susceptible to various kinds of kinetic instabilities

- Bump-on-tail distribution can drive instabilities through Cherenkov resonance.

$$\omega - k_{||} v_{||} = 0$$

- The anisotropy of the distribution can excite modes through anomalous Doppler resonance.

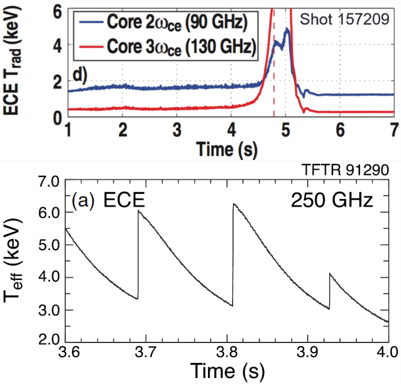
$$\omega - k_{||} v_{||} = n \frac{\omega_{ce}}{\gamma} \quad (n < 0)$$

- The excited modes can cause energy diffusion and pitch angle scattering of resonant electrons.

V.V. Parail and O.P. Pogutse, Nucl. Fusion 18, 303 (1978).
T. Fülöp, G. Pokol, P. Helander, and M. Lisak, Physics of Plasmas 13, 062506 (2006).
P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

Prompt growth of ECE signals are observed in RE experiments

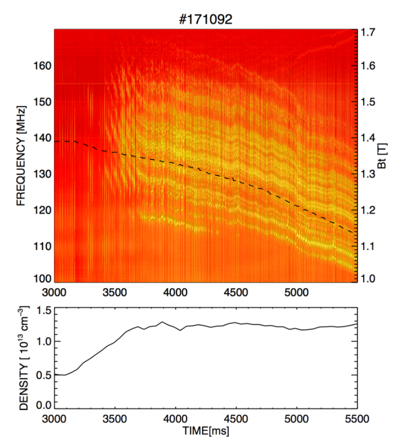
- In both quiescent runaway electron (QRE) experiments and post-disruption cases, fast growth of ECE signals (beyond the electron temperature value) are observed.
- This growth of ECE signals comes from fast increase of RE pitch angle, associated with kinetic instabilities.



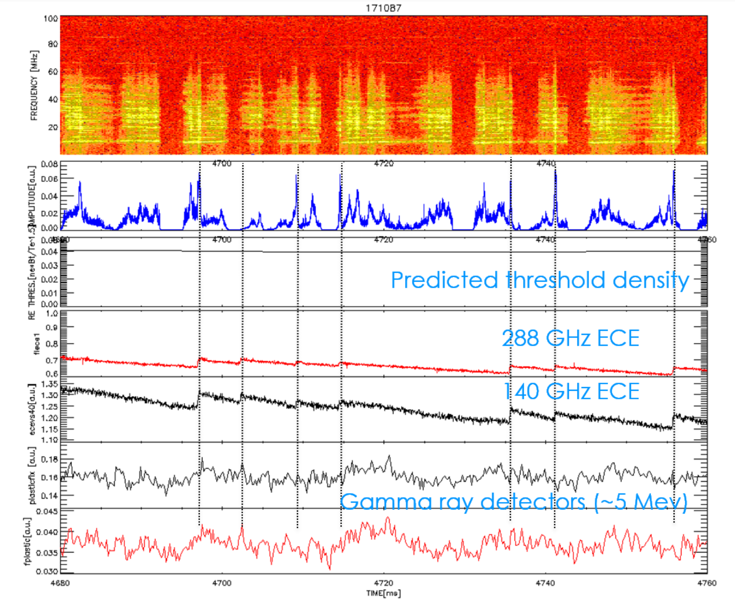
C. Paz-Soldan, et al., Nucl. Fusion 56, 056010 (2016).
E.D. Fredrickson, M.G. Bell, G. Taylor, and S.S. Medley, Nucl. Fusion 55, 013006 (2015).

Whistler waves are observed in recent QRE experiments on DIII-D

- Whistler waves are electron-driven waves with frequencies between ω_{ci} and ω_{pe}.
- Interact with RE through both Cherenkov and anomalous Doppler resonances.
- The whistler waves found in DIII-D QRE experiments have frequency range 100-200 MHz.



Whistler waves and ECE signals are correlated



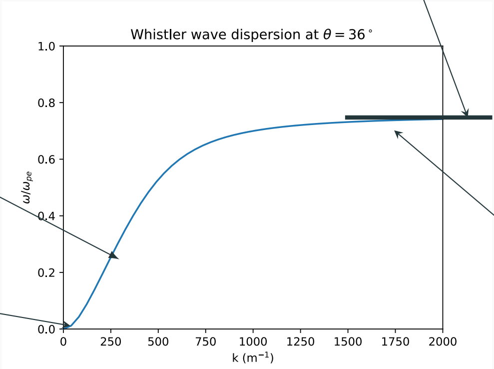
Both the whistler wave amplitudes and the ECE signals show cyclic behavior, and have strong correlations with each other.

Simulation model

Frequencies of whistler waves are calculated using cold plasma dispersion relation

$$\omega_{\max} = \frac{1}{\sqrt{2}}\sqrt{\omega_{ce}^2 + \omega_{pe}^2 - \sqrt{\omega_{ce}^4 + \omega_{pe}^4 - 2\omega_{ce}^2\omega_{pe}^2 \cos 2\theta}}$$

whistler wave



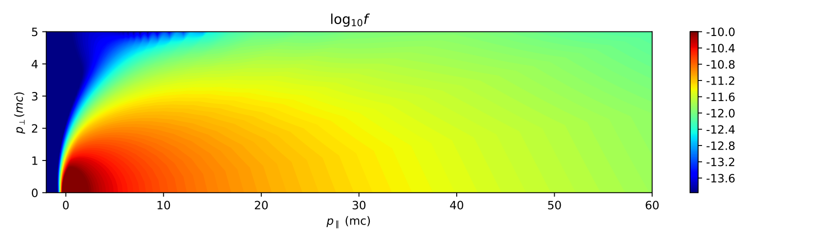
Alfven wave

magnetized plasma wave

RE distribution in momentum space is calculated dynamically

RE distribution evolution in momentum space is calculated using a finite element code solving the kinetic equation with time.

- Include the synchrotron radiation energy damping
- Include the secondary RE generation (runaway electron avalanche) using Chiu-Harvey source term



Growth of unstable modes are calculated according to RE distribution

$$\Gamma = \frac{\omega_{pe}^2}{D} \int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi \delta(\omega - k_\parallel v \xi - n\omega_{ce}/\gamma) \left[v\frac{\partial f}{\partial p} - \frac{v}{p} \frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi} \frac{\partial f}{\partial \xi} \right]$$

$$Q_n = \left[E_x \frac{n\omega_{ce}}{\gamma k_\perp v} J_n(k_\perp \rho) + E_z \xi J_n(k_\perp \rho) + iE_y \sqrt{1-\xi^2} J_n'(k_\perp \rho) \right]^2$$

$$D = \frac{1}{\omega} \mathbf{E}^* \cdot \frac{\partial}{\partial \omega}(\omega^2 \epsilon) \cdot \mathbf{E} \qquad \xi \text{ is the cosine of pitch angle}$$

- For $n = 0$: Γ depends on $\partial f/\partial p_\parallel$, Landau damping & bump on tail
- For $n > 0$: Anisotropic distribution ($\partial f/\partial \xi > 0$) stabilize the mode
- For $n < 0$: Anisotropic distribution gives positive growth rate

The growth rate Γ is subtracted by the damping rate due to collisions.

P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

Diffusion of RE in momentum space is addressed using quasi-linear model

$$\frac{\partial f_0}{\partial t} = \frac{1}{2} e^2 \sum_{n=-\infty}^{\infty} \int d^3\mathbf{k}\, \hat{L}[p_\perp \delta(\omega - k_\| v\xi - n\omega_{ce}/\gamma)|\psi(n,\mathbf{k},\omega)|^2 p_\perp \hat{L} f_0]$$

$$\hat{L}f = \frac{1}{p}\frac{\partial f}{\partial p} - \frac{1}{p^2}\frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi}\frac{\partial f}{\partial \xi}$$

$$\psi(n,\mathbf{k},\omega) = \frac{1}{2}(E_x + iE_y)J_{n-1}(k_\perp\rho) + \frac{1}{2}(E_x - iE_y)J_{n+1}(k_\perp\rho) + \frac{p_\|}{p_\perp}E_z J_n(k_\perp\rho)$$

We only take into account $n = 0$ and $n = -1$, which are the dominant resonances of whistler waves.

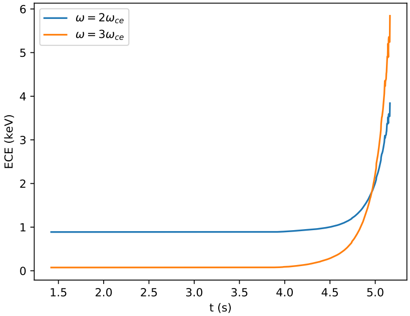
A. N. Kaufman, "Quasilinear diffusion of an axisymmetric toroidal plasma," Phys. Fluids 15, 1063 (1972).

Simulation results

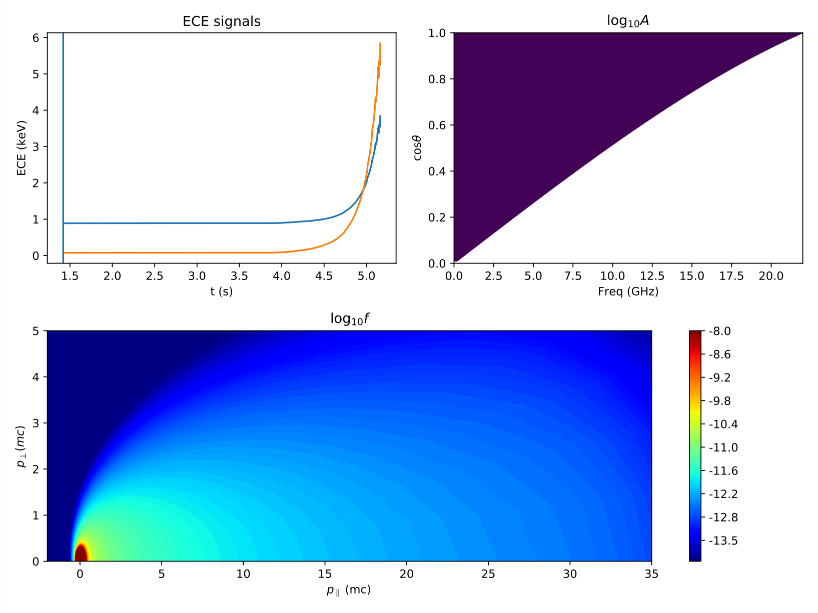
Stage 1: Initial RE growth with low electron density

$$n_e = 0.6 \times 10^{19}\mathrm{m}^{-3} \quad T_e = 1.1\mathrm{keV} \quad E/E_C = 9$$

- Whistler modes get excited after certain time of RE growth.
- Growth of $p_\perp$ results in a growth of ECE signals from RE, which agrees with experiments.
  - ECE signals are calculated using synthetic diagnostic assuming homogeneous RE distribution in space.



For details about ECE synthetic diagnostic of RE, see Chang Liu's PhD thesis.


ECE signals
ECE (keV)
t (s)
$\log_{10}A$
$\cos\theta$
Freq (GHz)
$\log_{10}f$
$p_\perp$ (mc)
$p_\parallel$ (mc)

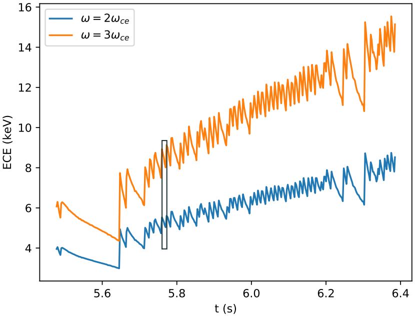
Highlights of Stage 1

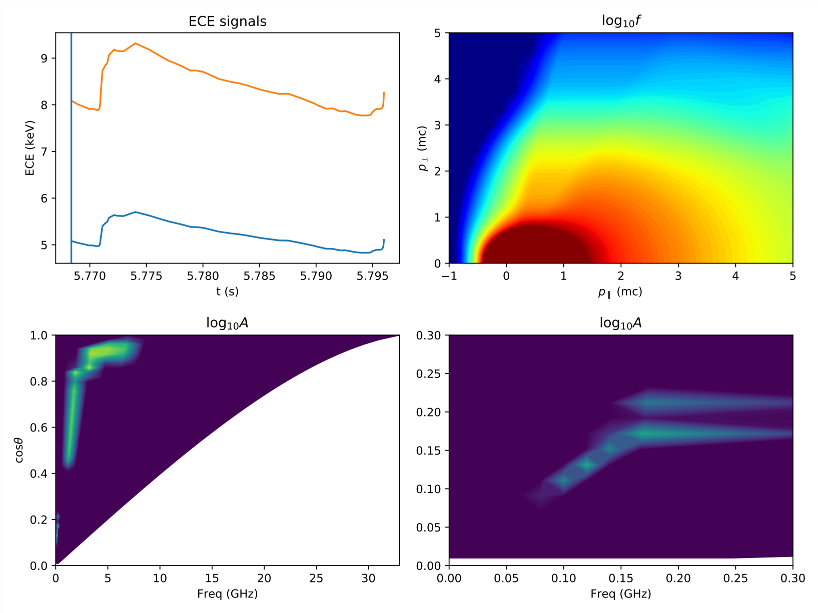
- Low frequency whistler waves (1GHz-5GHz) first get excited, and scatter RE in high energy regime ($15 < \gamma < 20$).
    - Stop RE from going into higher energy regime
- High frequency magnetized plasma waves (5GHz-20GHz) get excited later, and scatter RE in low energy regime ($2 < \gamma < 5$).
    - Result in fast growth of ECE signals

Stage 2: RE evolution after gas puffing

$$n_e = 1.35 \times 10^{19}\mathrm{m}^{-3} \quad T_e = 1.1\mathrm{keV} \quad E/E_C = 4$$

- The ECE signals keep growing, with inverse-sawtooth cycles (increases quickly then decreases slowly).
- A third branch of whistler wave with very low frequency (100-300 MHz) is excited.





ECE signals
ECE (keV)
t (s)
$\log_{10} f$
$p_\perp$ (mc)
$p_\parallel$ (mc)
$\log_{10} A$
$\cos\theta$
Freq (GHz)
$\log_{10} A$
Freq (GHz)

Highlights of Stage 2

- Low frequency whistler waves stay at high level during the cycle.
- High frequency magnetized plasma waves grow and decay during a cycle.
- Very low frequency whistler waves also oscillate and correlate with the high frequency magnetized plasma waves, which agrees with experiments.

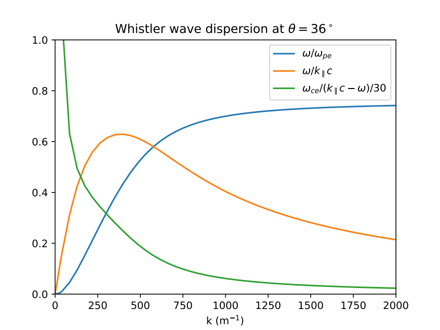
Explanations of whistler wave excitation

Resonance condition of Cherenkov and anomalous Doppler resonances

The main contribution to resonances are $n = 0$ and $n = -1$.

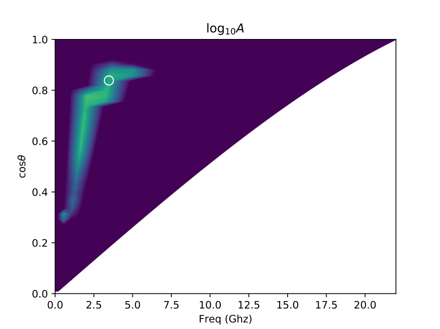
$$\omega - k_{\|} v_{\|} = 0, \quad \omega - k_{\|} v_{\|} = -\omega_{ce}/\gamma$$

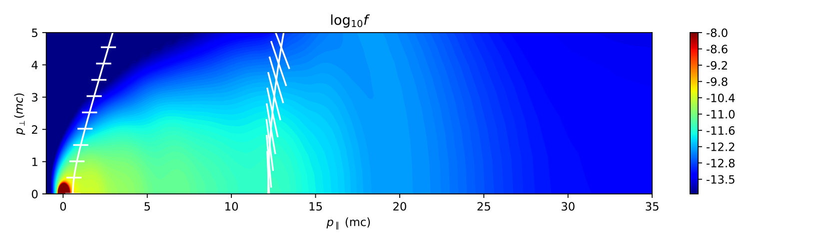
- For Cherenkov resonance, resonant velocity is a non-monotonic function of wave frequency.
- For anomalous Doppler resonance, resonant γ decreases as frequency increases.



Low frequency whistler wave excited by anisotropicity at high energy regime

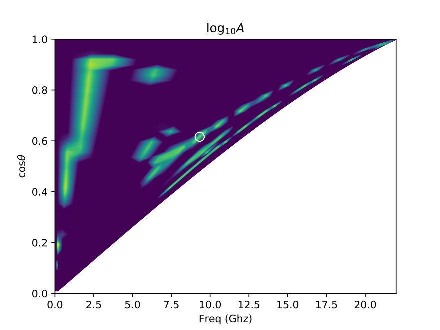
- Anomalous Doppler ($n = -1$) provides growth of the mode
- Cherenkov ($n = 0$) gives Landau damping.

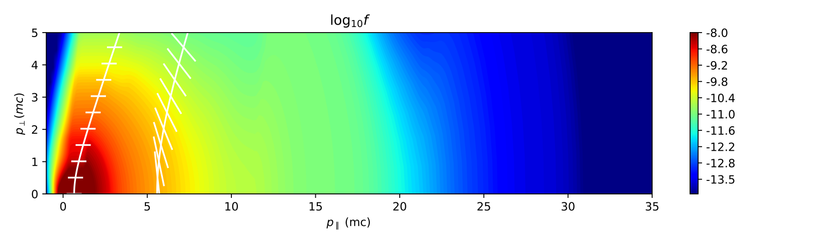




High frequency magnetized plasma wave excited by anisotropicity at low energy regime

- Anomalous Doppler ($n = -1$) provides growth of the mode
- Cherenkov ($n = 0$) gives Landau damping.





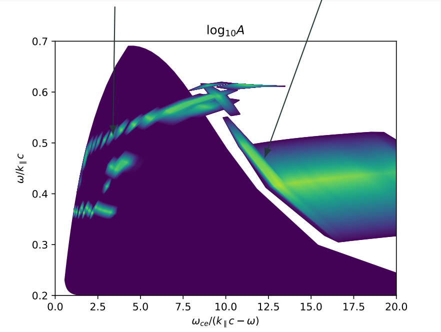
Two branches of waves are connected by Landau damping

$$\omega - k_{\|} v_{\|} = 0, \quad \omega - k_{\|} v_{\|} = -\omega_{ce}/\gamma$$

- Landau damping of two branches of waves happen at same energy regime.
- Excitation of whistler waves flatten the Cherenkov resonance region, thus suppress Landau damping of magnetized plasma waves.

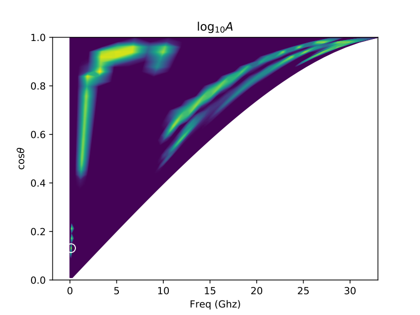
magnetize plasma wave

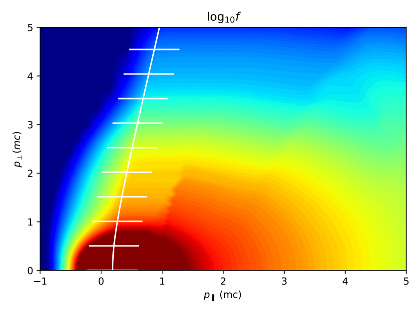
whislter wave



Very low frequency whistler waves excited by bump-on-tail

- The anomalous Doppler resonance of these waves happens at $\gamma > 80$, where RE population is very small.
- Instead, these waves are driven by Cherenkov resonance off the $\xi = 1$ axis, due to strong pitch angle scattering at low energy regime.





Summary

Summary

- By including the diffusion effect from excited modes in RE kinetic simulation using quasilinear model, the excitation of whistler waves and the growth of ECE signals are successfully reproduced.
- Low frequency whistler waves and high frequency magnetized plasma waves are excited by RE anisotropicity in high and low energy regimes respectively, and can lead to fast pitch angle scattering.
  - Most whistler interactions are not measured directly from experiments.
- The measured very low frequency whistler waves are result of bump-on-tail distribution as a side effect of inhomogeneous pitch angle scattering.
- Future work:
  - Trapping of RE with large pitch angle off the magnetic axis
  - Propagation of the excited waves in tokamak geometry

Thank you

Backup slides

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