Controlling Runaway Vortex Via Externally Injected High-Frequency Electromagnetic Waves

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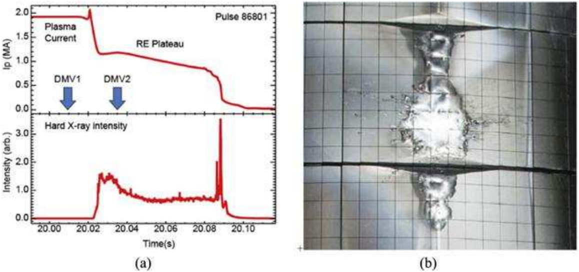
Motivation

- Damage of runaways to plasma-facing components is determined by runaway current ($j_{RE} \approx e n_{RE} c$) and runaway energy $\mathcal{E} = \gamma m c^2$.
- For given j_{RE} (essentially n_{RE}), higher runaway energy (larger γ) leads to
  - higher heat load (unit area) $\rightarrow$ melting, ablation, etc
  - deeper heat load deposition (longer stopping range) $\rightarrow$ unexpected subsurface damage.



(a) Be melting by RE in JET (G. Matthews 2016 Phys. Scr.)

Mitigation strategy

- Can we limit RE energy under a few MeV to control the extent of damage?
  - Physics basis: RE energy is determined by runaway vortex in momentum space $\sim E^2 B^{-2} n^{-1}$. [Guo, McDevitt, Tang, PPCF (2017)]
  - Approach: Phase-space engineering of runaway electrons by manipulating the runaway vortex.
    - This talk focuses on externally injected high-frequency electromagnetic waves

Runaway vortex is a robust feature in momentum space

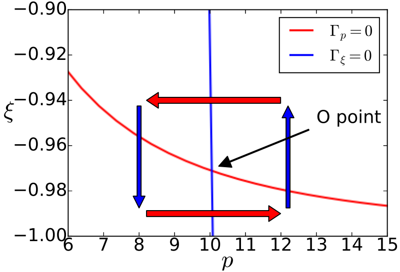
- The Fokker-Planck equation can be written in a conservative form

$$\frac{\partial f}{\partial(t/\tau_c)} + \frac{1}{p^2}\frac{\partial}{\partial p}p^2\Gamma_p + \frac{1}{p}\frac{\partial}{\partial \xi}\Gamma_\xi = 0\,,$$

- p normalized to $m_e c$, define $\alpha \equiv \tau_c/\tau_s = (2/3)\epsilon_0 B^2/n_e m_e \ln\Lambda$

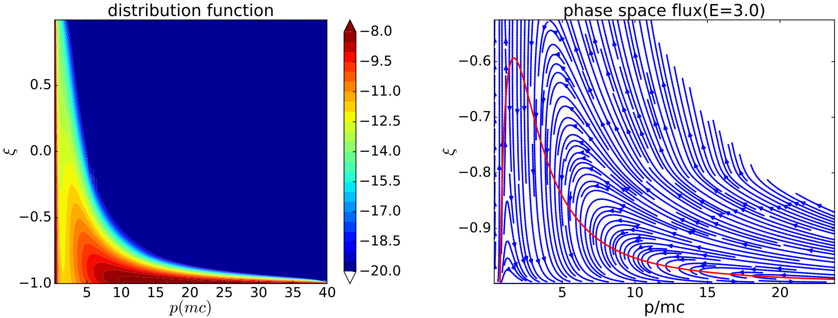
$$\boxed{\Gamma_p = \left[(-\xi)E - \frac{1+p^2}{p^2} - \alpha p\gamma(1-\xi^2)\right] f}$$

$$\boxed{\Gamma_\xi = -(1-\xi^2)\left(Ef - \alpha\frac{p}{\gamma}\xi f + \frac{1+Z}{2}\frac{\gamma}{p^2}\frac{\partial f}{\partial \xi}\right)}$$



- A runaway vortex forms around the O point where $\Gamma_p = \Gamma_\xi = 0$.

Runaway vortex → runaway energy distribution

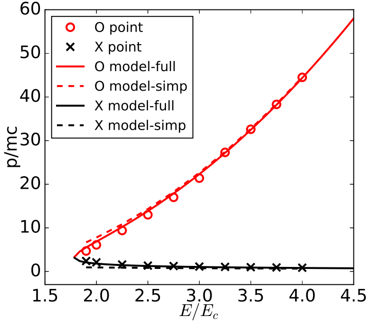


$E = 3.0, \alpha = 0.2, Z = 1$. The red curve corresponds to zero energy flux. Results computed with LAPS-RFP code (LANL).

- The spread of RE distribution in both energy and pitch-angle is associated with the phase-space vortex;
- A separatrix (X point) in phase-space separates RE into local and global circulating populations;
- Topological change (runaway vortex is present or not) is crucial for RE avalanche growth [Talk by C. Mcdevitt, this morning];

O/X merging $\rightarrow$ avalanche threshold

- At $p_{\rm O} \gg \sqrt{E/\alpha}$, an expression is derived for O point's energy [Guo-PPCF17]
$$p_{\rm O} = \sqrt{2}\frac{(E+\alpha)(E-1)}{(1+Z)\alpha},$$
- The energy of X point can also be modeled by
$$p_{\rm X} = \sqrt{\frac{1+(1+Z)/2\sqrt{2}}{E}}.$$



- Avalanche threshold electric field is determined by the merging of O and X points. (McDevitt talk this morning)

Z increases $\rightarrow p_O$ decreases & p_X increases

- At $p_{\mathrm{O}} \gg \sqrt{E/\alpha}$, an expression is derived for O point's energy [Guo-PPCF17]

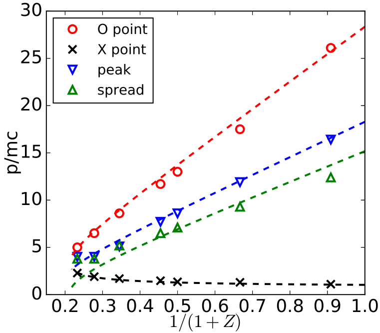
$$p_{\mathrm{O}} = \sqrt{2}\frac{(E+\alpha)(E-1)}{(1+Z)\alpha},$$

- The energy of X point can also be modeled by

$$p_{\mathrm{X}} = \sqrt{\frac{1+(1+Z)/2\sqrt{2}}{E}}.$$

- Increasing Z can effectively decrease the O point energy and the volume of the runaway vortex;
  - bound electrons $\rightarrow E_c$ and secondary source.
  - partial screening $\rightarrow$ higher effective Z of impurities $\rightarrow$ enhanced scattering

Talk by Fülöp this morning



Reduce p_O by enhancing pitch-angle diffusion

- Runaway vortex forms around an O point where $\Gamma_p = \Gamma_\xi = 0$.
- Approximate pitch-angle diffusion: $\frac{\partial \ln f}{\partial \xi} \sim (\sqrt{2}\Delta\xi_p)^{-1}$ with $\Gamma_p(\Delta\xi_p) = 0$

$$\boxed{\Gamma_p = \left[(-\xi)E - \frac{1+p^2}{p^2} - \alpha p\gamma(1-\xi^2)\right] f}$$

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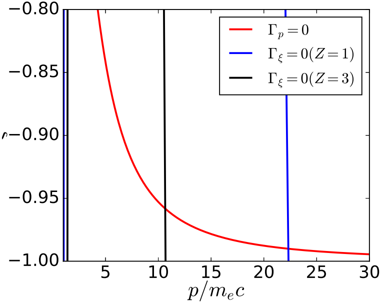

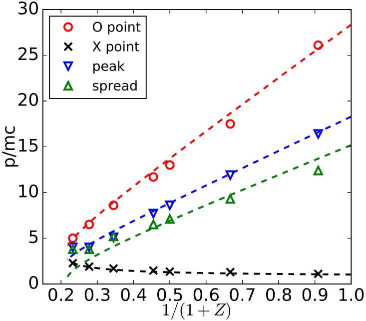

Enhance scattering via wave-particle interaction

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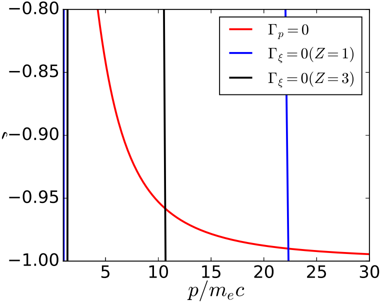


- Whistler wave is a good candidate for pitch-angle scattering of relativistic electrons [Lyon JPP1971; Guo PRL2012]
- We can use it to control runaway energy!?

Capture wave-particle interaction with quasilinear diffusion

- In general, the relativistic quasilinear diffusion of electromagnetic waves can be written as [Lerche POF68, Lyons JPP71]

$$\mathbb{C}(f) = \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[\mathbb{D}_{pp}\frac{\partial f}{\partial p} + \frac{\sqrt{1-\xi^2}}{p}\mathbb{D}_{p\xi}\frac{\partial f}{\partial \xi}\right] + \frac{1}{p}\frac{\partial}{\partial \xi}\left[\sqrt{1-\xi^2}\mathbb{D}_{\xi p}\frac{\partial f}{\partial p} + \frac{1-\xi^2}{p}\mathbb{D}_{\xi\xi}\frac{\partial f}{\partial \xi}\right]$$

- The diffusion coefficients ($\mathbb{D}_{p\xi} = \mathbb{D}_{\xi p}$) are

$$\mathbb{D}_{pp} = \frac{2\omega_{pe}^2\tau_c}{\omega_{ce}}(1-\xi^2)\sum_{n=-\infty}^{n=\infty}\int d^3\mathbf{k}\mathcal{E}_{n,k}\delta(\omega_{kr} - k_\parallel v_\parallel + \frac{n}{\gamma})$$

$$\mathbb{D}_{p\xi} = -\frac{2\omega_{pe}^2\tau_c}{\omega_{ce}}\sqrt{1-\xi^2}\sum_{n=-\infty}^{n=\infty}\int d^3\mathbf{k}\mathcal{E}_{n,k}\delta(\omega_{kr} - k_\parallel v_\parallel + \frac{n}{\gamma})(\xi - \frac{k_\parallel v}{\omega_{kr}})$$

$$\mathbb{D}_{\xi\xi} = \frac{2\omega_{pe}^2\tau_c}{\omega_{ce}}\sum_{n=-\infty}^{n=\infty}\int d^3\mathbf{k}\mathcal{E}_{n,k}\delta(\omega_{kr} - k_\parallel v_\parallel + \frac{n}{\gamma})(\xi - \frac{k_\parallel v}{\omega_{kr}})^2$$

  - ω normalized to ω_{ce}, k to ω_{ce}/c, t to τ_c;
  - For ITER-like plasma, $\omega_{pe}\tau_c \sim 10^{10}$ and $\omega_{pe} \sim \omega_{ce}$
  - The perturbation energy density normalized to electron inertial energy density

$$\mathcal{E}_{n,k} = \frac{1}{V(2\pi)^3}\frac{|\Psi_{n,k}|^2}{8\pi n_0 m_e c^2}\frac{\omega_{ce}^3}{c^3}.$$

for $\delta B_{n,k} \sim 10^{-5}B_0, \mathcal{E}_{n,k} \sim 10^{-10}$

Whistler wave dispersion and resonance condition

- Whistler waves are effective in inducing pitch-angle scatterings of electrons in space [Kennel& Engelmann PF66] and laboratory [Guo PRL12] plasmas;
- For $\omega_{ci} \ll \omega \ll \omega_{ce}$, $\omega_{ce}|k_\parallel/k| \gg \omega$, a simple dispersion for whistler wave

$$\omega_k = k|k_\parallel|\frac{v_A^2}{c^2}\frac{\omega_{ce}}{\omega_{ci}}$$

  - $\frac{v_A^2}{c^2}\frac{\omega_{ce}}{\omega_{ci}} \sim 1$ (ratio between magnetic and electron inertial energy densities) for ITER plasma;
- Two primary cyclotron resonances:
  - $n = -1$, normal Doppler resonance [Stix WIP83, Davidson BPP92]

$$\omega_{kr} - k_\parallel v_\parallel - \gamma^{-1} = 0$$

  - $n = 1$, anomalous Doppler resonance [Fülöp POP06, Aleynikov NF15]

$$\omega_{kr} - k_\parallel v_\parallel + \gamma^{-1} = 0$$

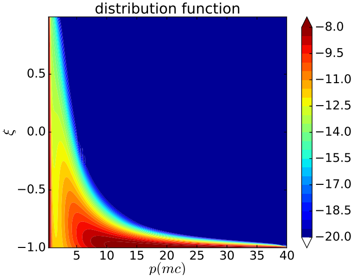
- Assume a spectrum with a fixed $k_\perp$ and a narrow band of $k_\parallel$, so that

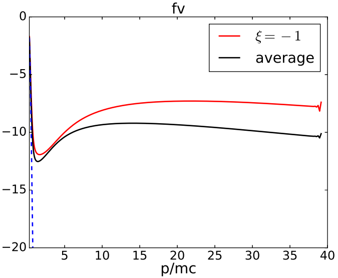
$$\mathcal{E}_{\pm 1,k} = \mathcal{E}_0 \frac{\delta(k_\perp - k_{\perp 0})}{(\sqrt{2\pi})^3 k_\perp \Delta k}\exp\left[-\frac{(k_\parallel - k_{\parallel 0})^2}{2\Delta k^2}\right]$$

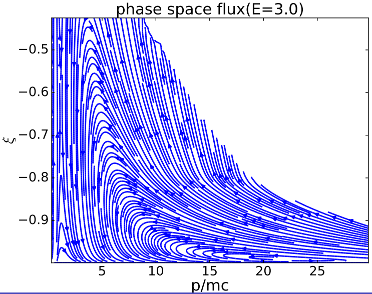
where $\int d^3\mathbf{k}\mathcal{E}_{\pm,k} = \mathcal{E}_0$, and only consider the primary harmonics $n = \pm 1$;

Chopping the runaway vortex

- No wave, $E = 3.0E_c$, $\alpha = 0.2$, $v_t = 0.1c$, a long high-energy tail forms;







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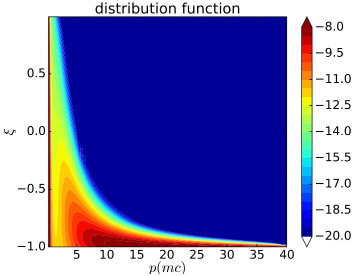
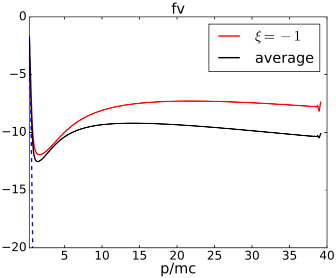
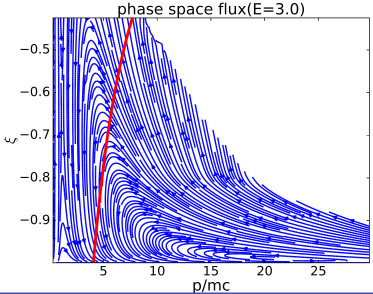


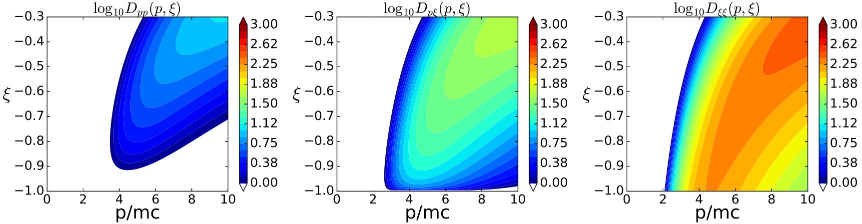




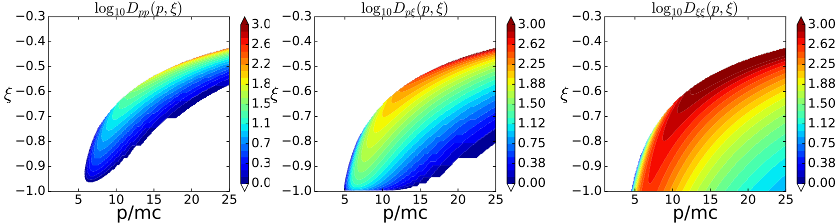
- Idea: locate the resonance to the left of O point & at low enough energy $\rightarrow$ chopping the vortex to kill high-energy runaways

Normal Doppler resonance is preferred

- Normal Doppler resonance, $k_{\parallel 0} = 0.2, k_{\perp 0} = 0.1, \mathcal{E}_0 = 10^{-9}, \Delta k = 0.05$



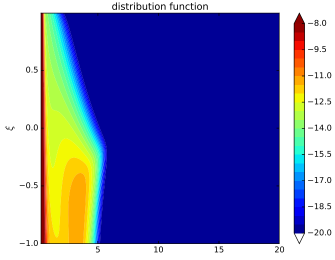
- Anomalous Doppler resonance, $k_{\parallel 0} = -0.2, k_{\perp 0} = 0.1, \mathcal{E}_0 = 10^{-9}, \Delta k = 0.05$

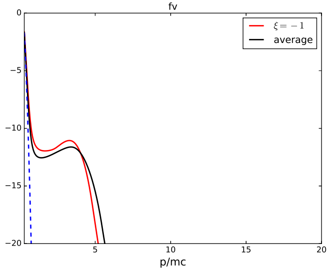


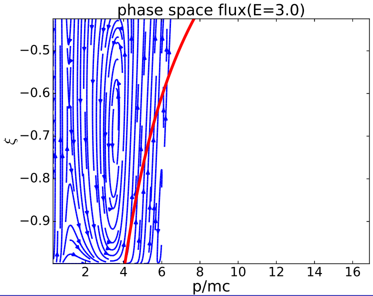
- The normal Doppler resonance tends to peak at lower energy comparing to the anomalous Doppler resonance, so the former is preferred for our purpose;

Whistler waves can limit the RE energy!

- With wave, $k_{\parallel 0} = 0.2, k_{\perp 0} = 0.1, \Delta k = 0.05, \mathcal{E}_0 = 5 \times 10^{-10}$;







- The long high-energy runaway tail above $\sim 2MeV$ is removed by the resonance;

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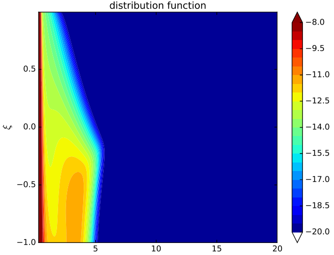
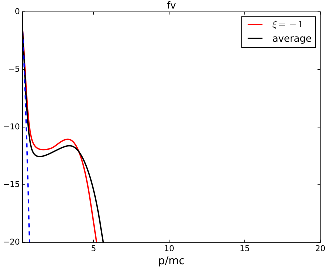
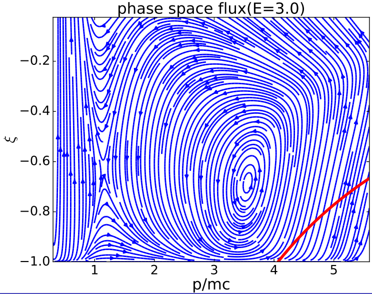
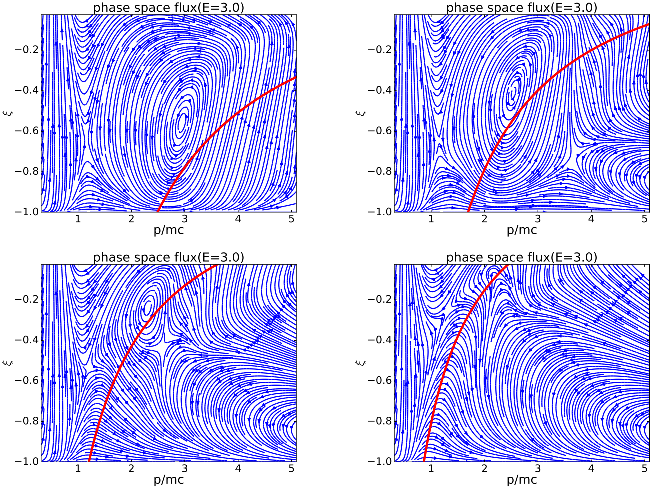






- The long high-energy runaway tail above $\sim 2MeV$ is removed by the resonance;
- A new vortex is created at lower energy below the center of resonance due to whistler waves;

Reduce resonance energy → the new vortex shrinks



- Pushing the resonance to lower energy $(k_{\|0} = 0.3, 0.4, 0.5, 0.6)$ → lower energy/higher pitch O point & vortex volume decreases;
- The X point moves to higher pitch quickly once the resonance is close to it;
- The new vortex eventually disappears when O/X points merge;

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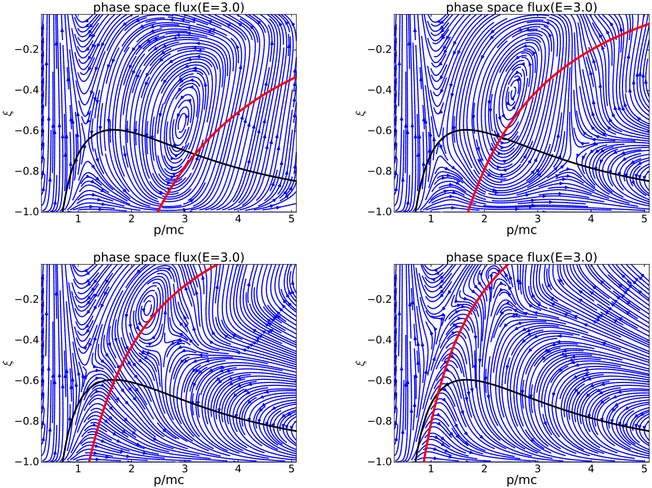

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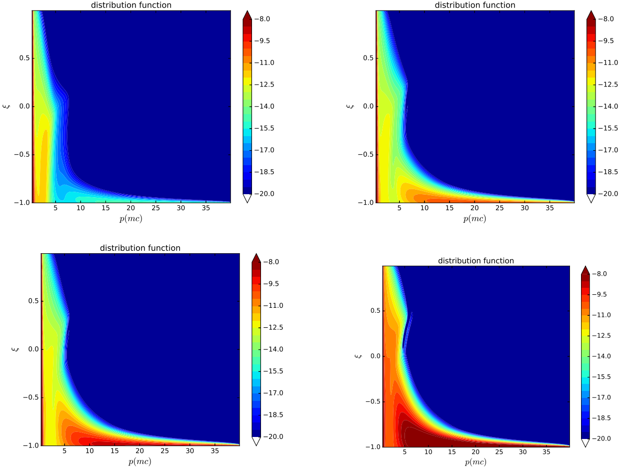
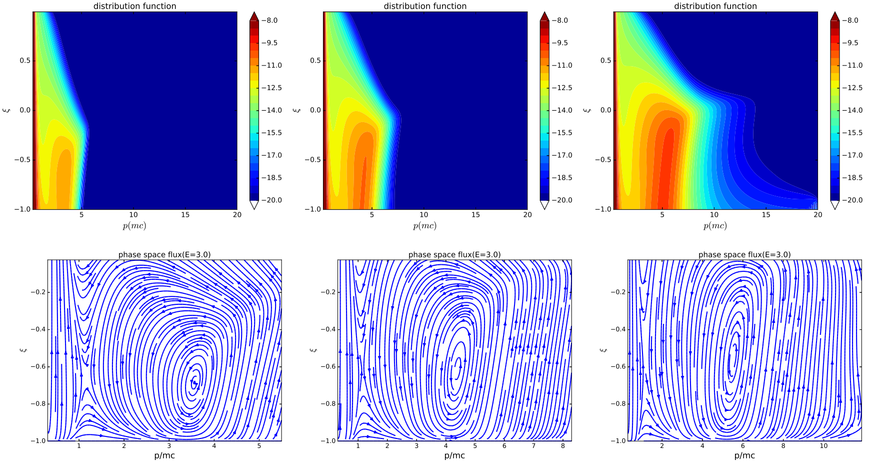
Resonance energy controls the RE distribution



- Runway electrons start to accumulate in the original vortex again as the new vortex's volume decreases; Don't overdo it!

Dependence on magnetic field strength

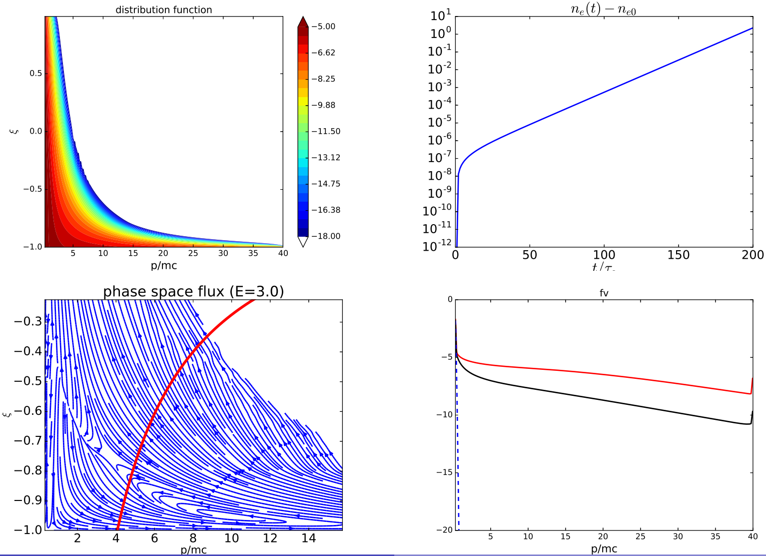
- The synchrotron damping parameter $\alpha \propto B^2 n^{-1}$
- Scan $\alpha = 0.2, 0.1, 0.05$, $k_{\|0} = 0.2, k_{\perp 0} = 0.1, \mathcal{E}_0 = 5 \times 10^{-10}, \Delta k = 0.05$



- Since the synchrotron damping is proportional to $\alpha p \gamma (1 - \xi^2)$, the O point energy only increases weakly with $\sqrt{\alpha}$;

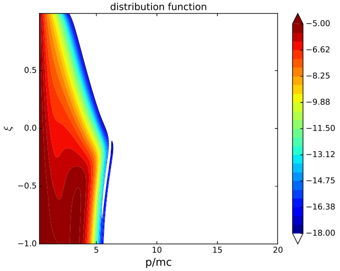
Chopping the vortex during avalanche growth

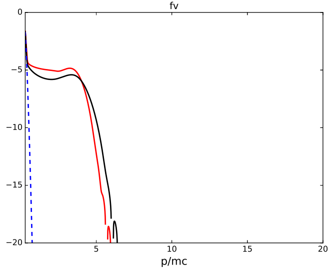
- Using chiu's avalanche source [Chiu NF98], snapshot at $J_{re} \simeq 2.5MA$;
- Unmitigated avalanche shows a long high-energy tail;

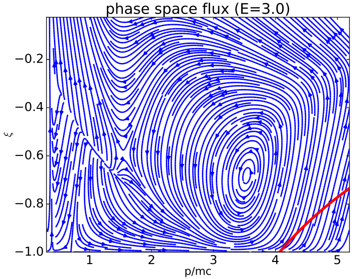


The mechanism works during avalanche growth!

- Whistler wave, $k_{\|0} = 0.2, k_{\perp 0} = 0.1, \Delta k = 0.05, \mathcal{E}_0 = 5 \times 10^{-10}$;

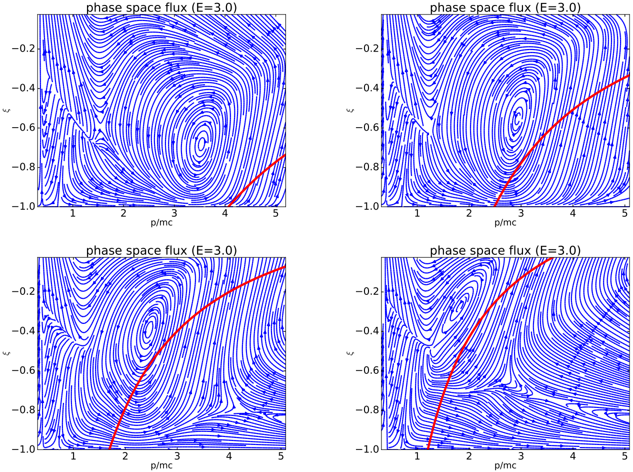






- A new vortex is created at lower energy below the center of resonance due to whistler waves;
- Avalanche electrons can be limited to lower energy by wave injections;

Reduce resonance energy $\rightarrow$ the new vortex shrinks



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- Runway electrons start to accumulate in the original vortex again as the new vortex's volume decreases; Don't overdo it!

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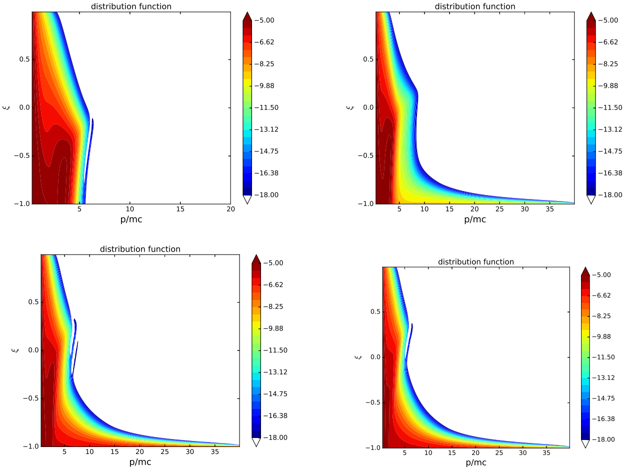

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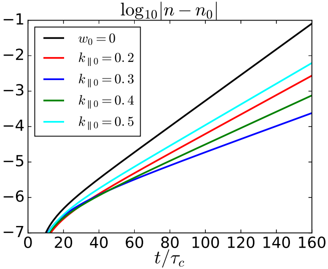
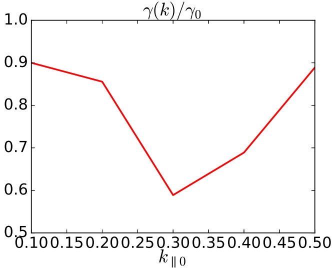
The avalanche growth rate is affected

- The low energy vortex is still present during avalanche, and keeps a large population of runaways at relatively low energy (MeV);
- The growth rate reaches a minimum near a particular $k_{\parallel 0} = 0.3$ with given parameters;
- The whistler waves can be applied to lift the avalanche threshold electric field;
- The vortex appears at larger pitch which may be favorable in tokamak geometry where trap electrons can not runaway;





Conclusion

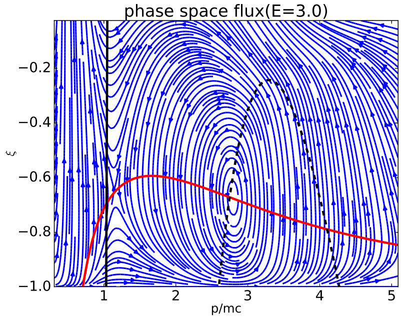
- The vortex is a local circulation of runaway electrons in momentum-space, which results from the competition between the parallel electric field acceleration, the Coulomb collision and the radiational damping force.
- The vortex determines the energy distribution of runaway electrons, and governs the threshold of avalanche growth;
- Pitch-angle diffusion due to Coulomb collisions alone becomes ineffective at relativistic energy $> MeV$, and thus the vortex extends to tens of MeV even with moderate electric fields;
- A narrow spectrum of small amplitude whistler waves ($\delta B < 10^{-4} B_0$) can effectively enhance the momentum scatterings in the MeV energy range through the normal Doppler cyclotron resonance. A new vortex forms below the peak of resonant energy $\sim MeV$;
- By properly choosing the wave (thus the resonance), the runaway electrons can be limited to below a few MeV in the presence of avalanche growth;
- Future work: the new vortex also appears at higher pitch, so the toroidal effect (finite trap-region) becomes very important;

Enhanced diffusion $\Rightarrow$ local "increase" of Z

- For simplicity, we can consider the pitch-angle diffusion alone;
- The O point is where $\Gamma_p = 0$ and $\Gamma_\xi = 0$ [Guo PPCF17]

$$\boxed{\Gamma_p = \left[(-\xi)E - C_F - \alpha p\gamma(1-\xi^2)\right] f}$$

$$\boxed{\Gamma_\xi = -(1-\xi^2)\left(Ef - \alpha\frac{p}{\gamma}\xi f + \frac{1+Z}{2}\frac{\gamma}{p^2}\frac{\partial f}{\partial \xi} + \mathbb{D}_{\xi\xi}\frac{\partial f}{\partial \xi}\right)}$$



- The X point is not affected as long as the resonance is away from it;
- The wave induced pitch-angle diffusion corresponds to a local increase of Z near the resonance $(1 \rightarrow \sim 6.5)$;

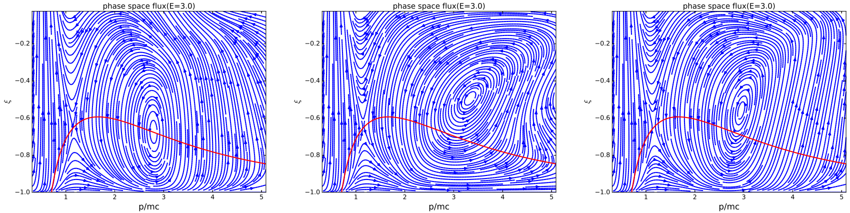
Energy/cross diffusions modify the O point location

- Energy and pitch-angle fluxes [Guo PPCF17]

$$\boxed{\Gamma_p = \left[(-\xi)E - C_F - \alpha p\gamma(1-\xi^2)\right] f}$$

$$\boxed{\Gamma_\xi = -(1-\xi^2)\left(Ef - \alpha\frac{p}{\gamma}\xi f + \frac{1+Z}{2}\frac{\gamma}{p^2}\frac{\partial f}{\partial \xi} + \mathbb{D}_{\xi\xi}\frac{\partial f}{\partial \xi}\right)}$$

- From left to right: only $\mathbb{D}_{\xi\xi}$, $\mathbb{D}_{\xi\xi} + \mathbb{D}_{pp}$, $\mathbb{D}_{\xi\xi} + \mathbb{D}_{pp} + \mathbb{D}_{p\xi}$



- Both energy and cross diffusions introduce additional energy flux, so the O point moves slightly to larger energy and higher pitch;

Energy/cross diffusions modify the O point location

- Energy and pitch-angle fluxes [Guo PPCF17]

$$\boxed{\Gamma_p = \left[E(-\xi) - C_F - \alpha p\sqrt{1+p^2}(1-\xi^2) - \mathbb{D}_{pp}\frac{\partial \ln f}{\partial p}\right] f}$$

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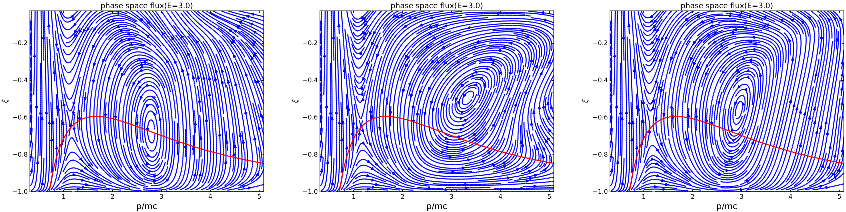

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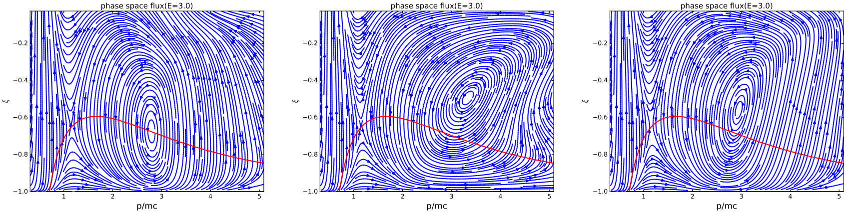

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