Energetic ion effects on disruptive instabilities with plasma rotation and a resistive wall

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Theory and Simulation of Disruptions Workshop
July 17-19, 2017

Princeton University

PPPL
PRINCETON PLASMA PHYSICS LABORATORY

MOTIVATION

The severe risk of disruptions should motivate us to redouble our efforts in avoidance
- The role of stability maps is crucial to machine learning based control.

Knowledge of the physics mechanisms leading to disruptions is key to control.
- What are the reasons for the mode growth and locking?
- Kinetic effects play a crucial role in mode onset, but how do we use that knowledge for control?

Understand and avoid.

Equilibrium states effective to calculate a stability map

A target discharge reconstruction can be used to build a "family" of equilibria around it that capture changes to the equilibrium state as experimental trajectories of the flattop state evolve.

Sometimes MHD alone determines the onset.

Sometimes it does not.

But, we have now come to the point where almost all known reasons for onset of disruptive instabilities are known and could be avoided with control.

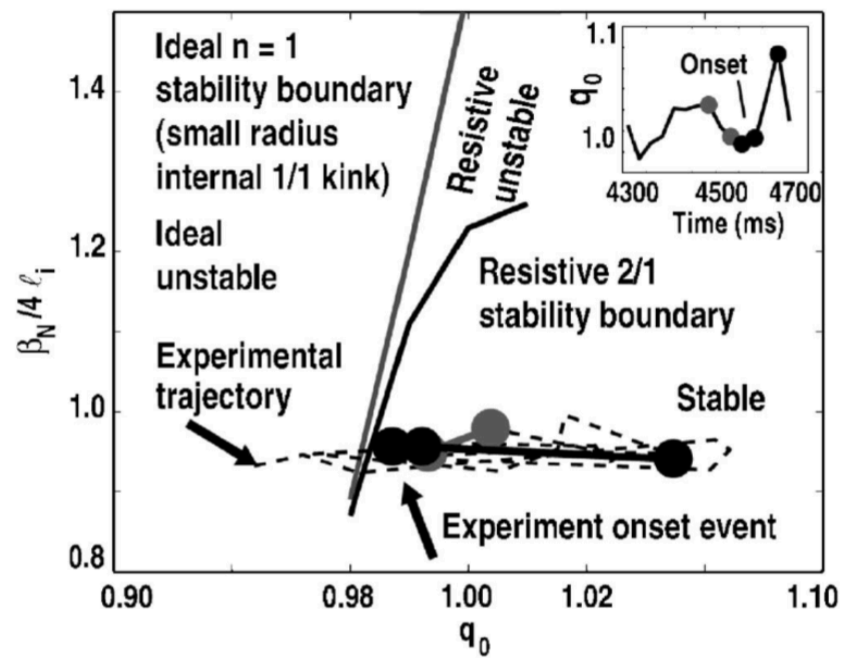

FIG. 9. The ideal and resistive stability boundaries in $(q_0, \beta_N/4\ell_i)$ space, and the experimental trajectory for the period leading up to onset of the resistive 2/1 mode. The inset shows the q_0 as a function of time at the point of onset, corresponding to the circles in the trajectory.

This talk: Magnetic shear and rotation affect both MHD and kinetic drive to resistive stability

OUTLINE

Puzzle: Why is 2/1 sometimes stabilized and others destabilized by energetic ions?

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- Brennan, Kim, La Haye Nucl. Fusion **52**, 033004 (2012). - 2/1 **destabilized** in reversed shear
- WHY? Need a reduced model to explain sims and experiments.

Extending a reduced MHD cylindrical model for the 2/1 tearing mode with energetic ions

- Brennan, Finn Phys. Plasmas **21**, 102507 (2014).
- Extended to include reversal in q, two pressure steps inside and out of sheared region.
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Energetic ion pressure contribution in core significantly effects the stability

- With positive shear particles are damping and stabilizing
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Concluding Remarks / Ideas for the Future

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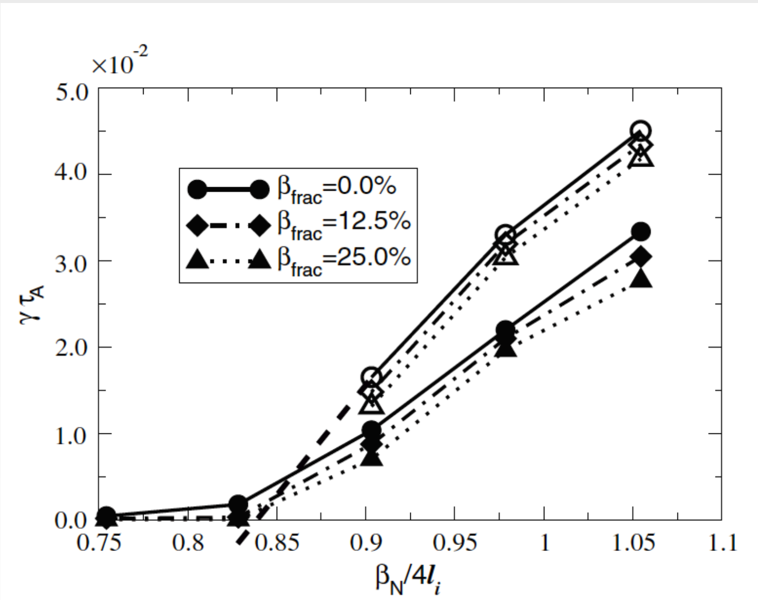
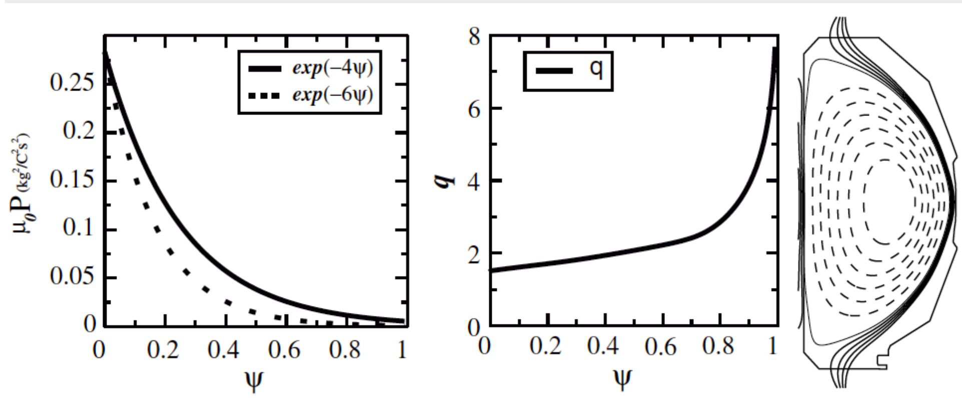
Slowing down distribution of energetic ions found to damp the 2/1 mode with monotonic shear





Results from δf PIC + MHD simulation in NIMROD show significant damping effect on resistive mode.
NOTE: q shear and pressure gradient extend to magnetic axis.

Takahashi, Brennan, Kim Phys. Rev. Lett **102**, 135001 (2009).

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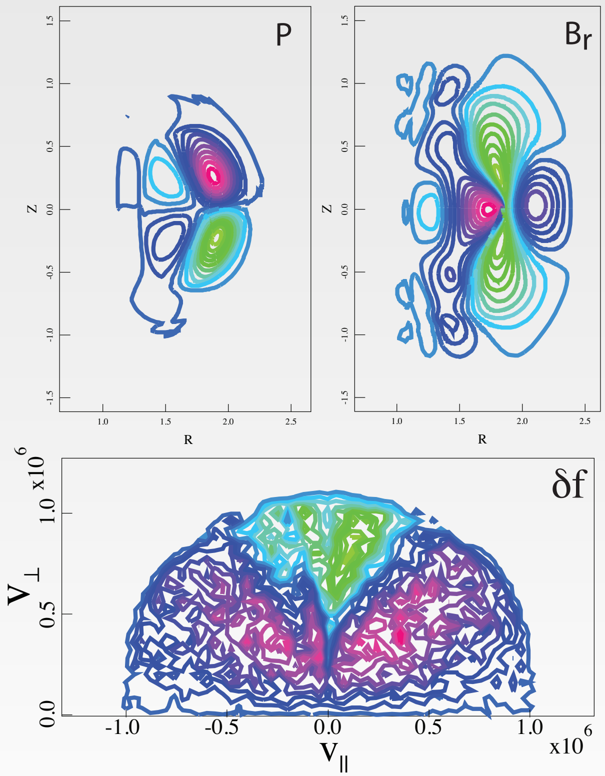


n=1 projection of δf shows effect dominantly from trapped and barely passing.

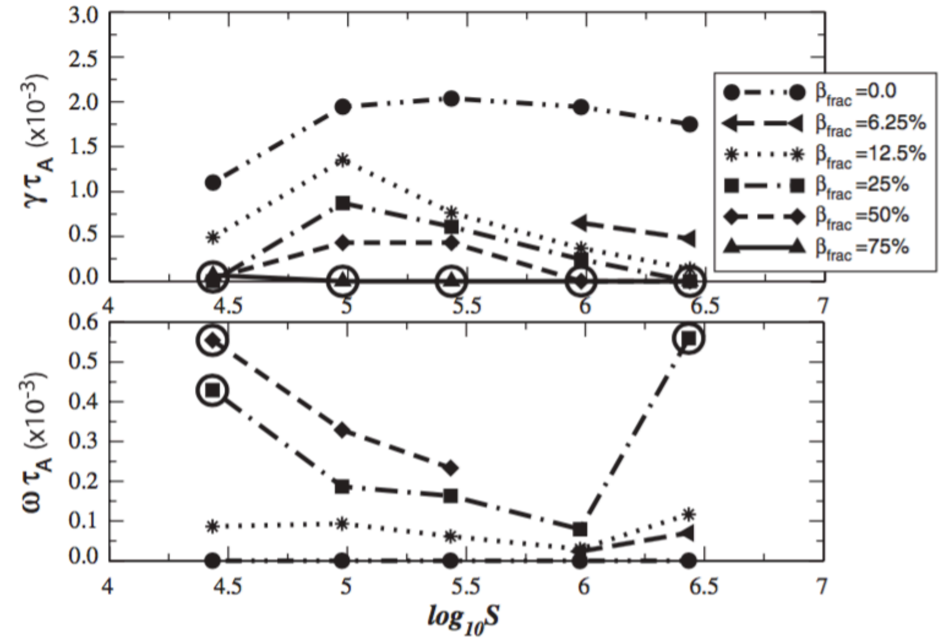


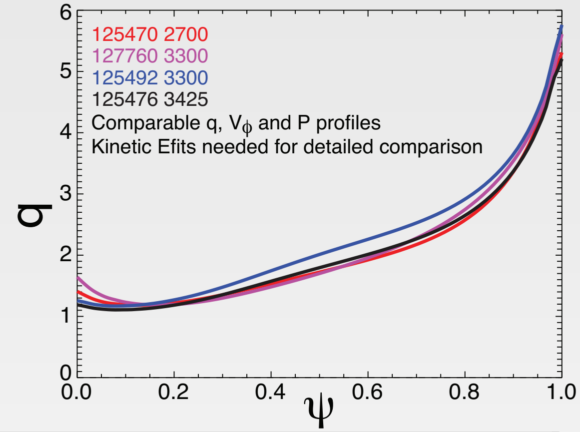
FIG. 4. The growth rates (upper panel) and the real frequency (lower panel) of the equilibrium $\beta_N/4l_i = 0.83$ for various $\beta_{\rm frac}$ as a function of S. Note that due to the adjustment of β_0, the equilibrium $\beta_N/4l_i$ does not change for various $\beta_{\rm frac}$.

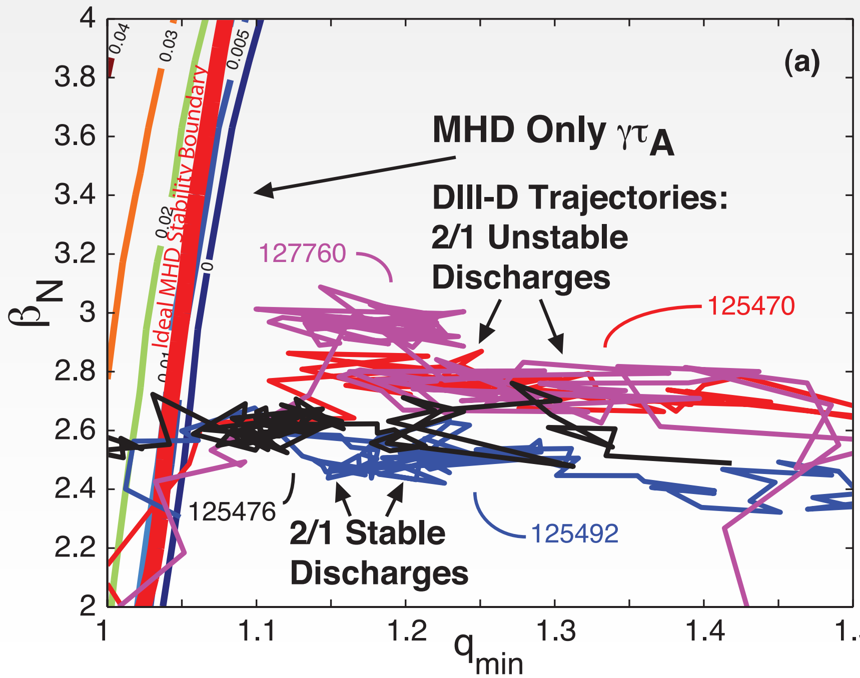
Takahashi, Brennan, Kim Phys. Rev. Lett **102**, 135001 (2009).

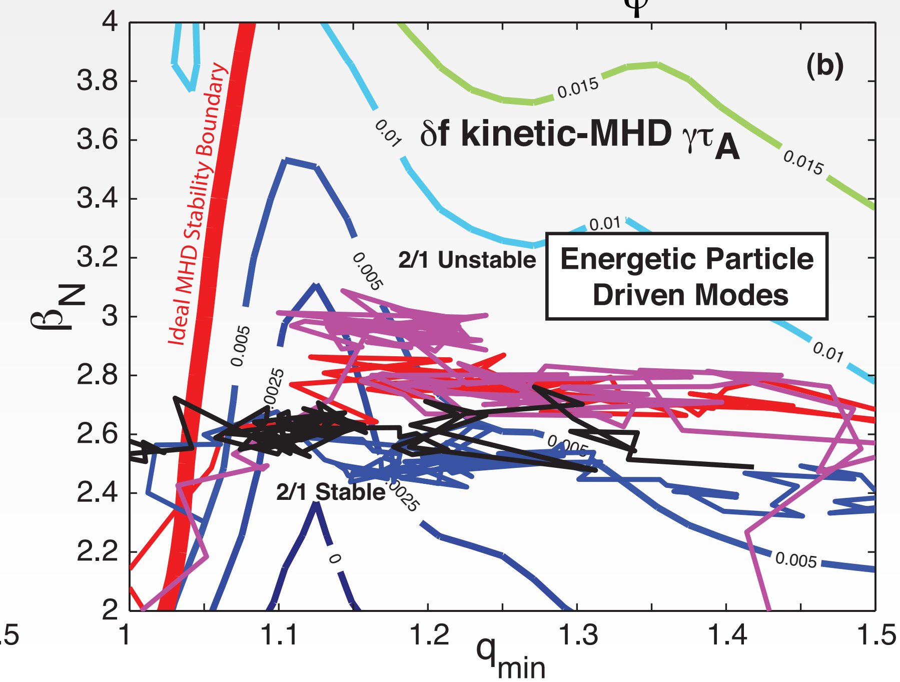
Stability map with δf kinetic – MHD shows 2/1 mode destabilized by energetic ions in reversed shear

Destabilization well into high q_{min} regime
Experimental trajectory in a low growth rate region
Gradient in increasing β_N direction, mode destabilized
Resistive instability significant at $\gamma\tau_A$~0.005

D.P. Brennan et al. Nucl. Fusion 2012



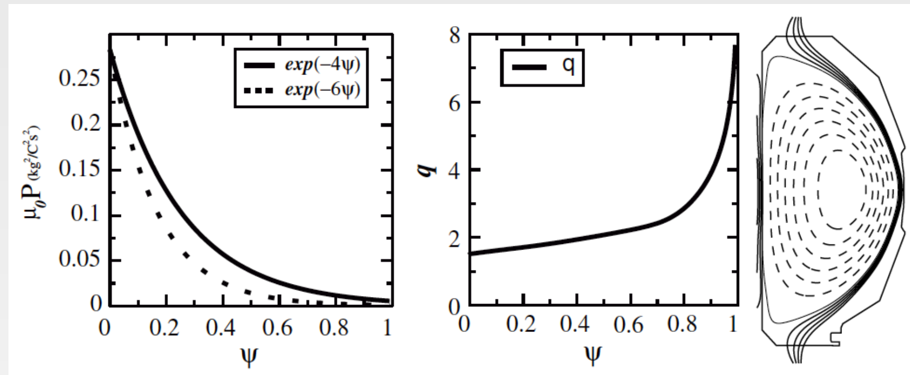




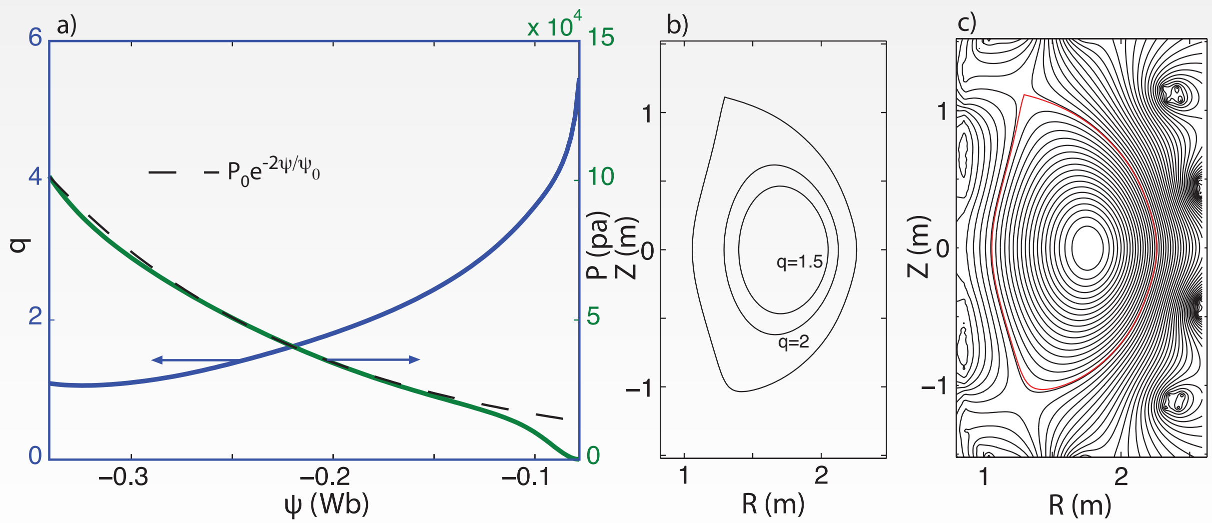
Puzzle: Why is it damping in sheared and destabilizing in the reversed shear case?

Robustly damping and stabilizing for equilibria with monotonic q.

Robustly destabilizing for weakly sheared or reversed q profiles.



Initiate a reduced modeling effort to address the physics.



R. Takahashi, D.P. Brennan and C.C. Kim PRL 102, 135001 (2009) vs.
D.P. Brennan et al. Nucl. Fusion 2012

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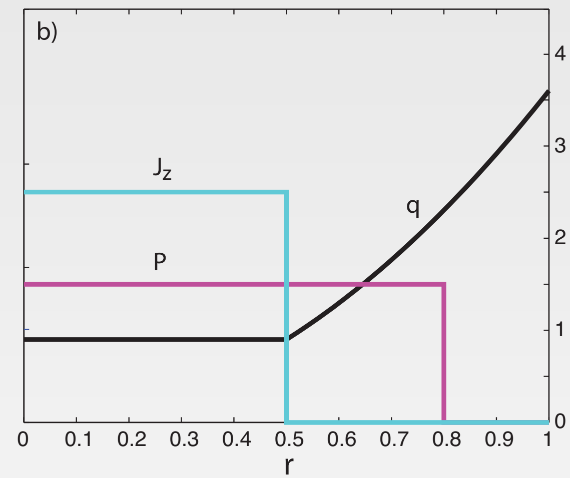
Consider single pressure step function profiles for P and J: simplest reduced MHD cylindrical model

The tearing stability equation becomes

$$\nabla_\perp^2 \psi = \frac{m j'_{z0}(r)}{rF(r)}\psi + \frac{2m^2 B_{\theta 0}^2(r) p'_0(r)}{B_0^2 r^3 F(r)^2}\psi$$

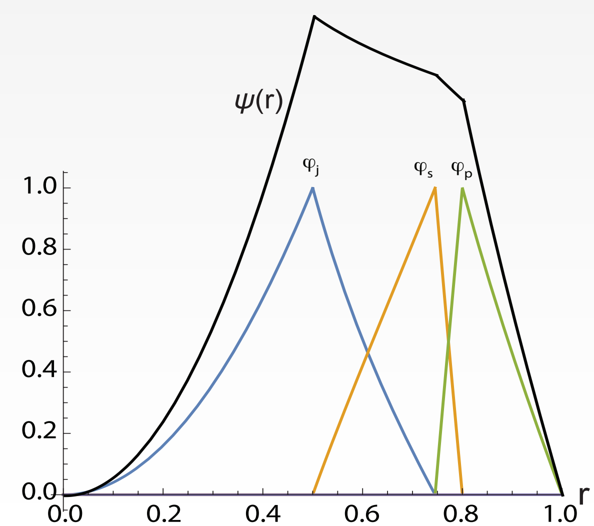
$$\nabla_\perp^2 \psi = -A\delta(r-a_1)\psi - B\delta(r-a_2)\psi$$



Solving for the eigenmode the Δ' becomes

$$\Delta' = [\psi'_s]_{a_s} + \frac{\varphi'_s(a_j^+)\varphi'_j(a_s^-)}{[\varphi'_j]_{a_j} + \dfrac{j_0 m}{a_j F(a_j)}}$$

$$+ \frac{\varphi'_s(a_p^-)\varphi'_p(a_s^+)}{[\varphi'_j]_{a_p} + \dfrac{2mB_\theta(a_p)\beta_0}{a_p^2 F(a_p)}\left(\dfrac{m}{a_p F(a_p)} - \lambda\beta_{frac}\right)}$$



More on this method: Brennan, Finn Phys. Plasmas **21**, 102507 (2014).

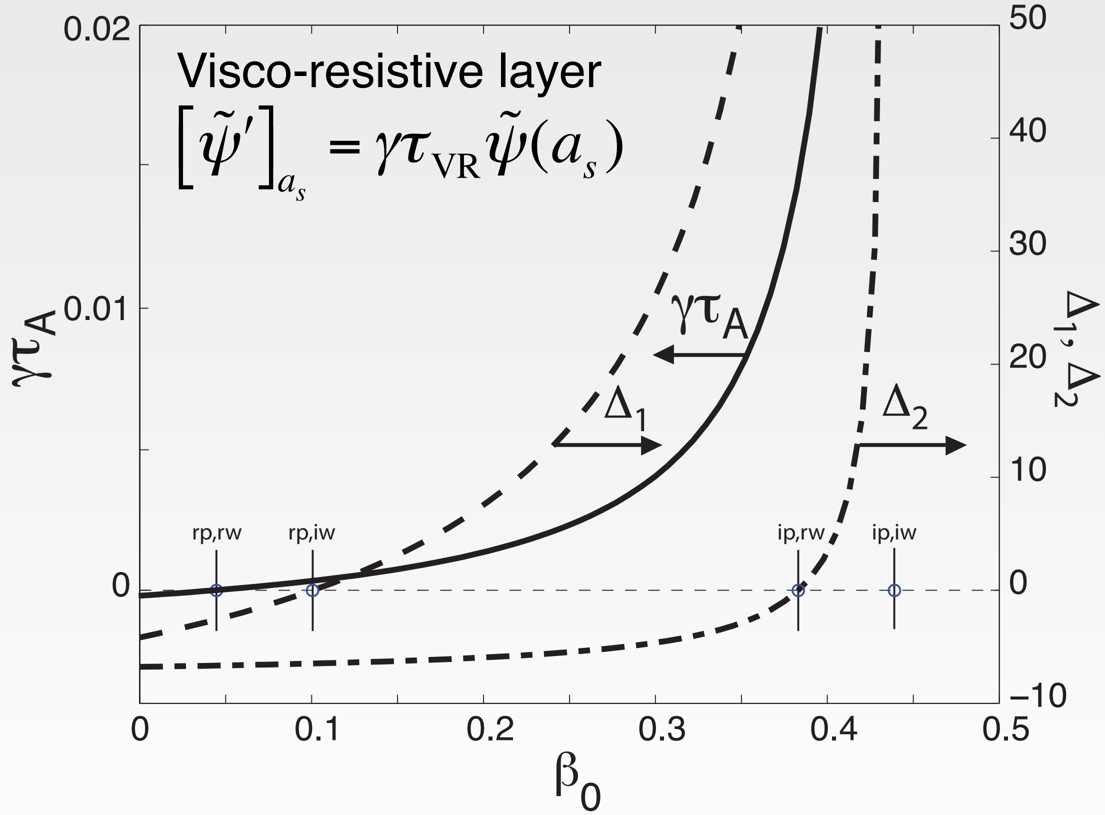
The effect of resistivity in the plasma and/or the wall can be seen in terms of 4 β limits: β_{rprw}, β_{rpiw}, β_{iprw} and β_{ipiw}

- Ip: ideal plasma
- Rp: resistive plasma
- Iw: ideal wall
- Rw: resistive wall

Here we focus only on the rpiw model and introduce the energetic ion effects.

The same model could easily be extended to include the rw effects and flow, etc.

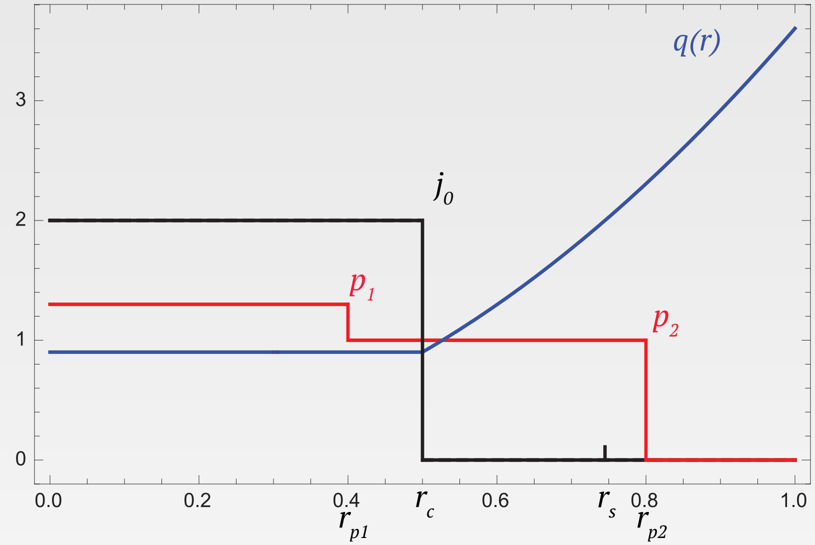
More: Brennan, Finn Phys. Plasmas **21**, 102507 (2014).

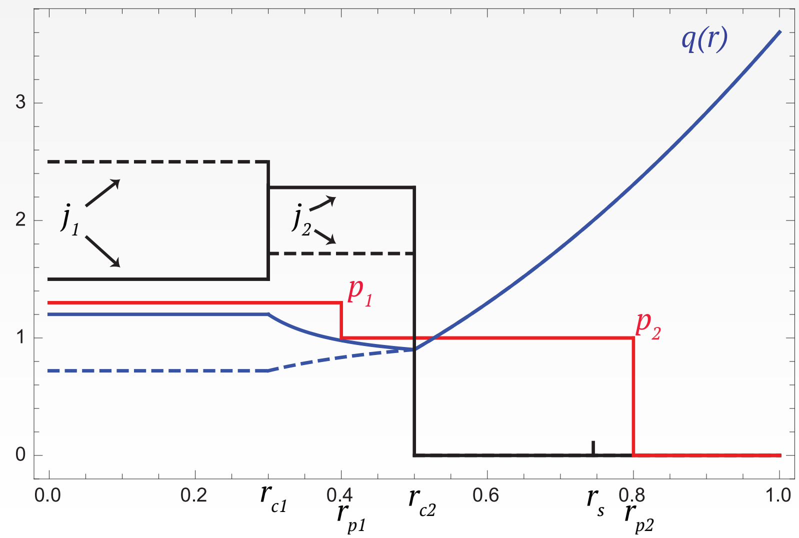


Δ_1, Δ_2 are the stability parameters for the rpiw and iprw models respectively

Consider an equilibrium configuration with a second pressure step inside the plasma column

- Second pressure step in zero shear region.
- Results with weak shear, reversed or not, from second current step indicate qualitative similar results
- Each pressure drive enters the stability equation separately due to the geometric configuration.
- Stability solver becomes more complicated but analogous to simple single step structure.
- At their separate step functions, local shear affects their contribution, -> stabilizing or destabilizing





Energetic ions: reduced model of EP effect on RWM studied by Hu & Betti (PRL 2004)

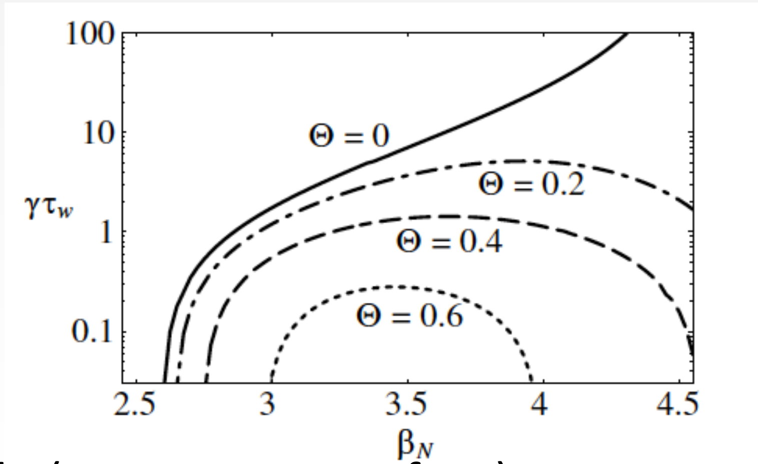
- The energetic particle pressure contribution takes the form of a scalar modification to the perturbed pressure.

$$\tilde{p}_j^m = \frac{2^{5/2}\epsilon^{1/2}}{5\pi^{3/2}} \int_0^\infty d\hat{v}\hat{v}^5 e^{-\hat{v}^2} \int_0^1 du K(u) \Pi_j \sigma_m \sum_{\ell=-\infty}^{+\infty} \sigma_\ell Y_\ell^j$$

- In their work, this was then placed into a δW calculation to determine the stability of the resistive wall mode.

$$\delta W_K = \frac{1}{2} \sum_{j=i,e} \int d\mathbf{r} (\tilde{\boldsymbol{\xi}}_\perp^* \cdot \boldsymbol{\kappa}) \tilde{p}_j^K$$

$$\gamma \tau_w \simeq -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$



- Did not take into account the tearing mode (no resonant surface)

EP pressure integral models the resonant interaction of trapped particles with mode structure

- Where

$$\Pi_j = -N_j \frac{R}{2}\frac{dT_j}{dr}\frac{\hat{v}^2 - \frac{3}{2} + \frac{l_{T_j}}{l_{N_j}} + 2\frac{l_{T_j}}{R} w_E^j}{w_E^j + \hat{v}^2 H(u)}$$

$$\sigma_m = \int_0^{\pi/2} d\chi \frac{\cos[2(m-q)\arcsin(\sqrt{u}\sin\chi)]}{K(u)\sqrt{1-u\sin^2\chi}}$$

$$w_E^j = \frac{\omega_E}{\bar{\omega}_B^j},\ l_{T_j} = -T_j/(dT_j/dr),\ l_{N_j} = -N_j/(dN_j/dr)$$

$$\bar{\omega}_B^j = \frac{q v_{th}^2}{\Omega_c R r},\ \ H(u) = (2s+1) + \frac{E(u)}{K(u)} + 2s(u-1) - \frac{1}{2}$$

- u is the pitch angle variable, q is charge, Ω_c is the cyclotron frequency, and s is the magnetic shear.
- The step function characteristic of the equilibrium pressure enters the pressure moment through the temperature gradient in Π_j

Approximation of Magnetic curvature is key component to capture trapped particle dynamics

- In the high aspect ratio, circular cross section model, the magnetic curvature has the form:

$$\vec{\boldsymbol{\kappa}} = -\frac{B_\theta^2}{rB_0^2}\hat{\boldsymbol{r}} - \frac{\widehat{\boldsymbol{R}_{\mathbf{0}}}}{R_0 + r\cos\theta}$$

- Which can be written as a flux average and poloidally varying term.

$$\langle \vec{\boldsymbol{\kappa}} \cdot \hat{\boldsymbol{r}} \rangle = -\frac{r}{R_0^2}\left(\frac{1}{2} - \frac{1}{q^2}\right),\ (\vec{\boldsymbol{\kappa}} \cdot \hat{\boldsymbol{r}})^{\sim} = -\frac{\cos\theta}{R_0}$$

$$\vec{\boldsymbol{\kappa}} = \vec{\boldsymbol{\kappa}}_{\mathbf{0}} + \vec{\boldsymbol{\kappa}}_{\mathbf{1}}\cos\theta$$

Particle pressure moment reduced to scalar coefficient on equilibrium pressure, to enter into tearing model

Using the coupling to curvature, we can determine the poloidal harmonics of the energetic pressure perturbation

$$Y_l^j = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-il\theta} (\hat{v}^2 \tilde{\xi}_\perp \cdot \kappa + \frac{Z_j e}{T_j} \tilde{Z})$$

Due to the poloidal dependence of $\tilde{\xi}_\perp$ this reduces to:

$$Y_l^j = \hat{v}^2 [(\hat{\xi}_r \kappa_0 + \frac{Z_j e}{T_j} \tilde{Z}) \delta_{l=m} + \hat{\xi}_r \kappa_1 (\delta_{l=m+1} + \delta_{l=m-1})]$$

Thus, the particle pressure has the form:

$$\tilde{p}_j^m = \int_0^\infty d\hat{v}^5 f_0(\hat{v}) \int_0^1 du K(u) \Pi_j \sigma_m \hat{v}^2 \left\{ \left(\tilde{\xi}_r \kappa_0 + \frac{Z_j e}{T_j} \tilde{Z} \right) \sigma_m + \tilde{\xi}_r \kappa_1 (\sigma_{m-1} + \sigma_{m+1}) \right\}$$

Which reduces to $\tilde{p}_j^m = \lambda p_0$

Where λ contains all the information of the energetic particle response to the fields

Energetic particle contribution enters stability equation at pressure step

In general, we solve the ideal outer region equations by solving for the jump conditions at the resonant surface, and pressure and current steps.

Particle pressure only enters at pressure steps, proportional to δP_0

For single pressure step, this is simply

Particle Pressure Term

β_{frac} is ratio of β's

$$[\tilde{\psi}']_{a_p} + \frac{2mB_\theta(a_p)\beta_0}{a_p^2 F(a_p)}\left(\frac{m}{a_p F(a_p)} - \lambda\beta_{frac}\right)\tilde{\psi}(a_p) = 0$$

$$\Delta' = [\psi_s']_{a_s} + \frac{\varphi_s'(a_j^+)\varphi_j'(a_s^-)}{[\varphi_j']_{a_j} + \frac{j_0 m}{a_j F(a_j)}} + \frac{\varphi_s'(a_p^-)\varphi_p'(a_s^+)}{[\varphi_j']_{a_p} + \frac{2mB_\theta(a_p)\beta_0}{a_p^2 F(a_p)}\left(\frac{m}{a_p F(a_p)} - \lambda\beta_{frac}\right)}$$

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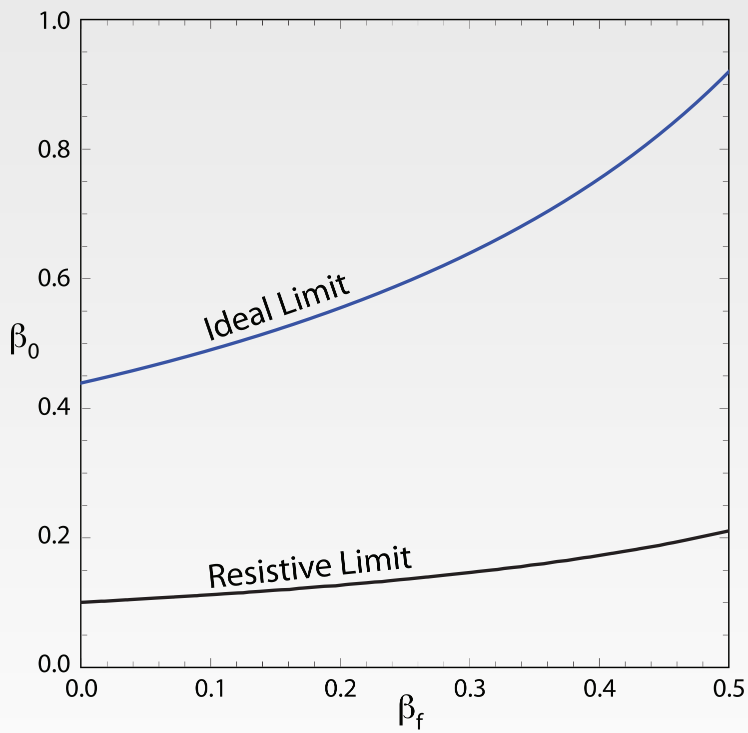
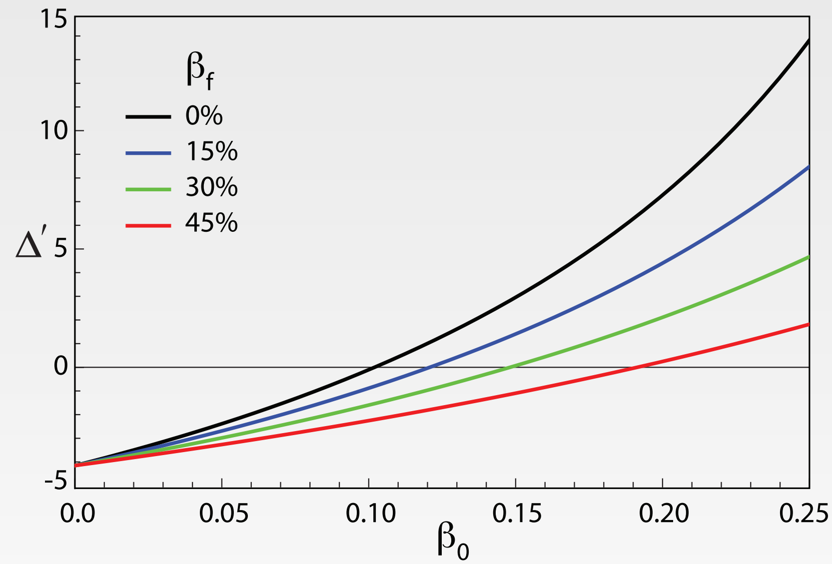
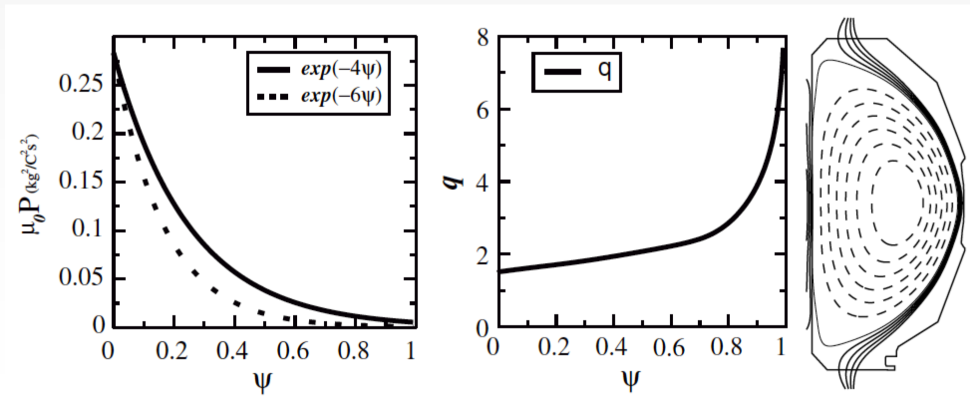
Single pressure step in outer radius (in shear) indicates damping and stabilizing effect

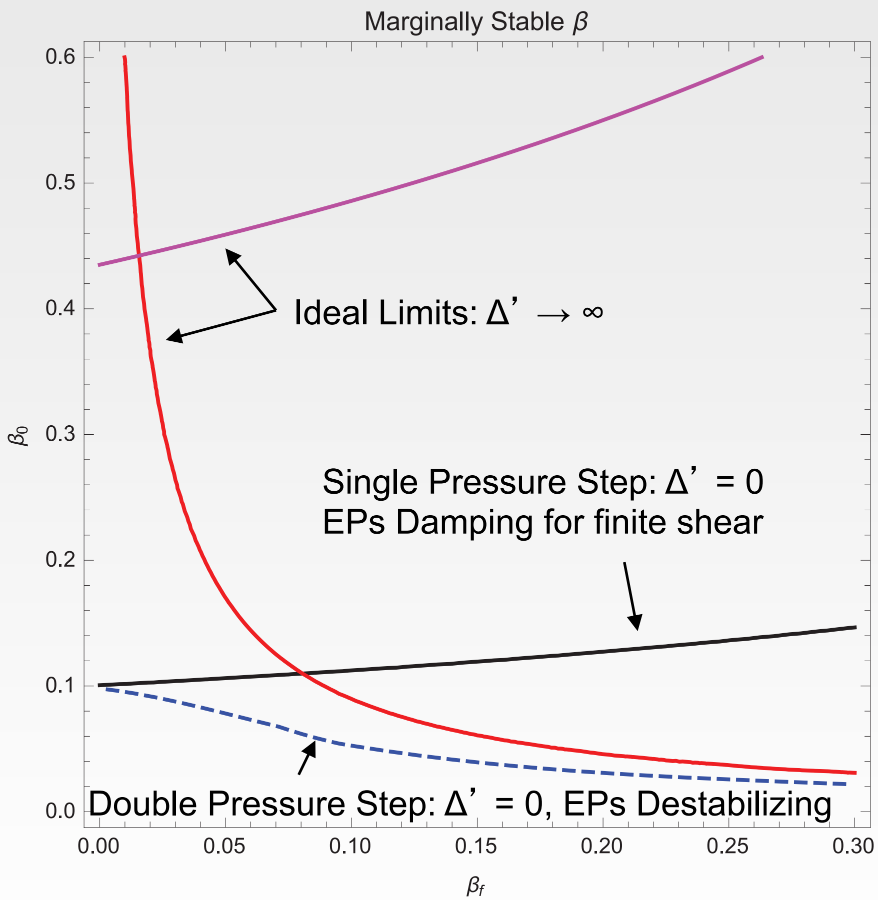


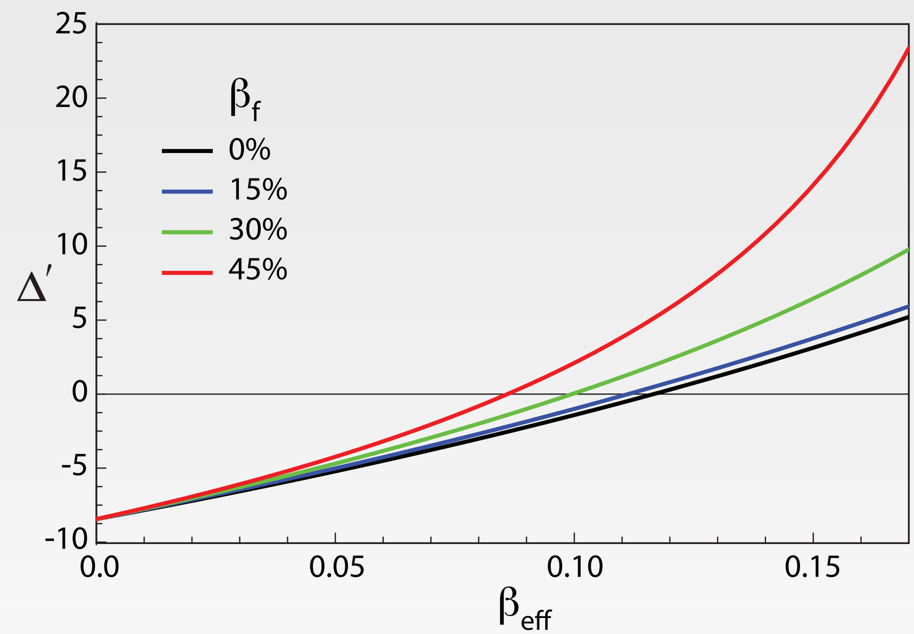


Result in qualitative agreement with simulations including monotonic q sheared to the core Takahashi PRL 09



Δ' calculation indicates a destabilizing effect for equilibria with internal P step in zero shear





Result in qualitative agreement with simulations including weakly reversed q shear in core. Brennan Nucl. Fusion 12

For the equilibrium configuration with an internal pressure step in zero shear, particles contribute to the growth of the 2/1 tearing mode.

WHY? λ Plays a Role in Determining the Effect that Particles Have on Mode Stability -> changes sign

Jump across a pressure step $\left[\tilde{\psi}'\right]_{a_p} + \frac{2mB_\theta(a_p)\beta_0}{a_p^2 F(a_p)}\left(\frac{m}{a_p F(a_p)} - \lambda\beta_{frac}\right)\tilde{\psi}(a_p) = 0$

where

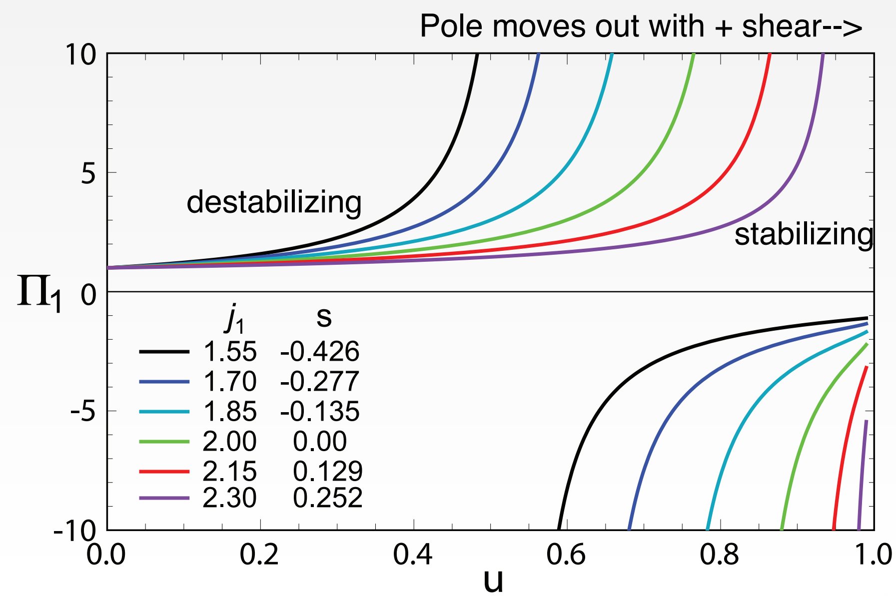
$$\tilde{p}_j^m = \lambda p_0 = \int_0^\infty d\hat{v}\,\hat{v}^5 f_0(\hat{v}) \int_0^1 du K(u)\Pi_j \sigma_m \hat{v}^2 \left\{\left(\tilde{\xi}_r\kappa_0 + \frac{Z_j e}{T_j}\tilde{Z}\right)\sigma_m + \tilde{\xi}_r\kappa_1(\sigma_{m-1} + \sigma_{m+1})\right\}$$

$$\Pi_j = -N_j \frac{R}{2}\frac{dT_j}{dr}\frac{\hat{v}^2 - \frac{3}{2} + \frac{l_{T_j}}{l_{N_j}} + 2\frac{l_{T_j}}{R}w_E^j}{w_E^j + \hat{v}^2 H(u)}$$

For low shear, Π_j in the λ integral has a pole where $H(u) = 0$

note $\omega_E = 0$ here

Sufficient amount of negative Π leads to integral switching sign, destabilizing effect



Magnetic Shear at Internal Pressure Step is Key Factor Determining Destabilizing Effect

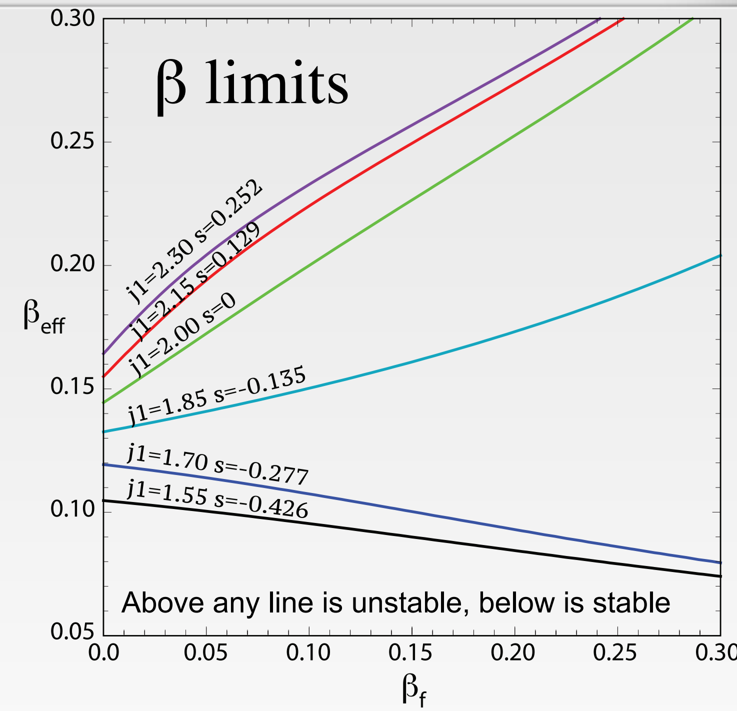
Series of q shear at internal pressure step r_{p1}, with fixed total current

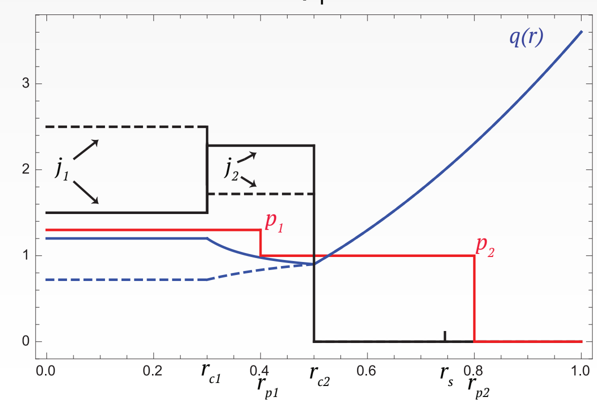
Pressure peaking held fixed

$$\delta_p \equiv (p_1 - p_2)/p_1 = 0.3$$

$$\beta_{eff} \equiv \beta_0 \frac{1-\delta_p + r_{p1}^2/r_{p2}^2}{1+r_{p1}^2/r_{p2}^2}$$ volume average

Energetic ions become destabilizing to the mode as shear is made negative



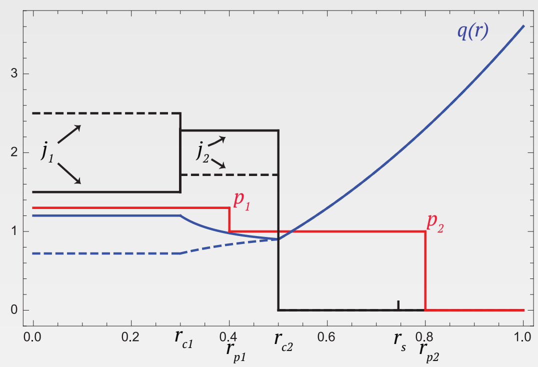


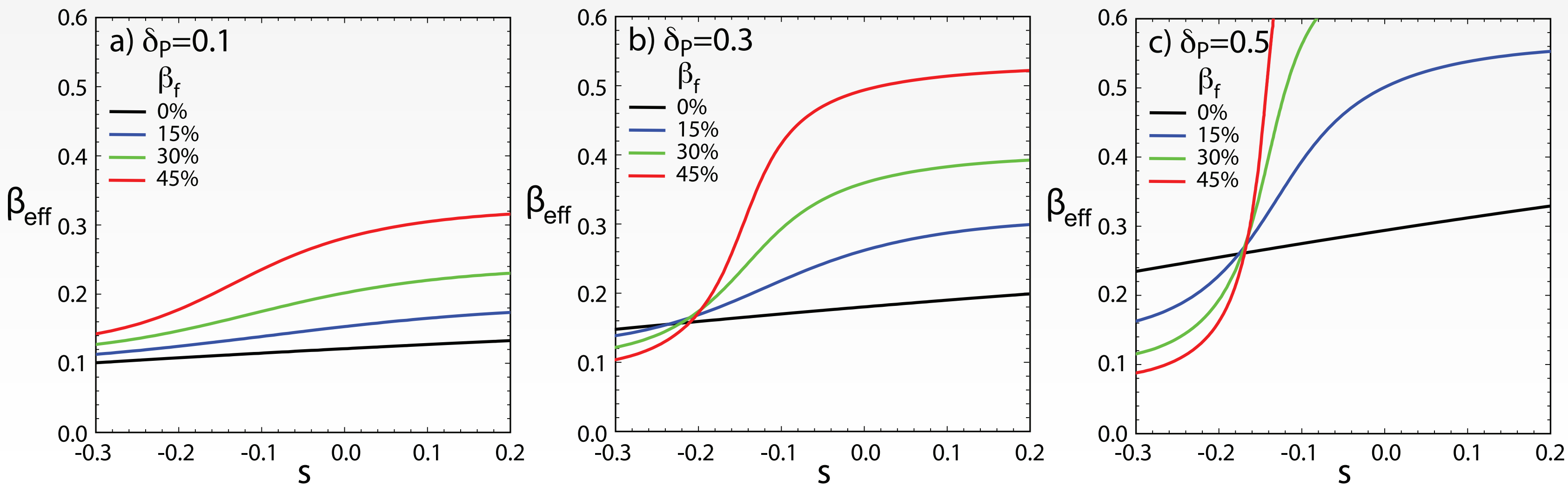
Magnetic Shear at Internal Pressure Step is Key Factor Determining Destabilizing Effect

As pressure peaking increased negative shear becomes destabilizing

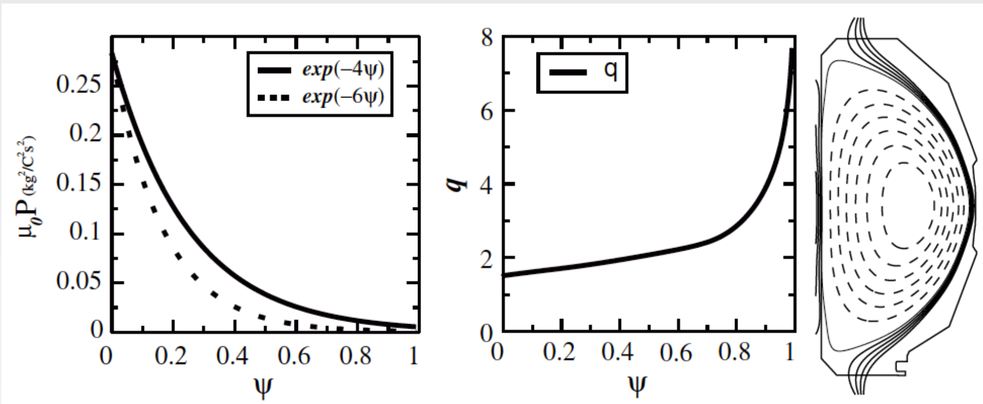
For low δ_p~0.1 stability dominated by outer pressure step

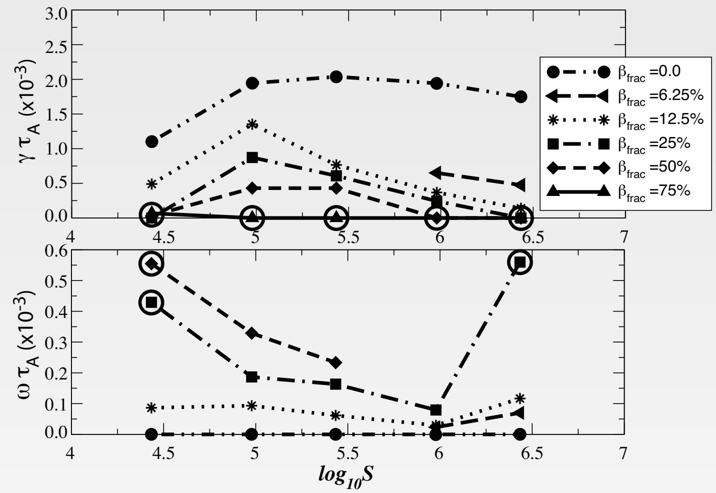
For large δ_p~0.5 both stabilizing and destabilizing effects are strong



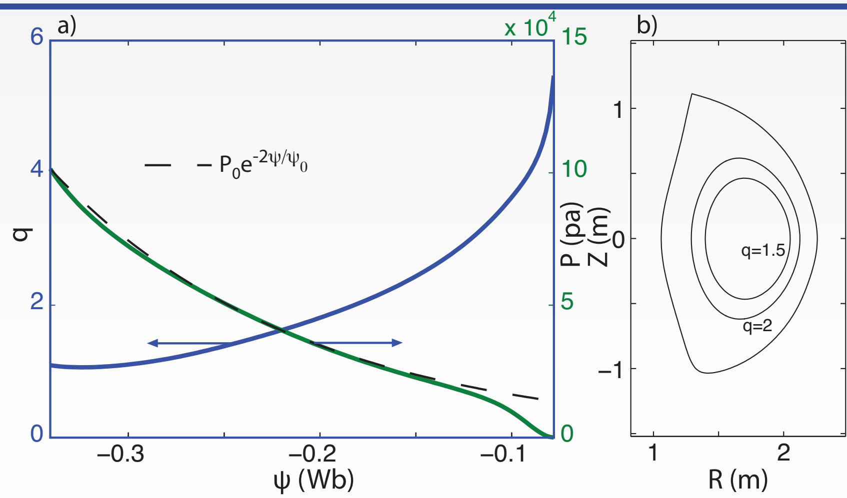


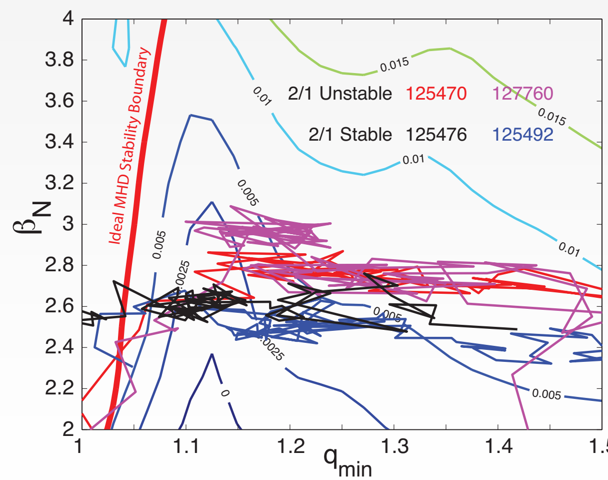
Summary: For experimental cases, pressure gradient and weak shear in the core can significantly affect 2/1 stability





R. Takahashi, D.P. Brennan and C.C. Kim PRL 102, 135001 (2009)





D.P. Brennan, C.C. Kim and R.J. La Haye, Nucl. Fusion 52, 033004 (2012)

Stability analysis with flow and resistive wall

Internal stability in D couples to wall via simple 2x2 matrix formalism:

$$\tilde{\psi}(r) = \alpha_1\psi_1(r) + \alpha_2\psi_2(r) + \alpha_3\psi_3(r)$$

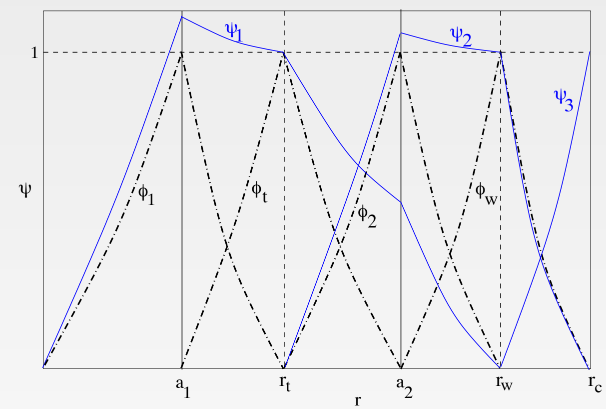
Resonant surface

$$\gamma_d \tau_t \psi(r_t) = \left[\psi'\right]_{r_t}$$

Resistive wall

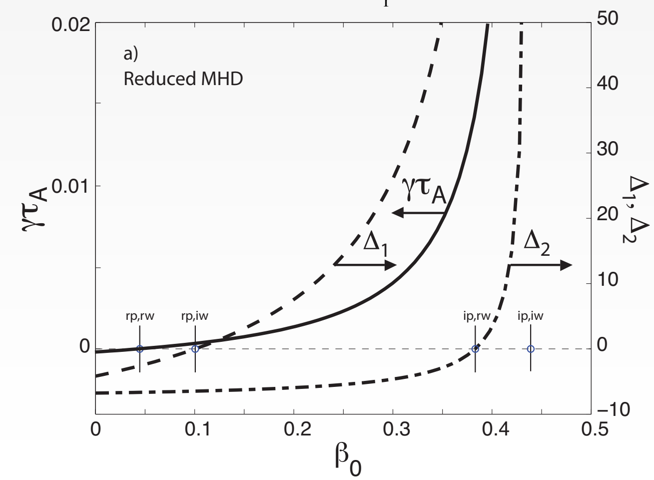
$$\gamma \tau_w \psi(r_w) = \left[\psi'\right]_{r_w}$$

Describes the resistive plasma resistive wall mode



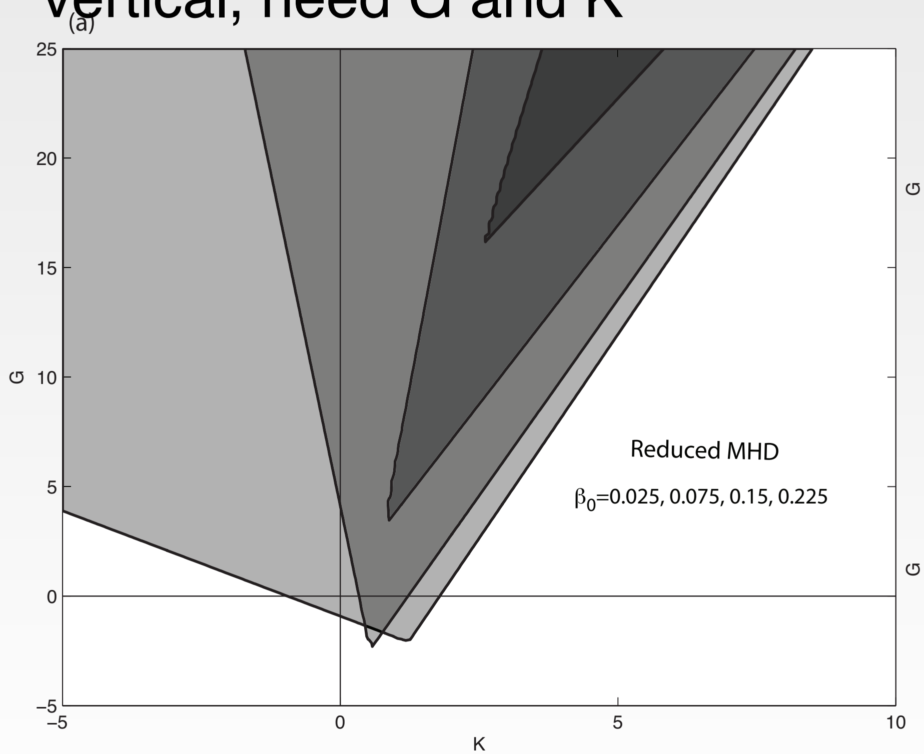
Including rotation at the surface can be approximated simply by a Doppler shift of the layer response.

$$\begin{pmatrix} \Delta_1 - (\gamma + i\Omega)\tau_t & l_{21} \\ l_{12} & \Delta_2 - \gamma\tau_w \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0$$

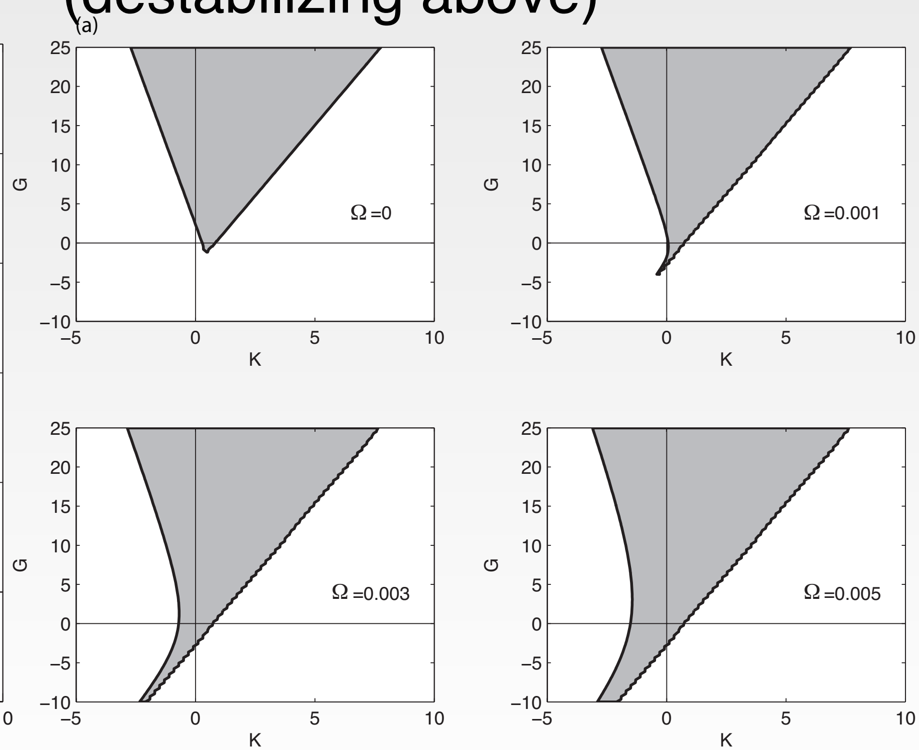


Control example: With flow and proportional gain, stable at higher radial and tangential gain as β increases

Crossing $\beta_{rp,iw}$ when left boundary vertical, need G and K



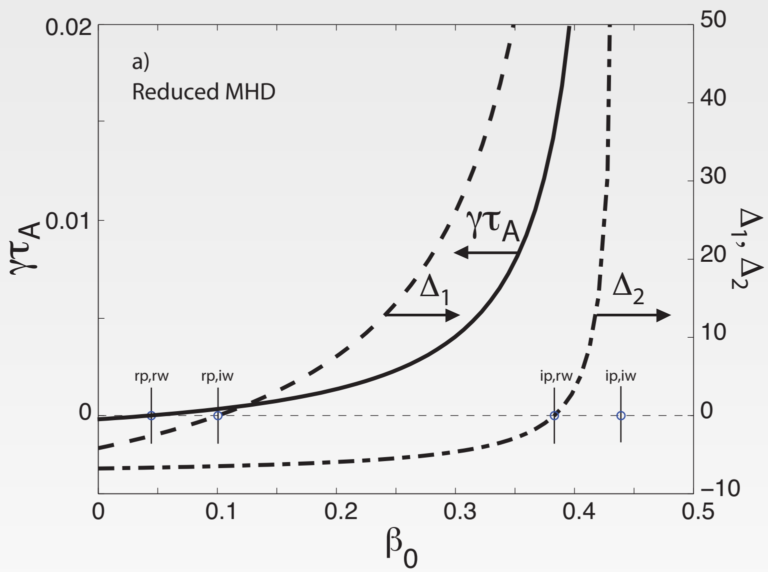
Effect of Ω stabilizing below $\beta_{rp,iw}$ (destabilizing above)



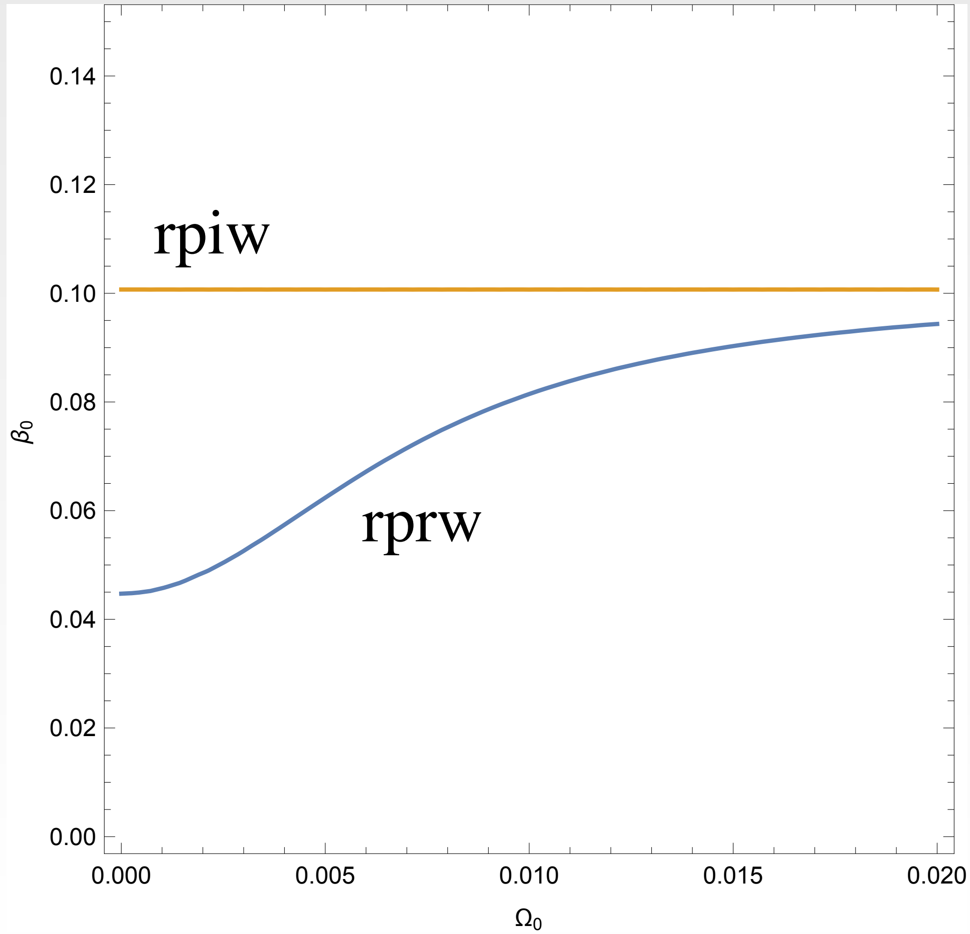
Several findings associated with optimal gains on radial K and tangential G sensors, and complex responses reported in:

D.P. Brennan and J.M. Finn, Phys. Plasmas 21, 102507 (2014).

Stability boundary approaches ideal wall boundary with increasing flow (no ions)



$$\begin{pmatrix} \Delta_1 - (\gamma + i\Omega)\tau_t & l_{21} \\ l_{12} & \Delta_2 - \gamma\tau_w \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0$$



With energetic ions the particle response also becomes flow dependent in addition to wall coupling

Flow enters the particle resonance condition directly

$$\Pi_j = -N_j \frac{R}{2}\frac{dT_j}{dr}\frac{\hat{v}^2 - \frac{3}{2} + \frac{l_{T_j}}{l_{N_j}} + 2\frac{l_{T_j}}{R} w_E^j}{w_E^j + \hat{v}^2 H(u)}$$

In addition to the coupling to the wall

We then solve the eigenvalue problem where each term depends on Ω

Current efforts are focused on understanding the effect on Δ_1

$$\begin{pmatrix} \Delta_1[\beta_f,\beta,j,\Omega]-(\gamma+i\Omega)\tau_t & l_{21}[\beta_f,\beta,j,\Omega] \\ l_{12}[\beta_f,\beta,j,\Omega] & \Delta_2[\beta_f,\beta,j,\Omega]-\gamma\tau_w \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0$$

OUTLINE

Puzzle: Why is 2/1 sometimes stabilized and others destabilized by energetic ions?

- Takahashi, Brennan, Kim Phys. Rev. Lett **102**, 135001 (2009). - 2/1 **stabilized** by particles
- Brennan, Kim, La Haye Nucl. Fusion **52**, 033004 (2012). - 2/1 **destabilized** in reversed shear
- WHY? Need a reduced model to explain sims and experiments.

Extending a reduced MHD cylindrical model for the 2/1 tearing mode with energetic ions

- Brennan, Finn Phys. Plasmas **21**, 102507 (2014).
- Extended to include reversal in q, two pressure steps inside and out of sheared region.
- Include energetic ions in analogous way to Hu Betti PRL 04, with toroidal field line curvature for trapped particle orbits in otherwise cylindrical model.

Energetic ion pressure contribution in core significantly effects the stability

- With positive shear particles are damping and stabilizing
- Near zero or negative shear in the core causes destabilizing influence
- Results consistent with simulations

Concluding Remarks / Ideas for the Future

Concluding Remarks

- Analytic reduced MHD modeling has shown energetic ions can have either a stabilizing or destabilizing effect on the growth of linear resistive tearing modes.

- The key to the variation is in the effect of magnetic shear on energetic ion orbital interactions with the mode structure. A resonance in phase space occurs in low to negative shear, driving a destabilizing influence.

- Stabilizing effect occurs with monotonic q and shear extending through the core region.

- Destabilizing effect occurs due to trapped particle resonance in a region of near zero or negative shear internal to the rational surface radial position.

- Results are consistent with simulations of sheared AT-like DIII-D equilibria and Hybrid-like equilibria with weakly reversed core shear, both of which have shown consistency with experimental results.

Ideas for Future Work (wider reduced modeling view)

Do the different β orderings relate to experimental observations of tearing vs. RWM onset? $\beta_{rp,iw} < \beta_{ip,rw}$ vs. $\beta_{rp,iw} < \beta_{ip,rw}$

Analysis of nonlinear simulations indicating finite frequency locking and driven flow
- two fluid layer responses, parallel dynamics toroidicity are key

Quasilinear mode locking between surfaces of different m

Rutherford modeling (ala Fitzpatrick 15)

Energetic particle effects on Resistive Plasma – Resistive Wall mode with flow

Study control with all these effects