M3D-C1 Simulation of a Current Ramp-down Disruption in NSTX

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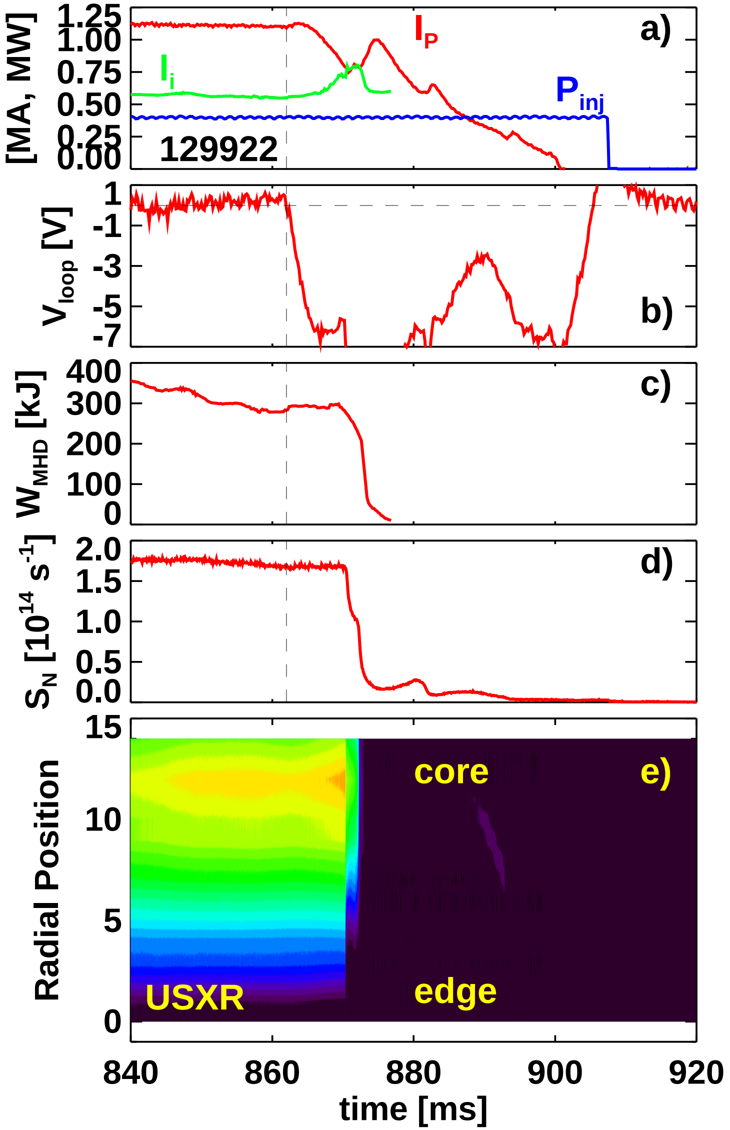
Unique Class of Major Disruptions Identified in NSTX

- Recipe:
  - Generate a stable low(er) q95 discharge.
  - Run it to the current limit of the OH coil.
  - Ramp the OH coil back to zero, applying a negative loop voltage, while leaving the heating on.
  - Watch l_i increase, then disruption occurs.
- Mechanism responsible for 21 for the 22 highest W_{MHD} disruptions in NSTX.
- Specific example in the general area of how unstable current profiles lead to catastrophic instability

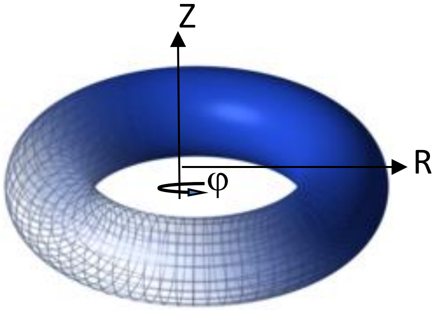


[S. Gerhardt, Nov. 2013]

3D Extended MHD Equations in M3D-C1

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \bullet D_n \nabla n + S_n$$

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi = -\nabla_\perp \bullet \frac{1}{R^2}\mathbf{E}$$



$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla \mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_i + \mathbf{S}_m$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \bullet \mathbf{\Pi}_e\right) - \frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \bullet \nabla \mathbf{V}_e\right) + \mathbf{S}_{CD}$$

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla \bullet \left(p_e \mathbf{V}\right)\right] = -p_e \nabla \bullet \mathbf{V} + \frac{\mathbf{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n + \mathbf{R}_c\right] + \nabla\left(\frac{\mathbf{J}}{ne}\right) : \mathbf{\Pi}_e - \nabla \bullet \mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t} + \nabla \bullet \left(p_i \mathbf{V}\right)\right] = -p_i \nabla \bullet \mathbf{V} - \mathbf{\Pi}_i : \nabla \mathbf{V} - \nabla \bullet \mathbf{q}_i - Q_\Delta + S_{iE}$$

$$\mathbf{V}_e = \mathbf{V}_i - \mathbf{J}/ne$$

$$\mathbf{R}_c = \eta ne\mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^\dagger\right] - 2(\mu_c - \mu)(\nabla \bullet \mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV}$$

$$\mathbf{q}_{e,i} = -\kappa_{e,i}\nabla T_{e,i} - \kappa_\parallel \nabla_\parallel T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B}/B^2)\nabla \bullet \left[\lambda_h \nabla\left(\mathbf{J} \bullet \mathbf{B}/B^2\right)\right], \qquad Q_\Delta = 3m_e\left(p_i - p_e\right)/\left(M_i \tau_e\right)$$

Kinetic closures extend these to include neo-classical, energetic particle, and turbulence effects.

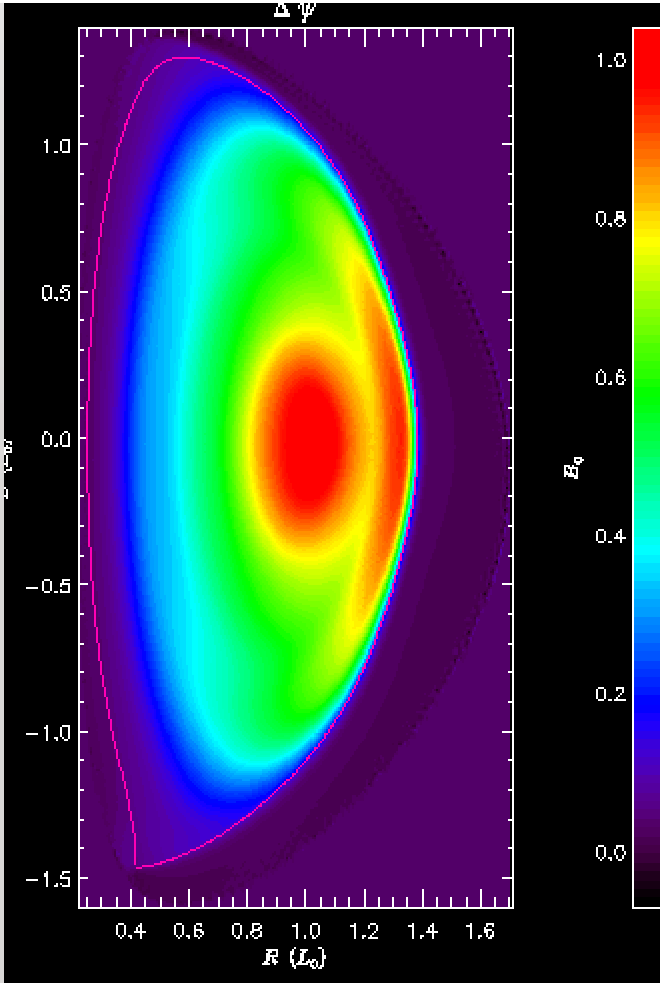
Difficulties in Disruption Modeling

- Multiple Timescales
  - Need to start calculation in stable state to be physical
  - Apply forcing (V_L), profiles change on diffusion timescale $\tau_D \sim \mu_0 a^2/\eta$
  - Once stability boundary is crossed, evolution is on ideal $\tau_A \sim R / V_A$
  - Since $S \equiv \tau_D/\tau_A >> 1$, many time-steps are required

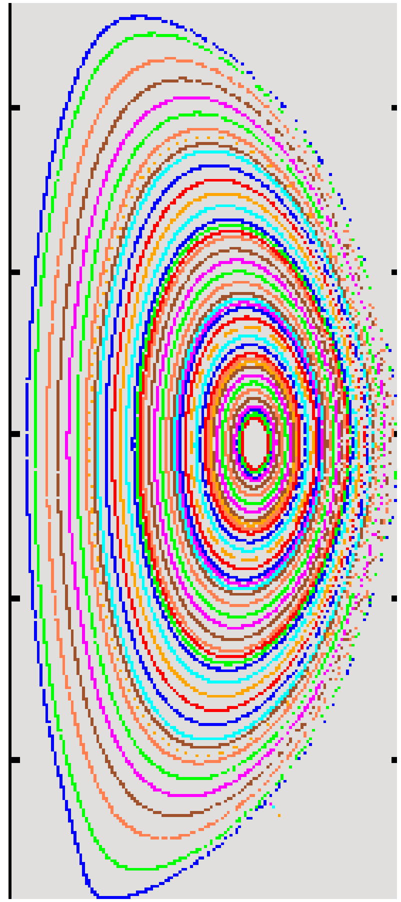
- Multiple Spatial Scales
  - First modes to go unstable are moderate $n \sim 10$ …. Multiple modes
  - These drive both higher and lower modes numbers
  - Eventually, some short wavelength modes are generated that cannot be resolved on the finite-element mesh

  ➔ some kind of sub-grid-scale model is required to deal with these

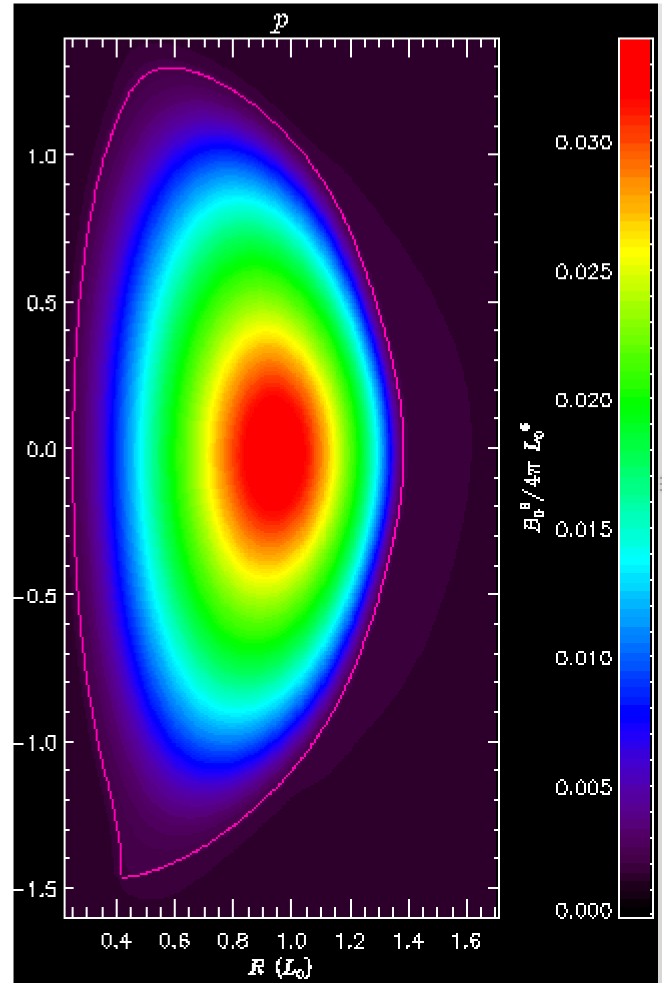
R J_φ



Poincare



Pressure



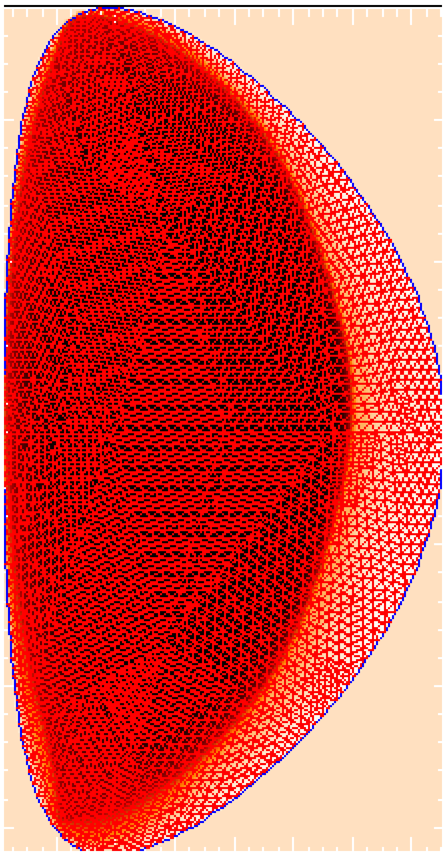
shot 129922
Time 860 ms
Diverted.

I_P ~ 1.1 MA
q_0 ~ 1.22
β ~ 6 %

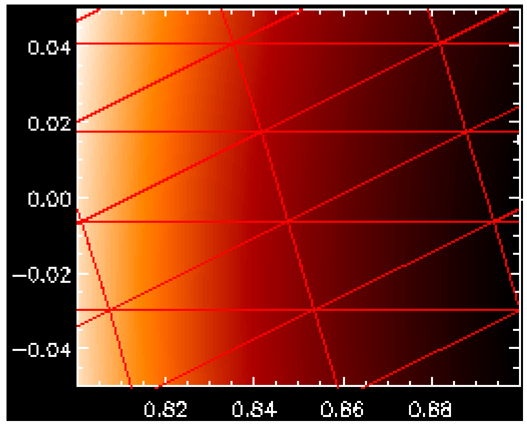
Te(0) = 1.14 keV
V_L = 0.36 Volts
χ = 1 m^2/sec

Numerical Parameters and Procedures :

Entire domain



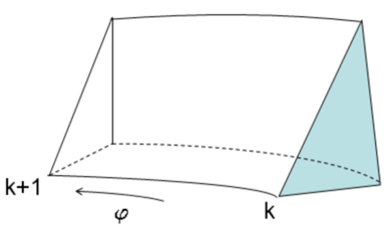
10 cm x 10 cm patch



$S = 10^7$ (in center)

2D triangle size: 2 – 4 cm

32 and 64 toroidal planes
With Hermite cubic elements: $E \sim 1/N^4$



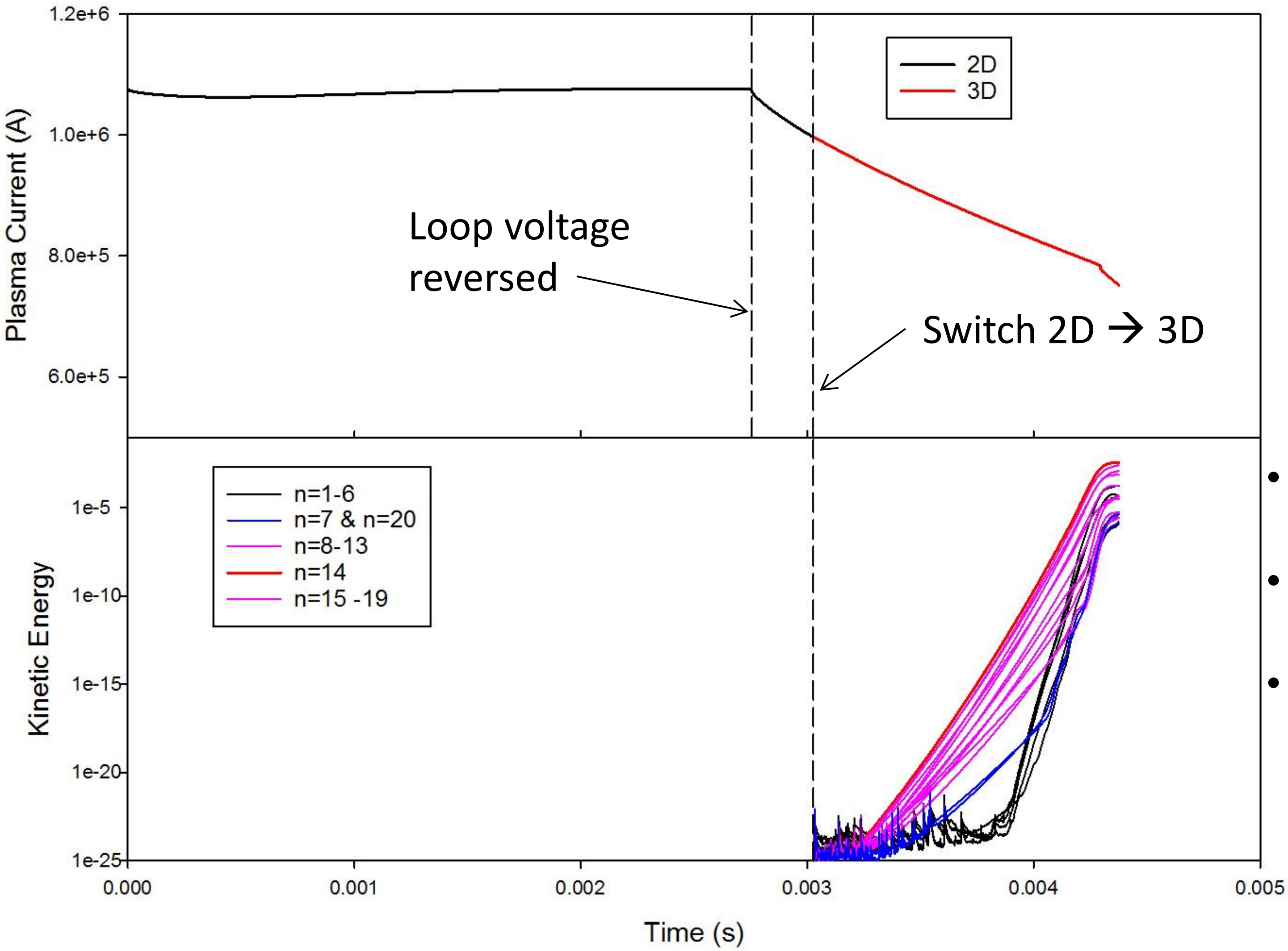
Triangular prism finite elements

Within each element, each scalar field is represented as a polynomial in (R,φ,Z) with 72 terms. All first derivatives are continuous.

Sequence of Calculation:

- Start calculation in 2D (axi-sym)
- Run for a few ms to establish stationary state with heating and particle sources
- Loop voltage prescribed at computational boundary
  - Control system to keep plasma current fixed before ramp-down
  - Switch to fixed negative value at start of current ramp-down
- Switch to full 3D geometry just before plasma becomes linearly unstable

Current and Harmonics Plots for typical calculation



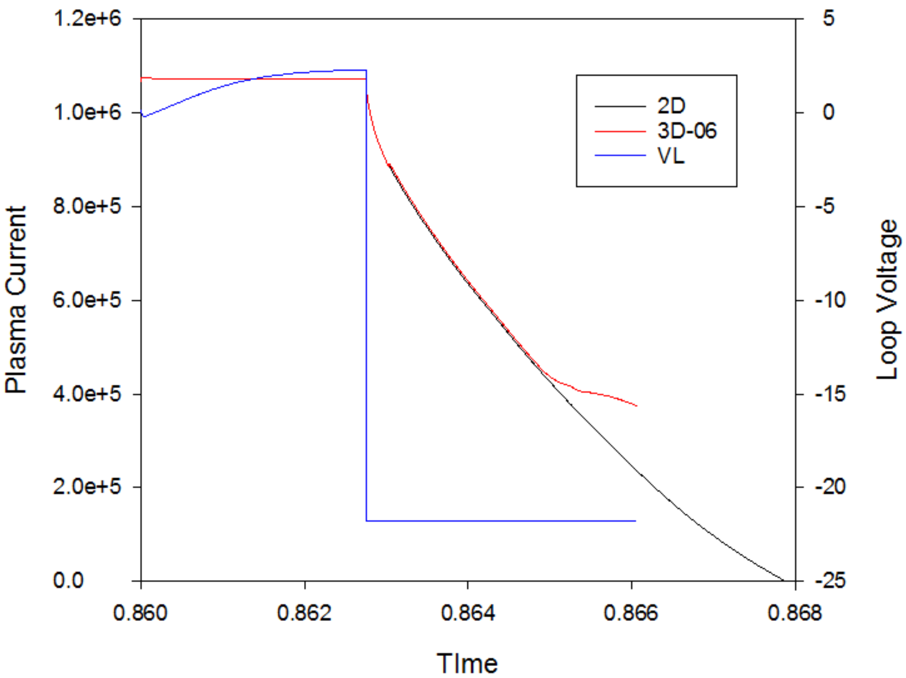
- All modes stable at start of 3D
- 7 ≤n ≤ 20 become linearly unstable
- Lower and higher modes driven non-linearly

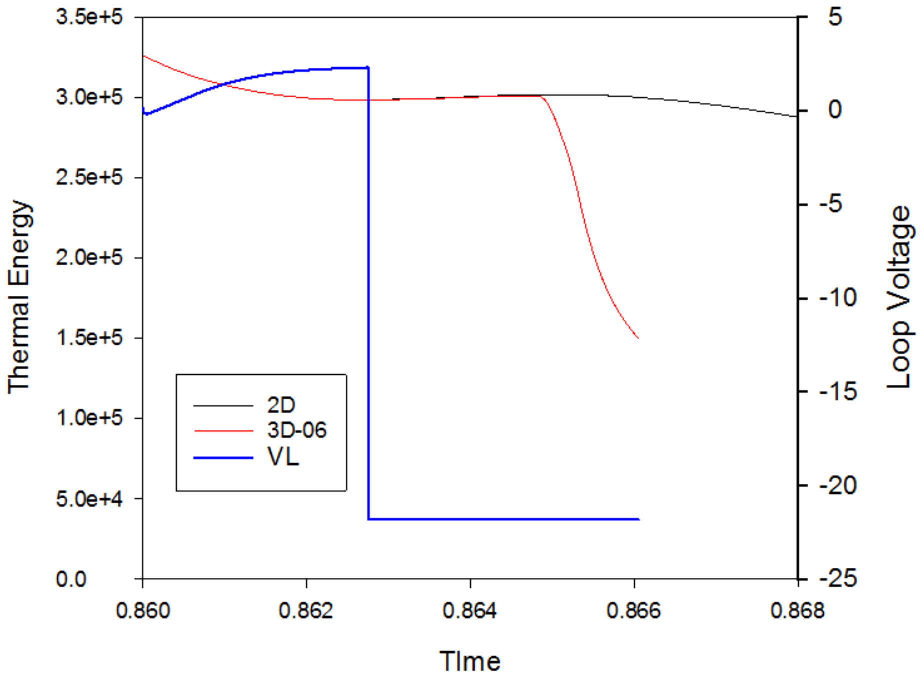
Run07

Time traces of Plasma Current, Thermal Energy, and Loop Voltage

Compare:

- 2D (axisymmetric) run (black)
- 2D -> 3D run (red)



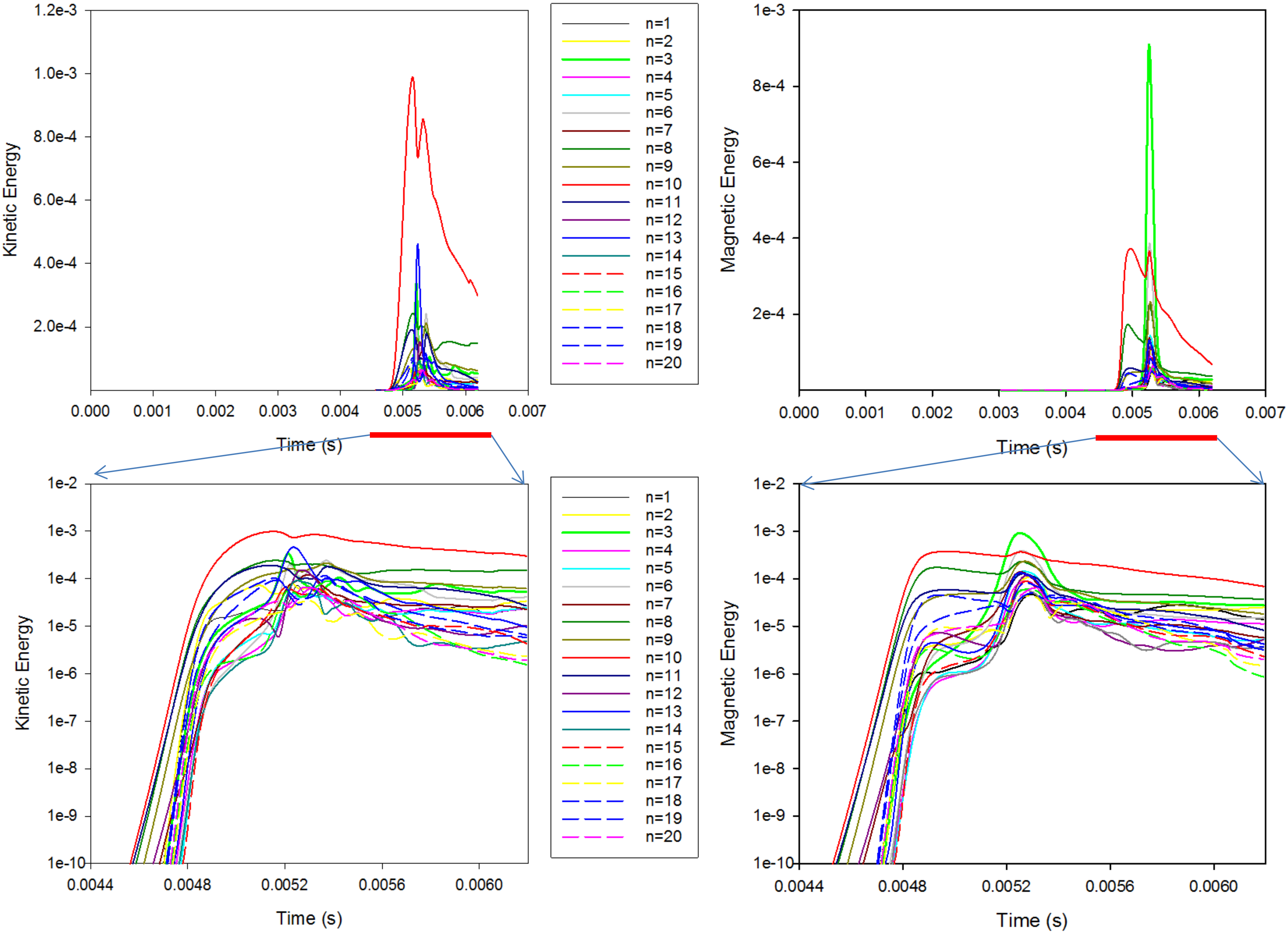


- Both runs have identical I.C. and boundary conditions (V_L)
- 3D run has slower current decay near end of calculation
- 3D run shows thermal energy loss, 2D run does not

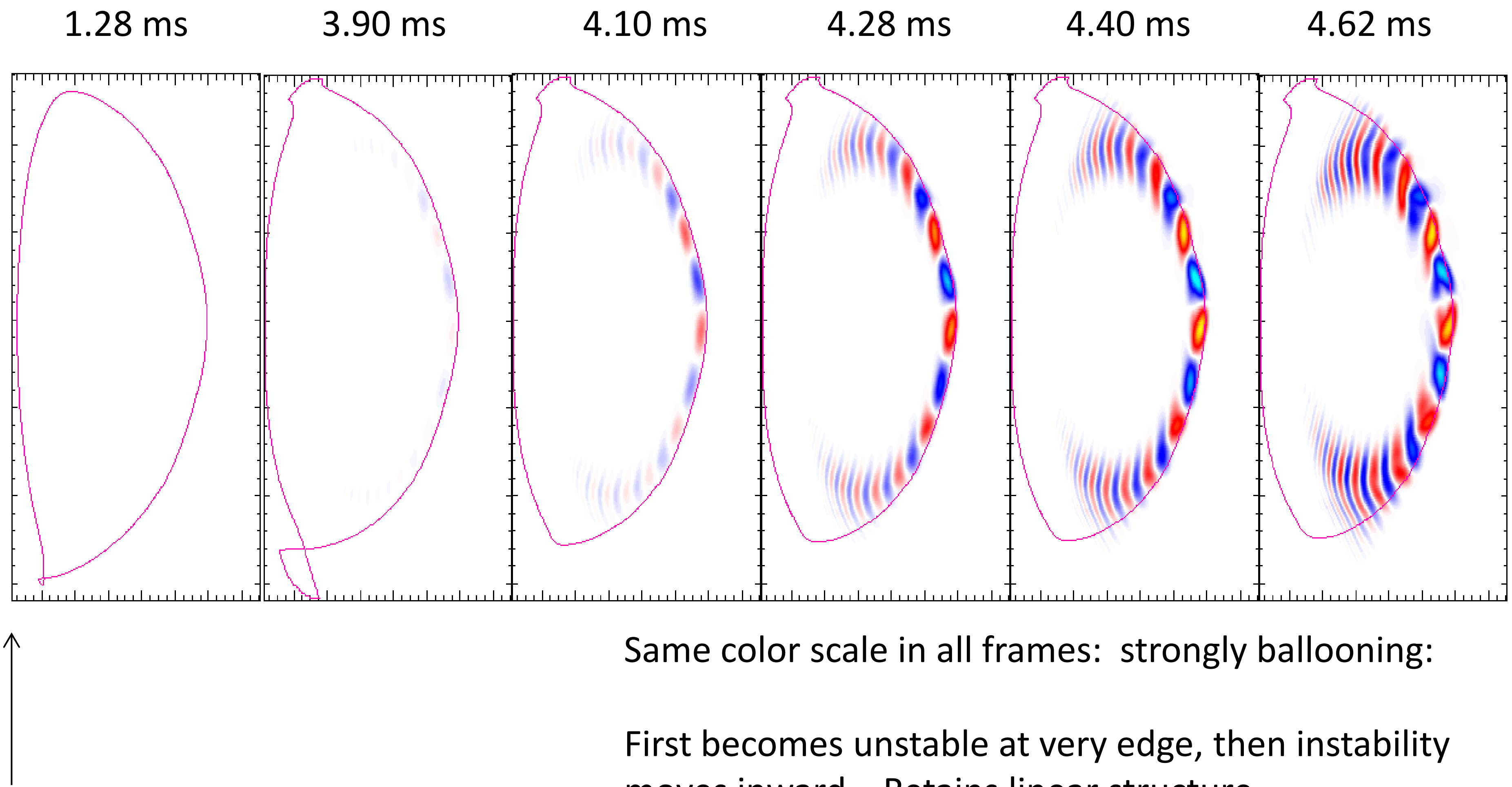
Run06b

Kinetic and Magnetic Energy Harmonics vs Time

Run06b



Toroidal derivative of pressure at several time slices



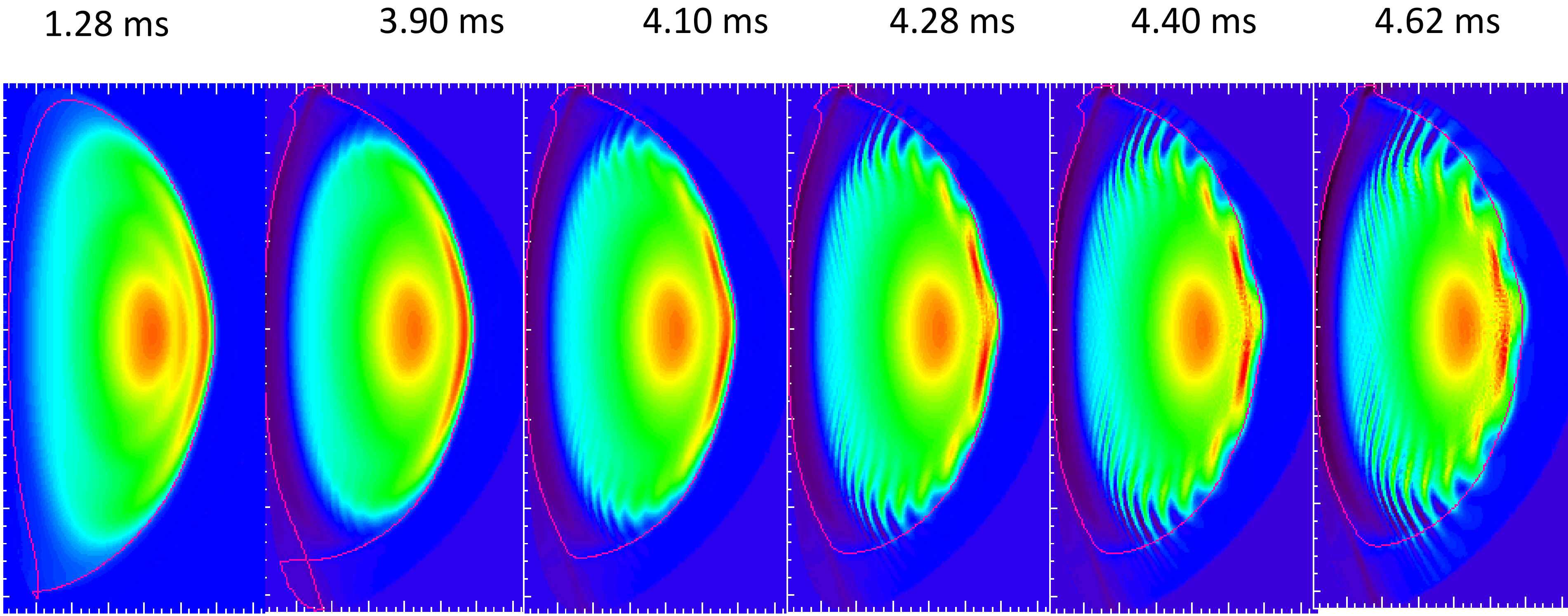
Voltage reversed at 1.28 ms

Same color scale in all frames: strongly ballooning:

First becomes unstable at very edge, then instability moves inward. Retains linear structure.

Becomes limited shortly after ramp-down starts. Impurity generation??

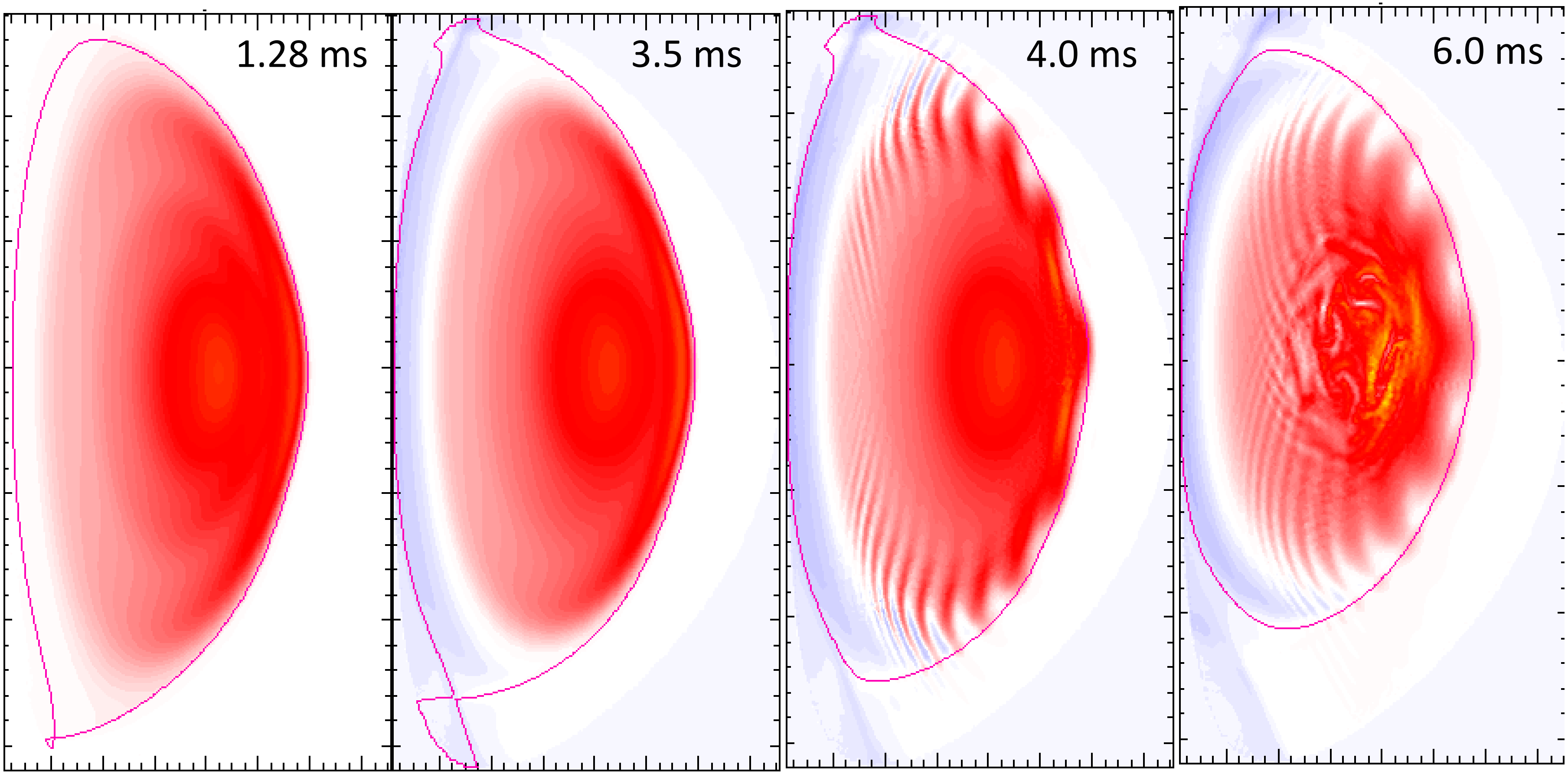
Plasma current density at several time slices



Same color scale in all frames

Current forms filaments all around, with shorter poloidal wave lengths on HFS

Plasma current density at several time slices

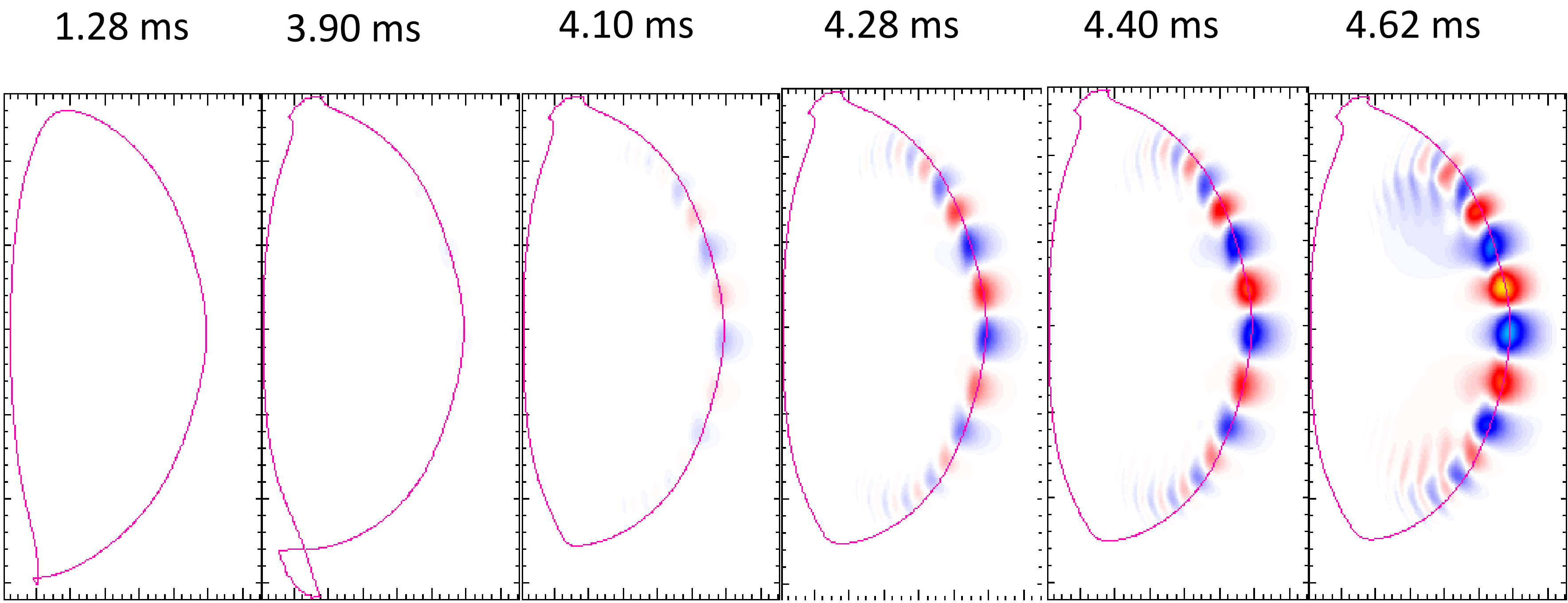


Different color scheme from previous viewgraph. Red and yellow are positive, blue is negative, zero is white.

Current is seen to reverse on HFS

Run05

Toroidal derivative of poloidal flux at several time slices

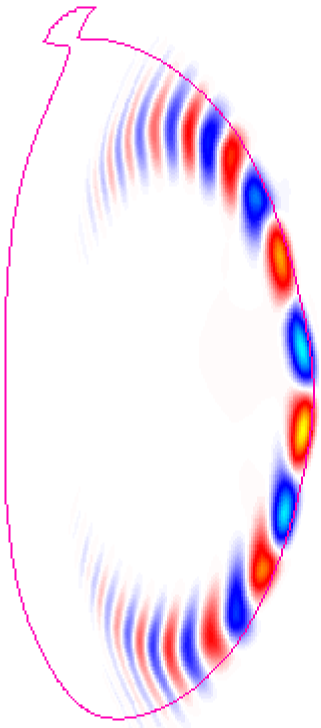


Same color scale for all times.   Same pattern, just grows.

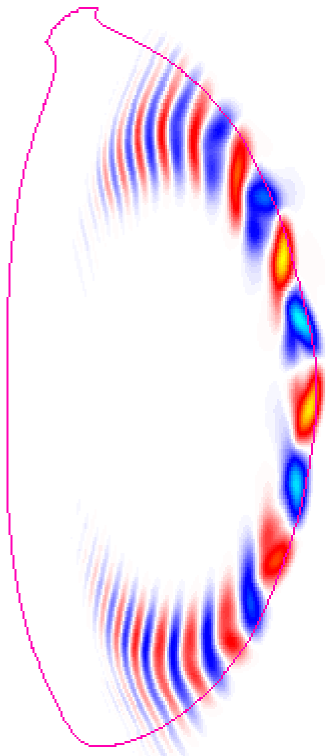
These should be observable on Mirnov loops

Perturbed pressure and currents at time of saturation are very similar for 32 plane and 64 plane cases

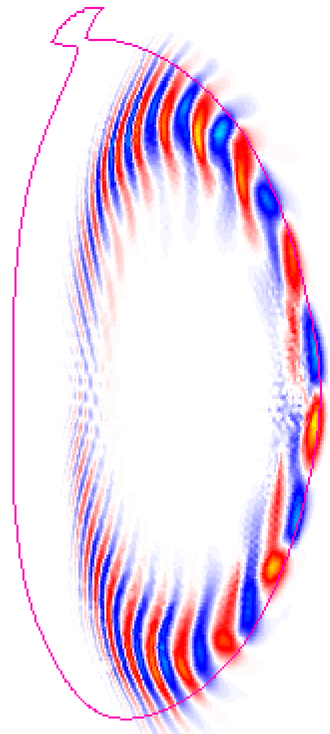
P_φ 32 planes



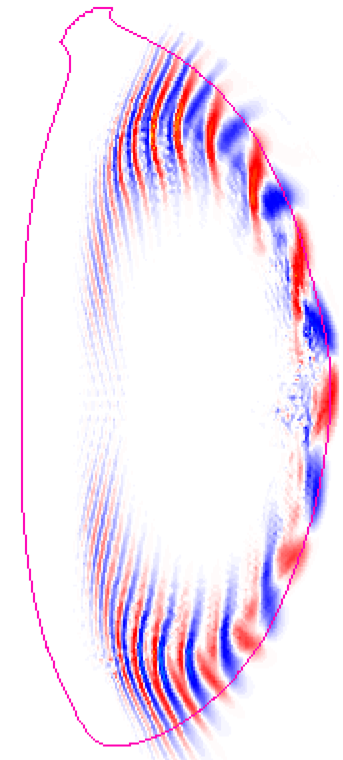
P_φ 64 planes



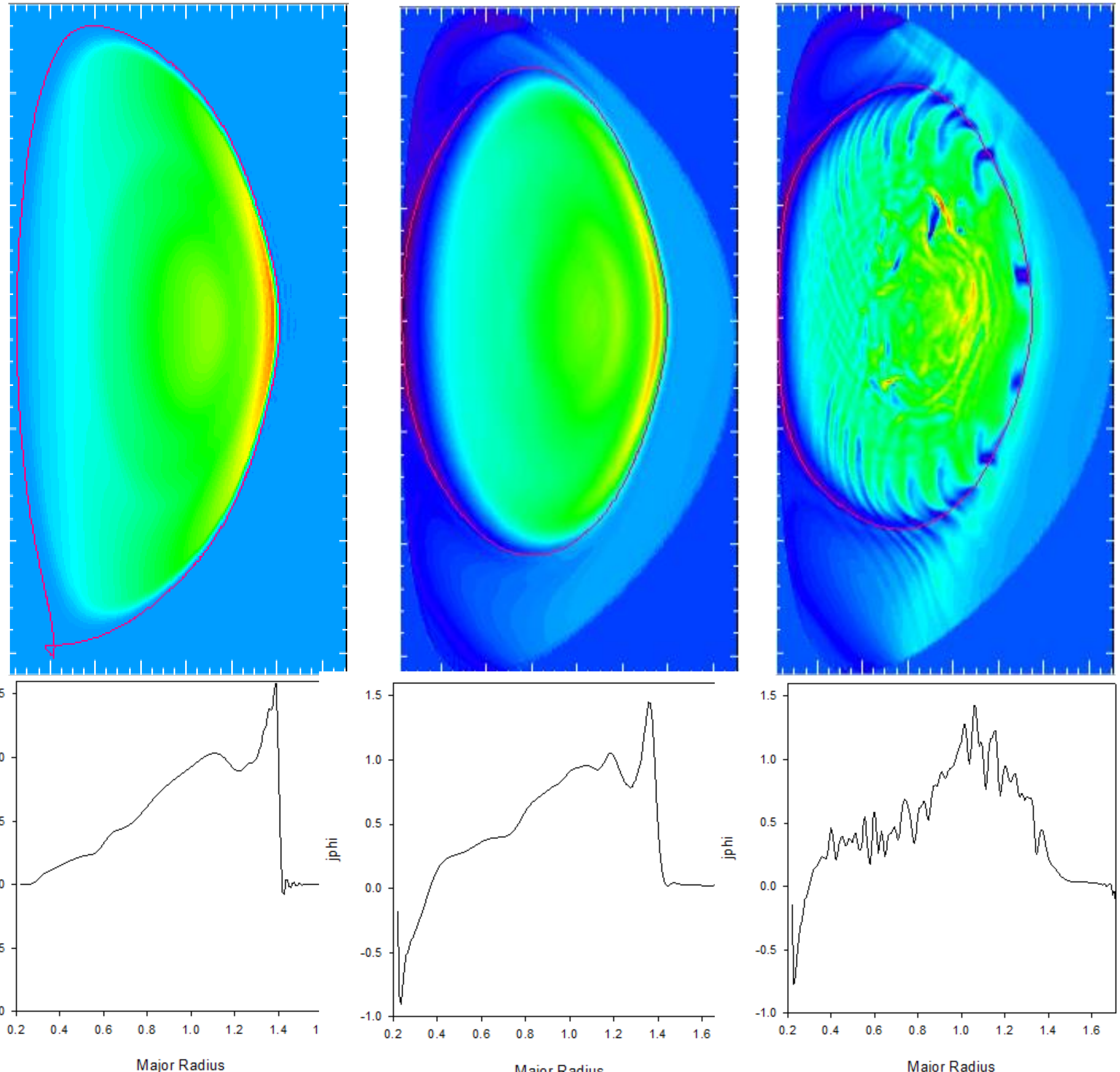
J_φ 32 planes



J_φ 64 planes



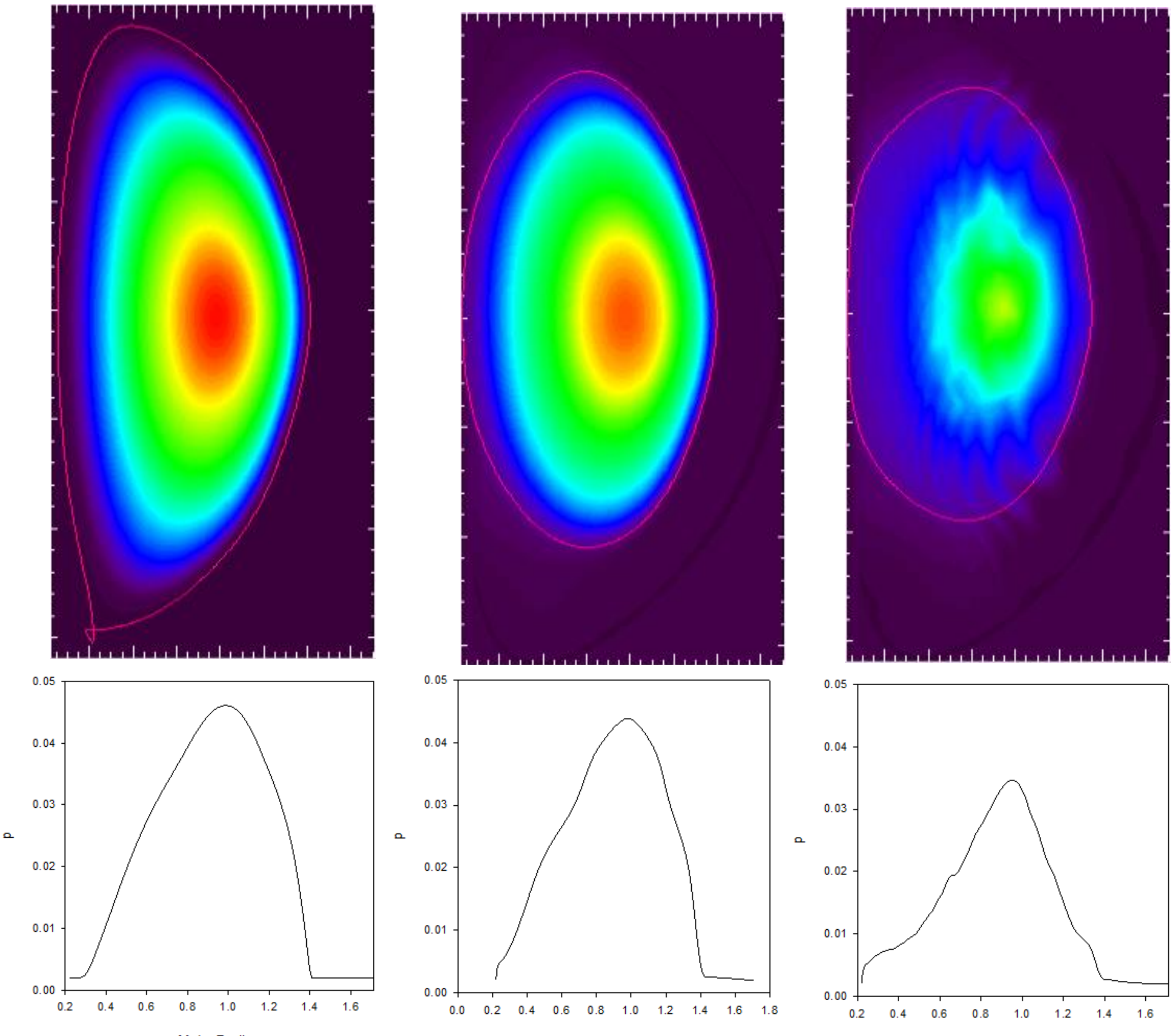
Jφ Initial Equilibrium | 2D – t= 6.0 ms | 3D – t= 6.0 ms



3D current distribution is slightly broader and much more spikey than 2D current at the same time

Run06b

Initial Equilibrium P 2D – t = 6.0 ms 3D – t = 6.0 ms



3D pressure is smaller and more peaked than 2D

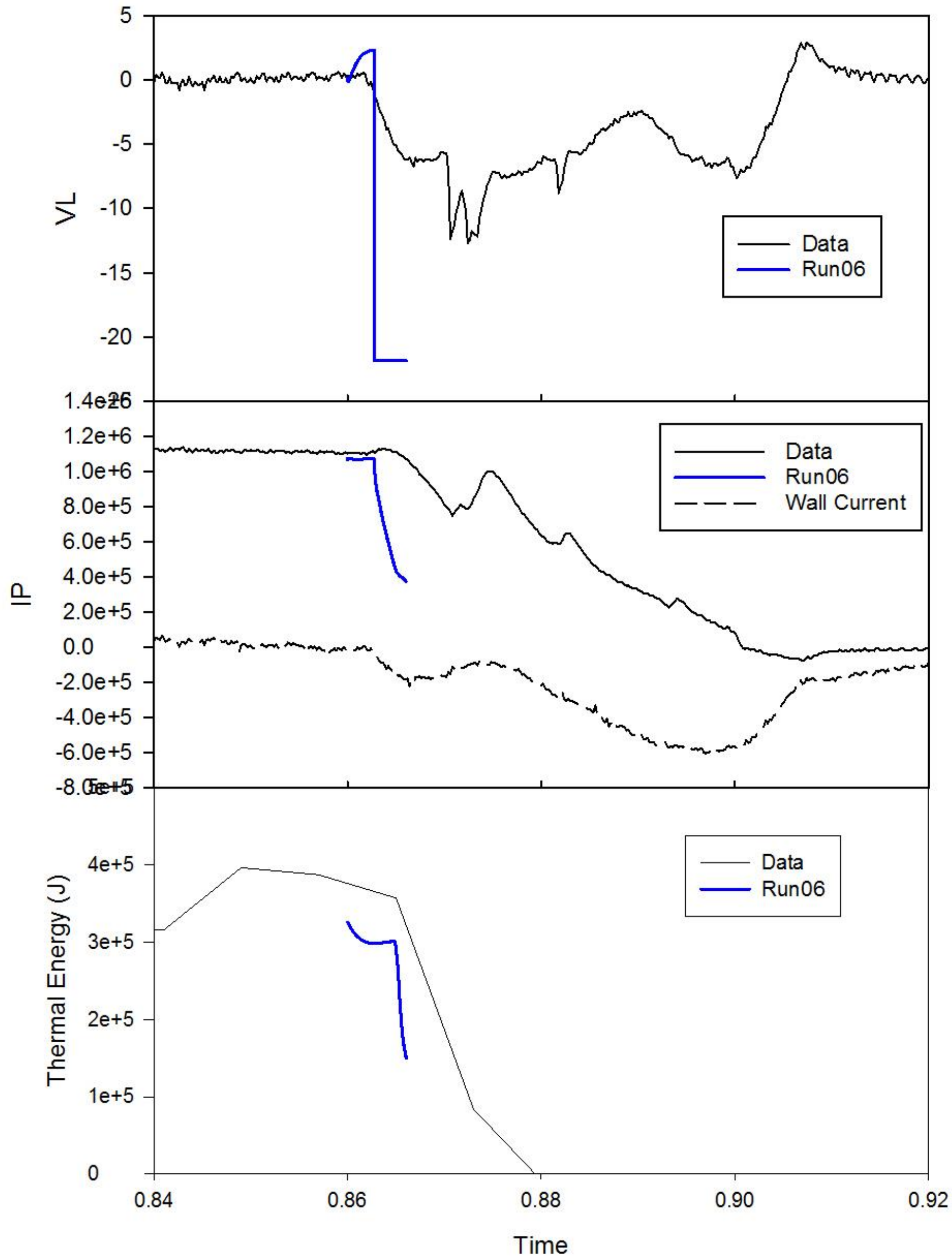
Run06b

Comparison with Experimental Data:

Run06: VL = -20 V

Current Quench
- Initial decay rate reasonable
- Can we see the current spike?

Thermal Quench
- Initial drop reasonable
- Need impurity radiation to get full drop?



Phases and Future Directions

- Phase I -- done
  - Demonstrate we can reproduce the basic physics of the current ramp-down disruption without sub-grid-scale model, vessel, or coils
- Phase II -- in progress
  - Can realism of model be improved by adding sub-grid-scale physics?
  - Does impurity radiation play a role in these disruptions?
- Phase III -- soon
  - Include NSTX vacuum vessel and coils and try and match experimental traces more closely
  - Add improved graphics and movies
  - Explore limits on rapid shutdown without causing a disruption.

Magnetic Helicity conserving sub-grid-scale model for current

Consider the new dissipative term in Ohm's law (hyper-resistivity):

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + \mathbf{R}_H \qquad \mathbf{R}_H = -\frac{\mathbf{B}}{B^2} \nabla \bullet \left[\lambda \nabla \left(\frac{\mathbf{J} \bullet \mathbf{B}}{B^2} \right) \right]$$

This term will always dissipate energy for $\lambda > 0$:

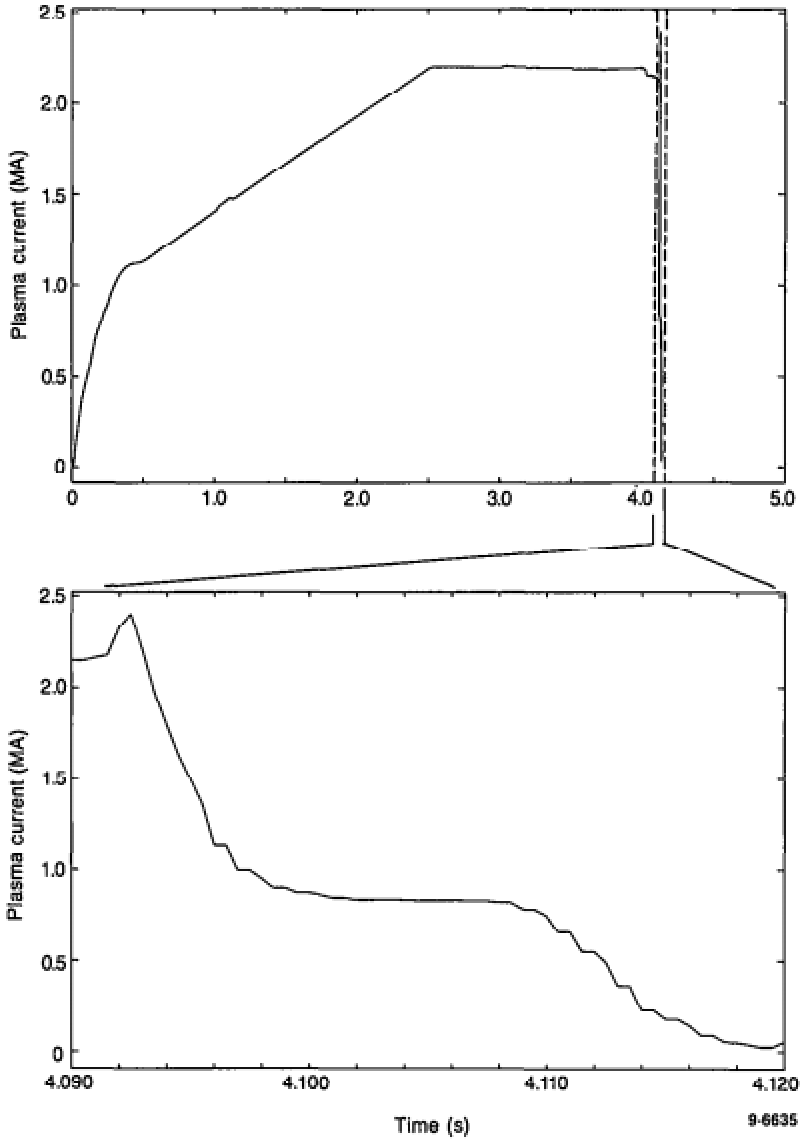
$$\int \mathbf{J} \bullet \mathbf{R}_H \, d\tau = -\int \left(\frac{\mathbf{J} \bullet \mathbf{B}}{B^2} \right) \nabla \bullet \left[\lambda \nabla \left(\frac{\mathbf{J} \bullet \mathbf{B}}{B^2} \right) \right] d\tau = \int \lambda \left| \nabla \left(\frac{\mathbf{J} \bullet \mathbf{B}}{B^2} \right) \right|^2 d\tau > 0$$

It will also conserve magnetic Helicity: $K = \int \mathbf{A} \bullet \mathbf{B} \, d\tau$

$$\begin{aligned}
\frac{\partial K}{\partial t} &= \int \left[\frac{\partial \mathbf{A}}{\partial t} \bullet \mathbf{B} + \mathbf{A} \bullet \frac{\partial \mathbf{B}}{\partial t} \right] d\tau \\
&= \int \left[\left[-\mathbf{E} + \nabla \Phi \right] \bullet \mathbf{B} - \mathbf{A} \bullet \nabla \times \mathbf{E} \right] d\tau \\
&= \int \left[-2\mathbf{E} \bullet \mathbf{B} + \nabla \bullet (\mathbf{B}\Phi) + \nabla \bullet (\mathbf{A} \times \mathbf{E}) \right] d\tau \\
&= -2 \int \left[\mathbf{E} \bullet \mathbf{B} \right] d\tau \\
&= 2 \int \nabla \bullet \left[\lambda \nabla \left(\frac{\mathbf{J} \bullet \mathbf{B}}{B^2} \right) \right] d\tau = 0
\end{aligned}$$

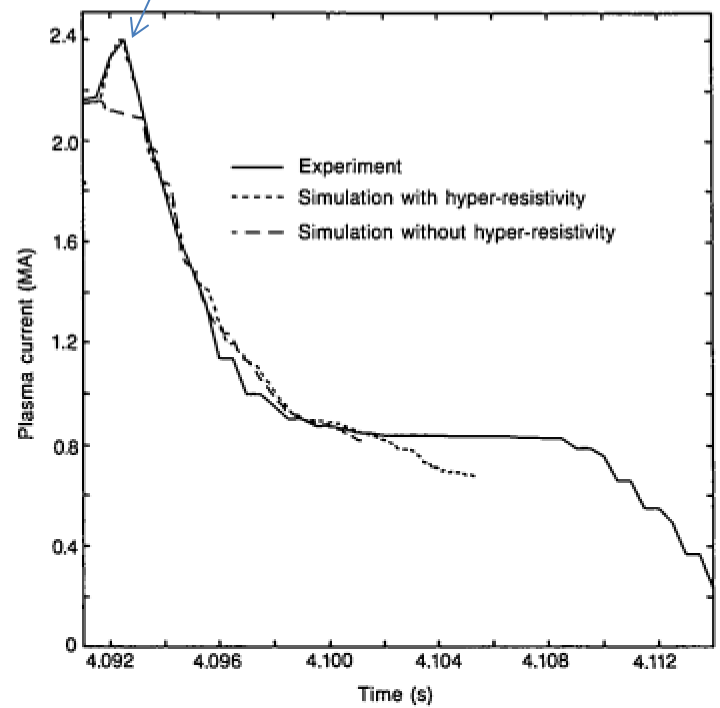
Boozer, 1986

This term has been used in the 2D TSC code to model disruptions

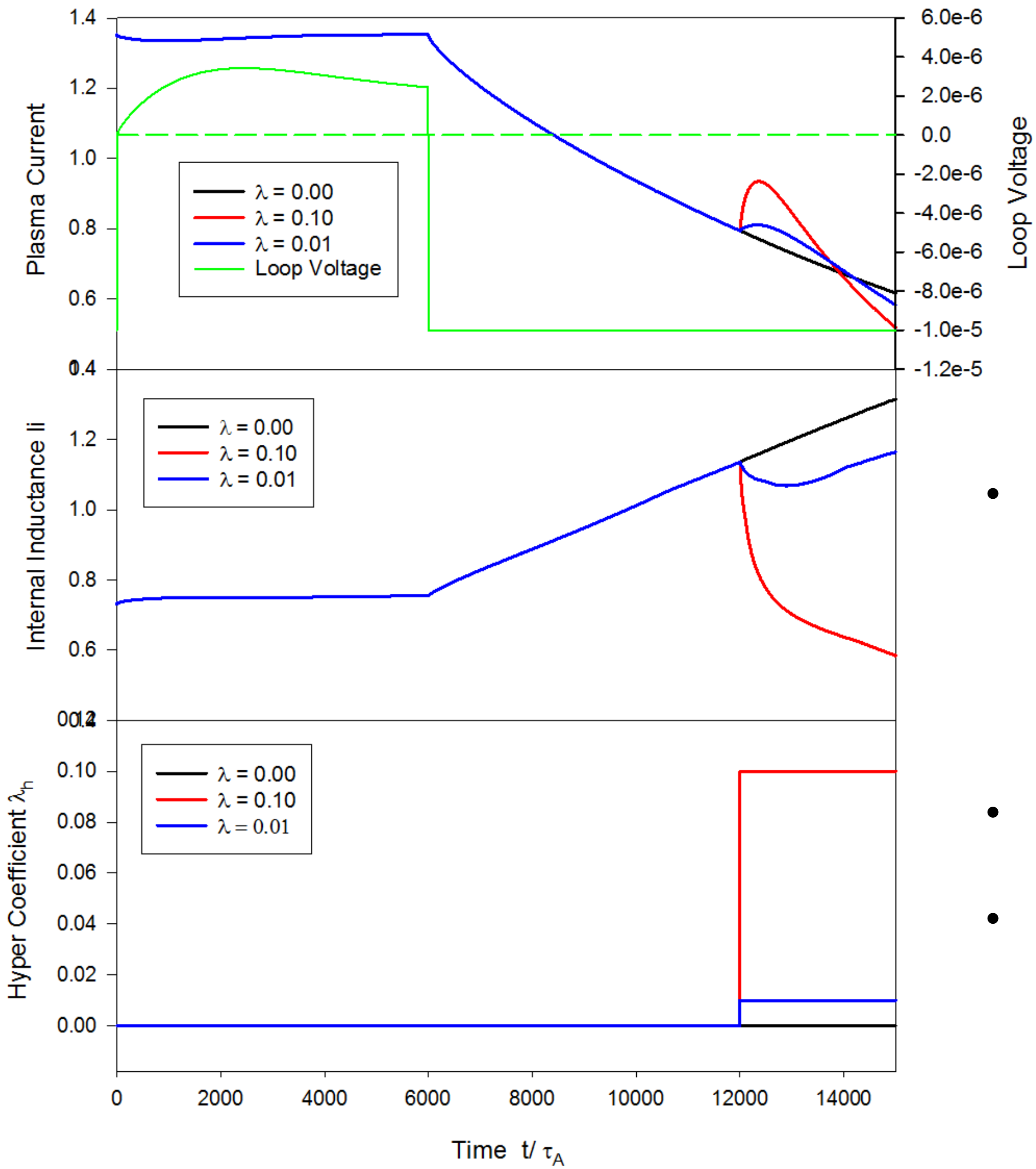


Plasma current in TFTR shot 19960

Can reproduce current spike with hyper-resistivity

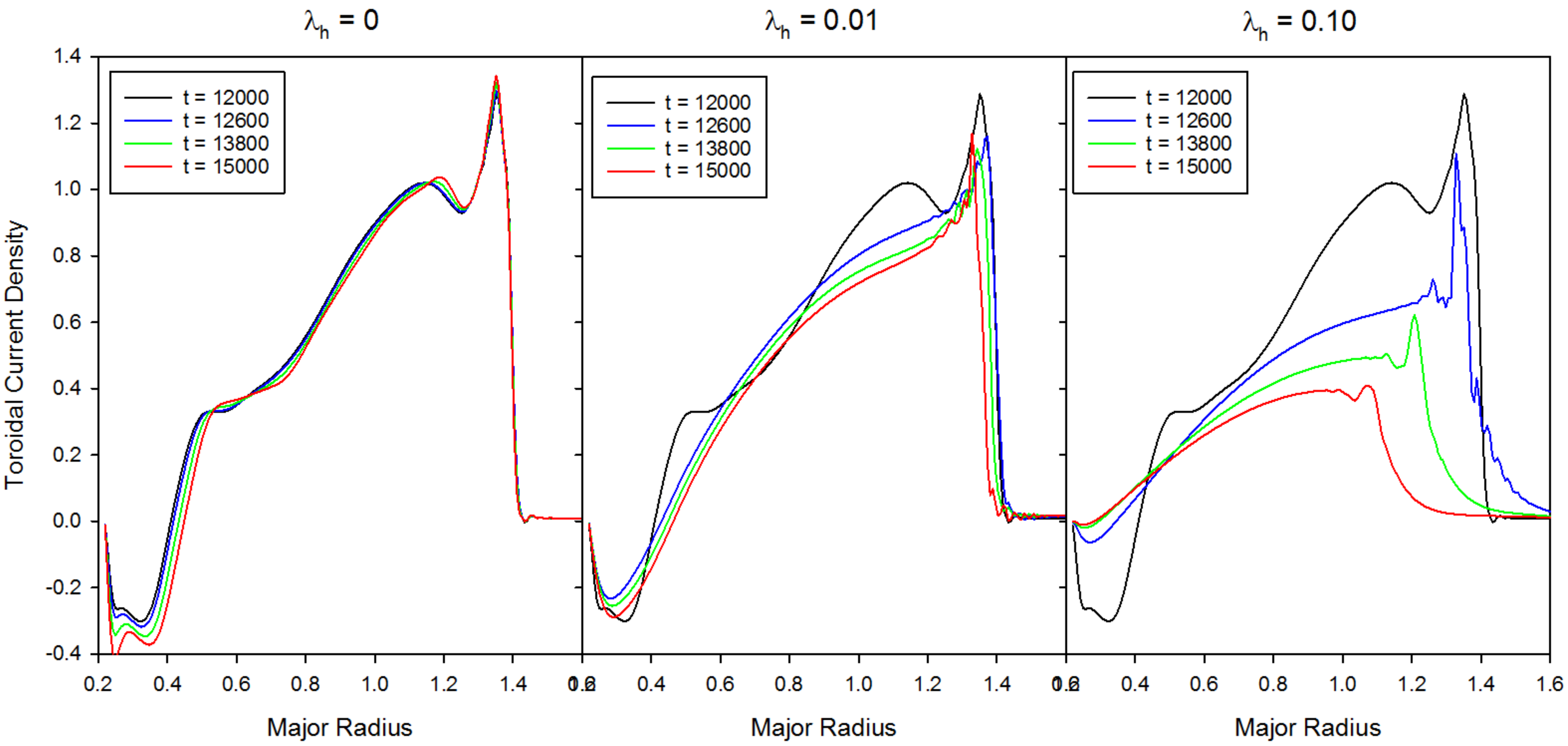


Addition of hyper-resistivity term to 2D M3D-C1 code



- Comparison of **2D** runs where hyper-resistivity is "turned on" at t=12000 τ_A
  - $\lambda = 0$
  - $\lambda = 0.10$ p
  - $\lambda = 0.01$ p
- Has the desired effect of increasing IP, lowering li
- When to turn it on?

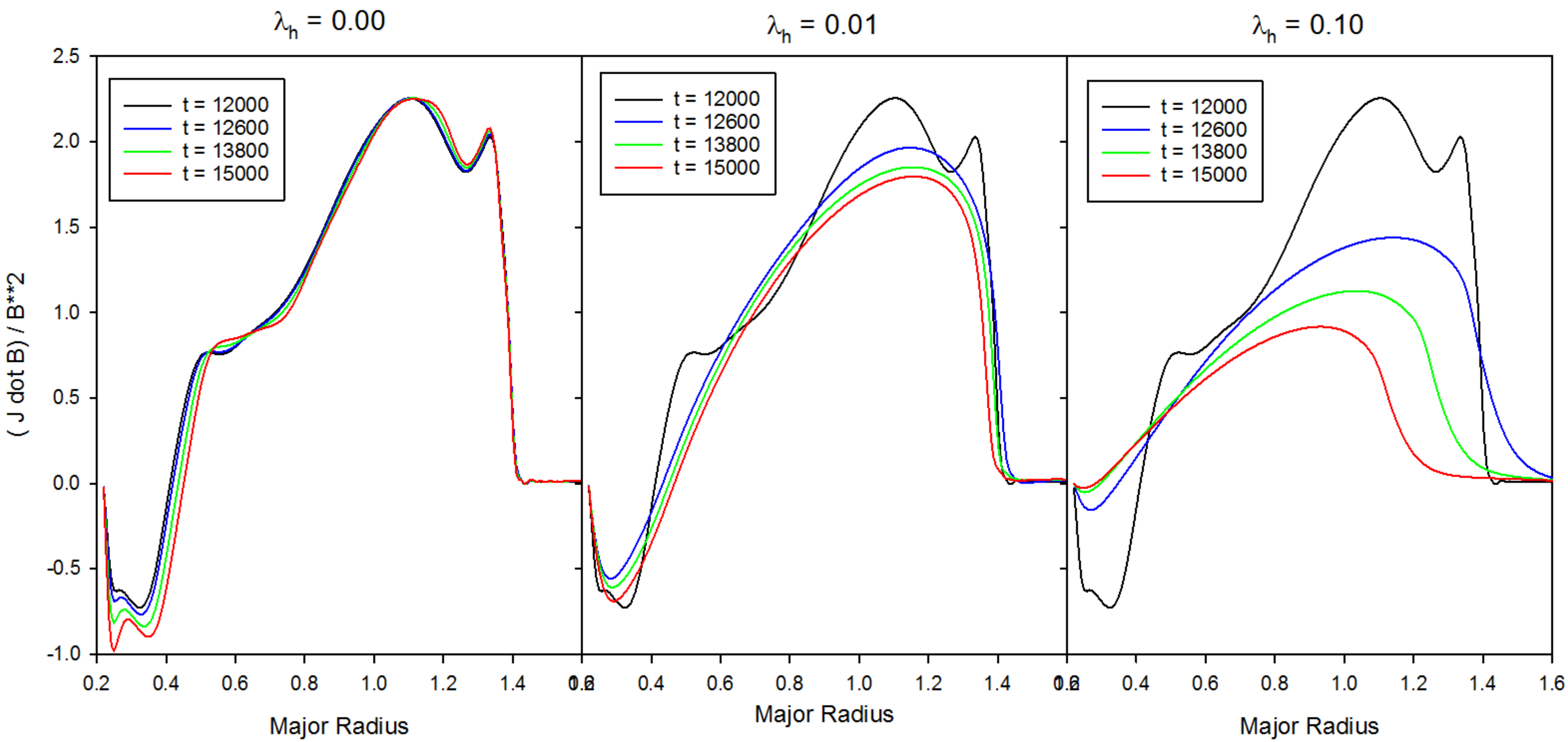
Comparison of current profiles after hyper-resistivity is applied



Clearly broadens current profile.

Comparison of σ profiles after hyper-resistivity is applied

$$\sigma \equiv \frac{\mathbf{J}\bullet\mathbf{B}}{B^2} \qquad \mathbf{R}_H = -\frac{\mathbf{B}}{B^2}\nabla\bullet\left[\lambda\nabla\sigma\right]$$



Summary

- Current ramp-down disruption in NSTX is caused by multiple ballooning modes becoming linearly unstable and nonlinearly interacting
- Modes with $6 < n < 21$ all become linearly unstable and grow
- Thermal quench caused by parallel conductivity on destroyed surfaces
- Reasonable agreement with experimental thermal quench time

But

- Have not been able to reproduce "current spike" in 3D simulation without hyper-resistivity
- May need to include hyper-resistivity proportional to magnitude of shortest wavelength being resolved….looks promising from 2D

And

- Now preparing to include resistive vessel and coils, and impurities to more closely model the experimental conditions