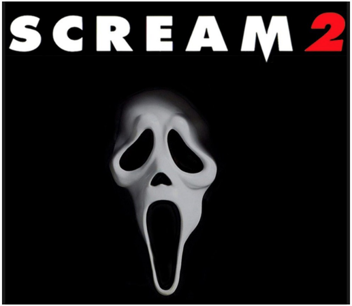
Recent progress and future directions in global runaway electron modeling

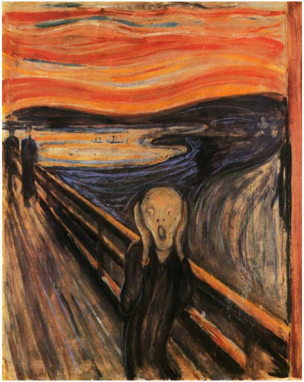
D.P. Brennan[⋆], E. Hirvijoki[†], C. Liu[†]

July 16, 2018

[⋆]Princeton University
[†]Princeton Plasma Physics Laboratory

What is SCREAM Really?



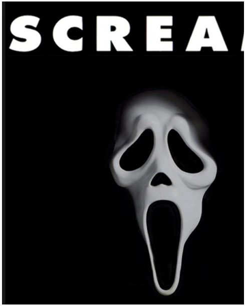


Are we this?

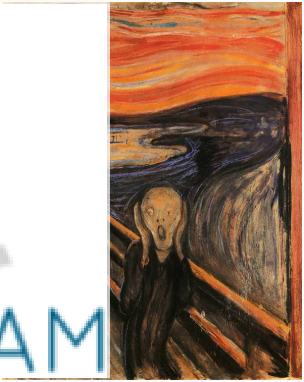
Or are we this?

Simulation Center for Runaway Electron Avoidance and Mitigation

What is SCREAM Really?



SCREAM



We're this

Simulation Center for Runaway Electron Avoidance and Mitigation

SCREAM is addressing all the primary issues identified in REs

Critical questions on runaway electron physics are driving the research

- Scattering of runaways by whistler waves, kinetic instabilities (could increase cyclotron losses and facilitate mitigation)
- Magnetic surface break up and and reformation.
- Impurity penetration to core of plasma
- The poloidal flux change required for an e-fold in the number of energetic electrons when $E_{||} \gg E_{ch}$.
- The spatial and temporal localization of relativistic electron losses.
- What can be learned about runaways during the non-nuclear phase of ITER operations.
- Mitigation methods (injection of particles, injection of cyclotron waves, induced currents in walls)

Great overview of RE issues: A.H. Boozer,"Pivotal issues on relativistic electrons in ITER," Nuclear Fusion 58,036006 (2018).

SCREAM SciDAC center structure in theory and simulation is poised to address the primary issues with REs

- Runaway electron generation
  - Full orbit simulations of runaway electrons with KORC
  - Backward Monte Carlo and Adjoint methods for runaway probability
  - Lifetime of runaways
  - Runaway vortex
- Thermal quenches and magnetic surface breakup
  - Reduced modeling of VDEs
  - Disruptions and RE Modeling with NIMROD and RE Orbit Modeling
- Mitigation via impurity injection
  - In MHD simulations of MGI and SPI with RE tracers
  - Theoretical efforts to better understand experiments
- Whistler wave scattering of runaway electrons
  - Explaining experimental observations of spontatneous onset
  - Exploring the use of wave launching to mitigate REs
- Advanced Vlasov-Fokker-Planck solvers
  - Conservative adaptive algorithms and solvers
  - Conservative Hamiltonian Vlasov integrators

Focus here on the coupling problem between MHD and RE simulation

Here we review a few selected highlights in some of these areas relevant to the self consistent coupling of MHD and RE generation.

We then present a theoretical basis for perhaps the most urgent near term need in development: self consistent coupling between MHD and RE generation modeling in full 3D.

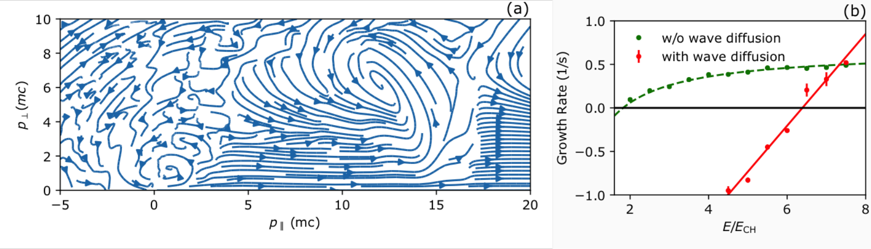
For a more comprehensive review of SCREAM research:
https://theory.pppl.gov/news/rrseminars/20180615Brennan.pdf

Table of contents

Recent progress and results

Quasilinear whistler interaction and phase space vortices explain critical E field puzzle

C. Liu et al, "Role of kinetic instability in runaway electron avalanche and elevated critical electric fields," arXiv:1801.01827, Phys. Rev. Lett. 120, 265001 (2018)

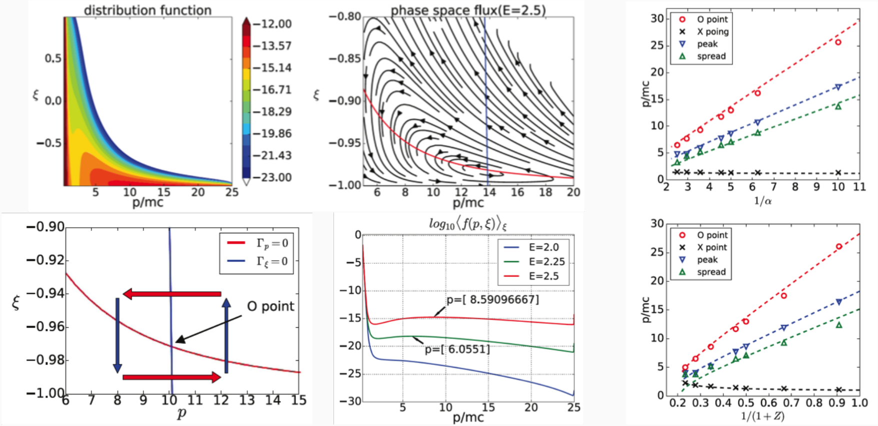


Experimental measurements of critical electric field for runaway have consistently been found to be 2x-3x higher than theory predicts with radiation and avalanche effects alone.

Explained by quasilinear whistler wave interaction, could be critical effect for ITER.

Runaway vortex captures the physics of energy and pitch distribution saturation

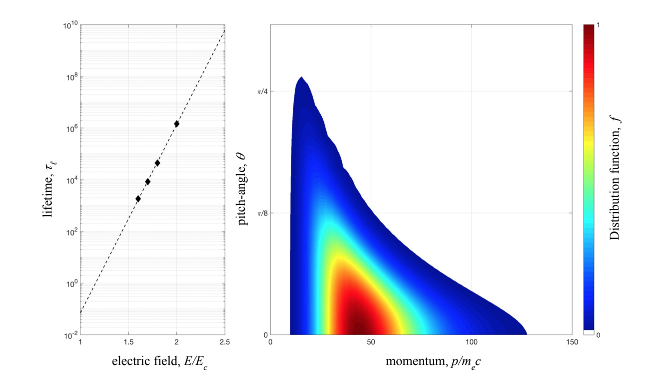
Due to radiation damping, runaways can saturate in energy and pitch $\rightarrow$ Runaway vortex $\rightarrow$ sets the primary runaway mean energy, the energy spread, and the bump in energy for pitch-integrated distribution.



Guo, McDevitt,Tang, PPCF 59, 044003 (2017)

$\alpha = \tau_c/\tau_s$ =coll/sync

Lifetime and universal distribution of seed runaway electrons elucidate the physics of avalanche

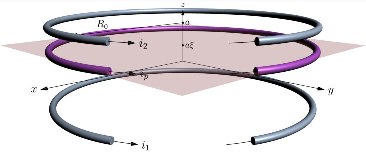


The lifetime (left) of runaway electrons increases exponentially with the inductive field which facilitates the onset of avalanche. The runaway seed forms a quasi-stationary distribution (right) in momentum-space due to the balance between the inductive drive, collisional friction, and synchrotron drag.

A.K. Fontanilla, B.N. Breizman, Phys. Plasmas 24, 112509 (2017).

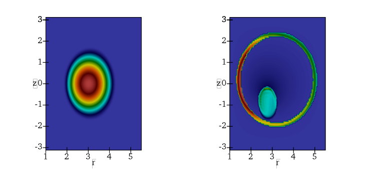
Model of vertical plasma motion during The current quench clarifies physics during RE generation

1D model



- The plasma current decay time is shorter than the wall resistive time in cold VDEs.
- Cold VDEs are characterized by a monotonic relation between the plasma current and plasma vertical displacement.

2D simulation



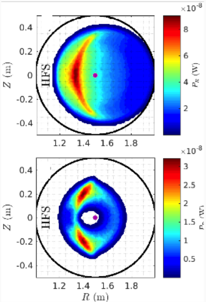
2D cold VDE code is being developed to

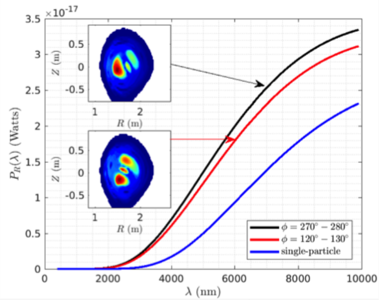
- Highlight key physics of 2D force-free plasma motion
- Amend numerical codes accordingly

D.I. Kiramov and B.N. Breizman, Phys. Plasmas 24, 100702 (2017)

Numerical simulation of runaway electrons with KORC: 3-D effects on synchrotron radiation (SR)

- **Understanding SR is important**: Radiation damping is a main energy loss mechanism of RE and SR is routinely used as experimental diagnostic.
- Shows key role played by trapped-particle dynamics and 3-D magnetic field effects including magnetic islands and stochasticity.
- Applications to DIII-D quiescent plasmas used to validate and point out potential limitation of current models of pitch angle dynamics.

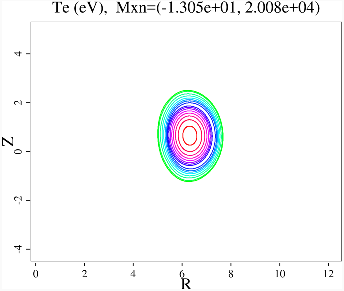


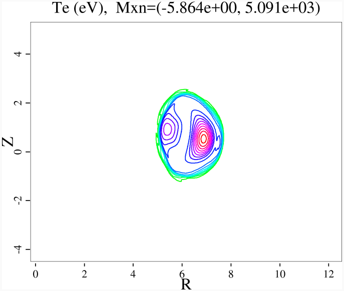


- Study of 3-D magnetic field geometry and orbit effects on synchrotron radiation (SR)
- Quantified radiation emission of trapped particles
- Studied role of magnetic islands and stochasticity on the 3-D spatial distribution of SR and power spectra
- Modeled SR on DIII-D quiescent plasmas.

D. del-Castillo-Negrete, L. Carbajal, D.Spong, and V. Izzo. Invited presentation APS-DPP 2017. Physics of Plasmas 25, 056104 (2018).

Disruptions and RE Modeling with NIMROD: Simulations of SPI and runaway confinement in ITER thermal quench

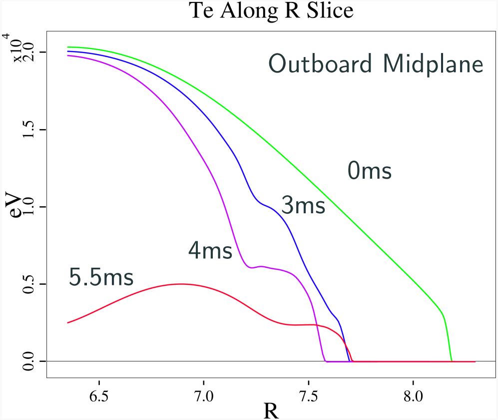




Initially 15MA Q=10 ITER Equilibrum

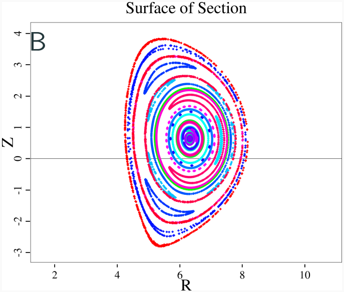
0.5kPam3 Ne pellet shatters into 125 fragments

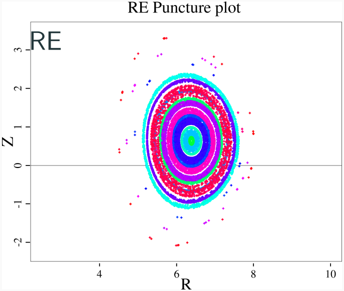
Spread out along single 1.5m outboard beam



60% T quench, all fragments ablate by 4.5ms

REs are found to be insensitive to islands and somewhat robust to stochasticity



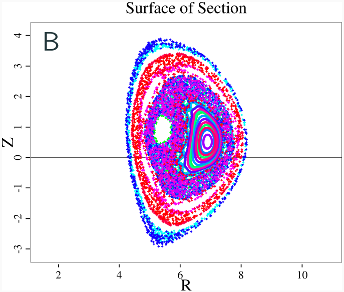


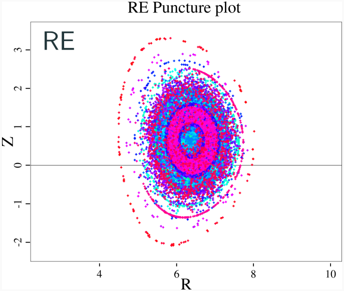
RE tracers accelerated in full fields for 30μs. Builds on work of V. Izzo.

Caveat: pitch scatter in $v_{||}$ but not in μ.

Note: Poincare points are at fixed toroidal angle ϕ crossing RE trace plots are at fixed time intervals.

C.C. Kim, 2018.





New directions

SCREAM plan includes exciting new directions in global runaway electron modeling

- Theoretical investigation of runaway physics and mitigation
- Relativistic Vlasov-Fokker-Planck simulations
- Interactions of RE with Whistler Waves
- Runaway Electron Modeling in MHD Simulations (M3D-C1 and NIMROD)
- Kinetic modeling of runaway electron and plume interaction
- Particle-based simulation of RE using KORC
- Computation of RE production rate using BMC probabilistic method
- Uncertainty quantification for statistical validation and verification
- Connecting with disruption SciDAC centers CTTS and TDS
- Self consistent coupling of MHD and RE simulations

Standard methods are inadequate for coupling Kinetic RE and MHD

Common theoretical practice: represent bulk distribution as Maxwellian parametrized by fluid quantities and advance remainder of distribution with kinetic principles.

Works given a source of a small population of highly energetic particles (mono-energetic beam injection, cosmic rays), where the source of the kinetic population is independent of the physics of the bulk population.

However, if an imbalance of effective forces generates the kinetic population from the bulk, and the interaction between the particles and the fluid is a key aspect in determining the outcome, then the two populations must be treated together to obtain a fully self consistent model of the evolution.

How to address this type of problem in general largely unsolved.

The challenge presented in this paper is to do it rigorously.

A fluid-kinetic framework for self-consistent runaway-electron simulations

Address the problem by dividing the electron population into a bulk and a tail.

Adopt probabilistic closure to determine the source and sink between the bulk and the tail populations

- preserving them both as genuine, non-negative distribution functions.

Derive macroscopic one-fluid equations and the kinetic equation for the runaway-electron population with source and sink terms.

Applicable to coupling particle codes (KORC) to MHD codes (M3D-C1,NIMROD) via a closure source/sink calculation (BMC).

E. Hirvijoki, C. Liu, G. Zhang, D. del Castillo-Negrete, and D. Brennan, Submitted to Phys. of Plasmas, arXiv:1802.02174, 2018.

Split of kinetic equations

Start from the kinetic equation for species α

$$\frac{df_\alpha}{dt} = \sum_\beta C_{\alpha\beta}[f_\alpha, f_\beta], \tag{1}$$

where d/dt refers to a linear phase-space advection operator, such as the Vlasov operator, and $C_{\alpha\beta}$ is a bilinear collision operator between the species α and β.

Use linearity of d/dt, bilinearity of $C_{\alpha\beta}$ and split the equations according to

$$\frac{df_{\alpha 0}}{dt} = \sum_\beta C_{\alpha\beta}[f_{\alpha 0}, f_{\beta 0}] + \sum_\beta C_{\alpha\beta}[f_{\alpha 0}, f_{\beta 1}] - I_\alpha, \tag{2}$$

$$\frac{df_{\alpha 1}}{dt} = \sum_\beta C_{\alpha\beta}[f_{\alpha 1}, f_{\beta 1}] + \sum_\beta C_{\alpha\beta}[f_{\alpha 1}, f_{\beta 0}] + I_\alpha. \tag{3}$$

Where I_α is an as-of-yet unknown interaction or closure term.

Probabilistic closure

For each species, we consider a closure term I of the form

$$I(\boldsymbol{z}, t) = \frac{f_0}{\tau}\Big(1 - \mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}]\Big) - \frac{f_1}{\tau}\mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}], \quad (4)$$

- τ: characteristic time scale, such as collision time
- Ω_0: characteristic domain for f_0
- $\boldsymbol{z}$: phase-space coordinates, e.g. $(\boldsymbol{x}, \boldsymbol{v})$
- $\boldsymbol{Z}_s$: with $s \in [t, t+\tau]$, particle trajectory in phase-space.
- $\mathbb{E}$: expectation value
- $\mathbf{1}$: indicator function

Specifically, $\mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}]$ is the probability for finding a particle with an initial position $\boldsymbol{z}$ at time t within the domain Ω_0 after the time interval τ.

Properties of the closure

Form of interaction term $I(\boldsymbol{z}, t)$ guarantees the non-negativity of f_0, f_1.

- given that d/dt and C do so.

For any point $\boldsymbol{z}_\star$ where $f_0(\boldsymbol{z}_\star, t) = 0$, the contribution from $I(\boldsymbol{z}, t)$ to the evolution of f_0 is $f_1(\boldsymbol{z}_\star, t)\mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}_\star]/\tau \geq 0$, increasing the value of f_0.

Similarly, if $f_1(\boldsymbol{z}_\star, t) = 0$, the contribution from $I(\boldsymbol{z}, t)$ to the evolution of f_1 is $f_0(\boldsymbol{z}_\star, t)(1 - \mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}_\star])/\tau \geq 0$, increasing the value of f_1. This stems from the fact that $\mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}] \in [0, 1]$.

Both f_0 and f_1 can thus be interpreted as genuine distribution functions.

We also expect $I(\boldsymbol{z}, t)$ to provide a stable splitting scheme devoid of unphysical oscillations and exponentially growing modes: assuming f_0 and f_1 to be driven only by $I(\boldsymbol{z}, t)$ with a fixed value for $\mathbb{E}$, both f_0 and f_1 would relax exponentially with a time-scale τ to an equilibrium determined by the initial values for f_0 and f_1 and $\mathbb{E}$ and its complement.

Coupling of MHD and runaway physics

Equations for a bulk and a tail

Start from the kinetic equations and split the distribution functions

$$\frac{df_\alpha}{dt} = \sum_\beta C_{\alpha\beta}[f_\alpha, f_\beta] \qquad\qquad f_\alpha = f_{\alpha 0} + f_{\alpha 1} \tag{5}$$

Invent an "interaction term" to obtain linearly independent equations

$$\frac{df_{\alpha 0}}{dt} = \sum_\beta C_{\alpha\beta}[f_{\alpha 0}, f_{\beta 0}] + \sum_\beta C_{\alpha\beta}[f_{\alpha 0}, f_{\beta 1}] - I_\alpha, \tag{6}$$

$$\frac{df_{\alpha 1}}{dt} = \sum_\beta C_{\alpha\beta}[f_{\alpha 1}, f_{\beta 1}] + \sum_\beta C_{\alpha\beta}[f_{\alpha 1}, f_{\beta 0}] + I_\alpha, \tag{7}$$

$$I(\boldsymbol{z}, t) = \frac{f_0}{\tau}\left(1 - \Phi(\boldsymbol{z}, t)\right) - \frac{f_1}{\tau}\Phi(\boldsymbol{z}, t), \tag{8}$$

The transition probability $\Phi(\boldsymbol{z}, t) = \mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau}) | \boldsymbol{Z}_t = \boldsymbol{z}]$ to model the relabeling of particles between the bulk and the tail.

Fluid-kinetic equations

Continuity equation

$$\partial_t \varrho + \nabla \cdot (\varrho \boldsymbol{u}) = -\int m_e I_e d\boldsymbol{v}, \tag{9}$$

Momentum equation

$$\begin{aligned}\varrho(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}) = -\nabla \cdot \mathbf{p} + \mu_0^{-1} \nabla \times \boldsymbol{B} \times \boldsymbol{B} \\ - \sum_\alpha \boldsymbol{F}_{e1,\alpha 0} - \int m_e (\boldsymbol{v} - \boldsymbol{u}) I_e d\boldsymbol{v} + e n_{e1} (\boldsymbol{E} + \boldsymbol{v}_{e1} \times \boldsymbol{B}).\end{aligned} \tag{10}$$

Ohm's law

$$\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = \eta(\mu_0^{-1} \nabla \times \boldsymbol{B} + e n_{\mathrm{e1}} \boldsymbol{v}_{\mathrm{e1}}), \tag{11}$$

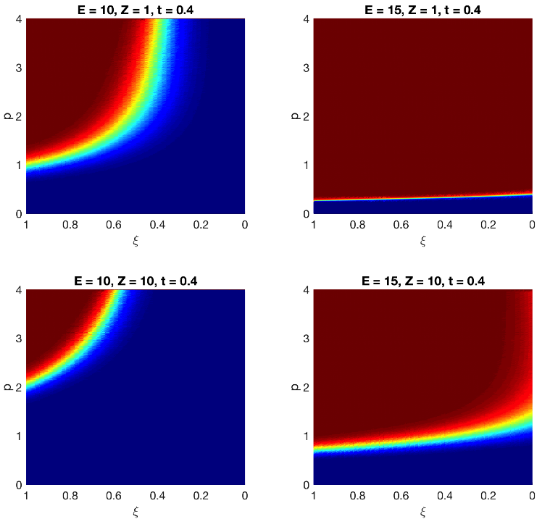
Kinetic equation for runaways

$$\frac{d f_{e1}}{dt} = C_{ee}[f_{e1}, f_{e1}] + \sum_\beta C_{e\beta}[f_{e1}, f_{\beta 0}] + I_e. \tag{12}$$

Backward Monte Carlo can be used to calculate the transition probabilities

Near Term Target: Couple single fluid MHD and KORC, calculate the transition probabilities using the Backwards Monte Carlo method.

Key aspect: time-dependent transition probabilities computed deterministically from the stochastic trajectories of particles



G. Zhang and D. del-Castillo-Negrete, Phys. Plasmas 24, 092511 (2017), ArXiv:1708.00947.

Lowest-hanging fruit

Number of seed electrons available for runaway and avalanche during a thermal quench

- Before significant runaway population forms, seed electrons remain with a kinetic energy above the critical energy
- Among the most important questions
- Effects of 3D fields of MHD during a disruption on this process is largely unexplored.
- Can make progress with self-consistent fluid-kinetic framework.

Seed electron distribution in momentum and pitch angle, along with electric field and dissipative effects, can lead to three outcomes; sub-criticality, avalanche, or fast transfer.

Simulating generation in 3D fields is critical to predictive capability.

Summary and Conclusions

SCREAM is making significant progress in addressing the runaway electron problem.

- Advanced understanding of confinement, transport and energetics of relativistic electrons
- Whistler scattering effects identified in experimental data
  - Critical electric field and mechanisms for mitigation strongly affected
- Multiple new mitigation ideas being explored, including wave launching
- At least 37 journal articles published with team member authorship since 7/16 initiation, including 3 PRLs. Broad range of topics.

The fluid-kinetic framework offers a new and exciting mechanism to explore coupling MHD and runaway electron simulations.

- Best path forward to understand effects of 3D fields on runaway generation and make progress toward predictive capability.