



**CHALMERS**  
UNIVERSITY OF TECHNOLOGY



# Modeling the anisotropic radiation of runaway electrons

**Mathias Hoppe<sup>1</sup>**

**O. Embréus<sup>1</sup>, R. A. Tinguely<sup>2</sup>, R. S. Granetz<sup>2</sup>, T. Fülöp<sup>1</sup>**

<sup>1</sup> Chalmers University of Technology, Gothenburg, Sweden

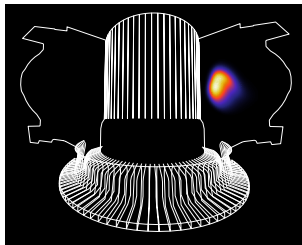
<sup>2</sup> Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA, USA

- How to measure the runaway distribution?
- Runaways emit bremsstrahlung & synchrotron
- Measured in most larger tokamaks
  - ▶ **Bremsstrahlung:** hard X-rays ( $\sim$  MeV range)
  - ▶ **Synchrotron:** visible & IR
- Give rise to “spots” in images

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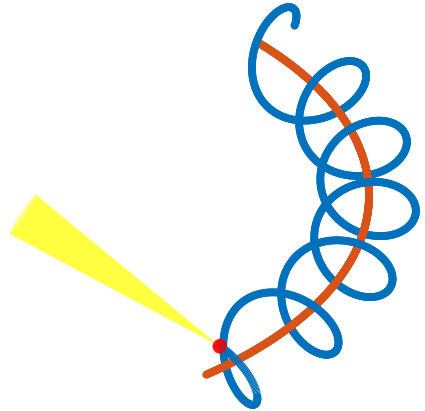
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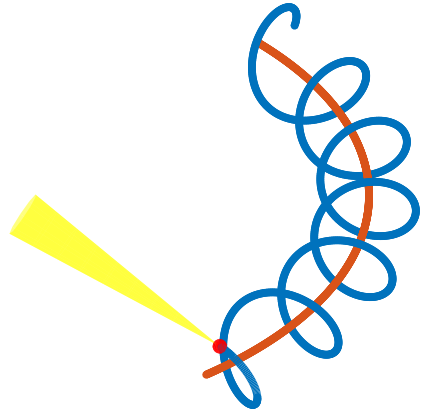
Continuous acceleration  
+ relativistic energy

⇒ **Radiation directed  
along velocity vector**



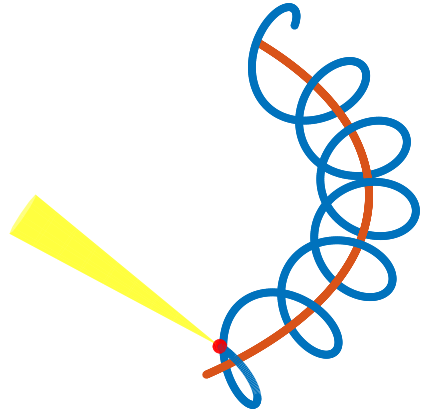
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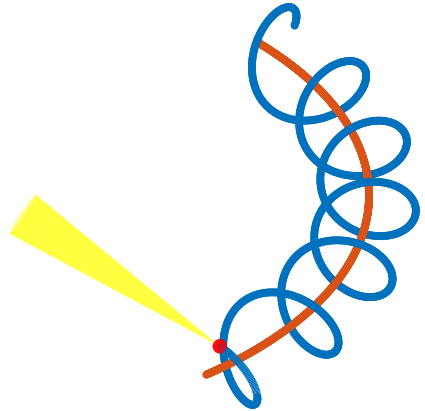




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⇒ **Radiation depends  
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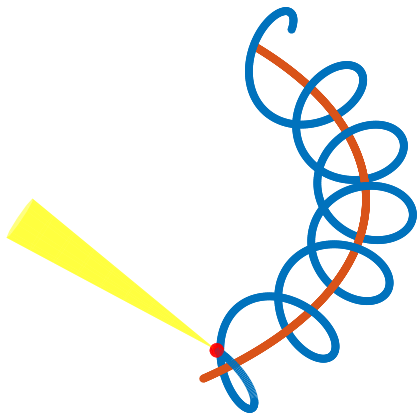


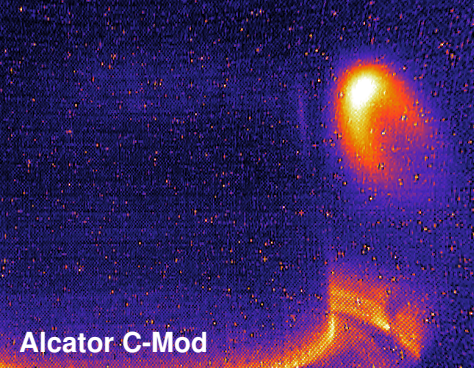
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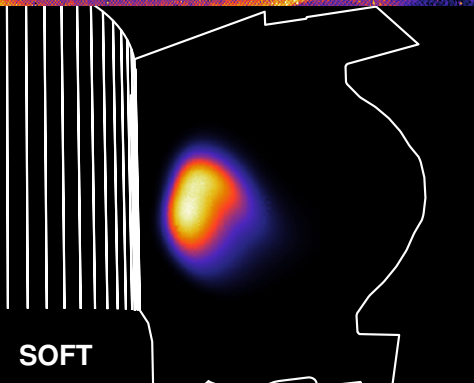
⇒ **Probes**

- Radial position
- Pitch angle
- Energy





Alcator C-Mod



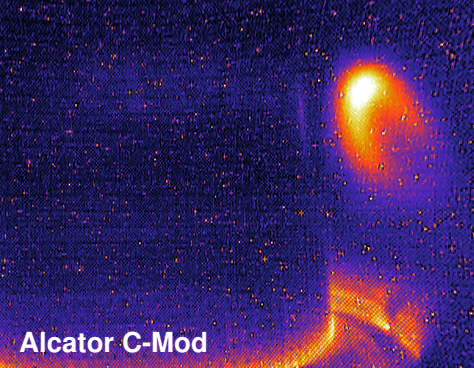
SOFT

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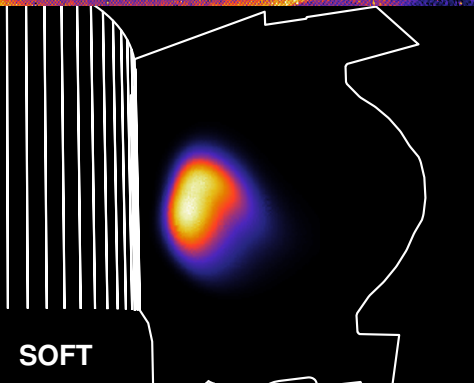
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- Synthetic diagnostic for radiation from runaways
- Real(istic) magnetic geometry
- Includes RE distribution
- Image/spectrum/Green's function

$$\text{Image/spectrum} = \int W(\mathbf{x}, \mathbf{p}) f(\mathbf{x}, \mathbf{p}) d\mathbf{x} d\mathbf{p}$$



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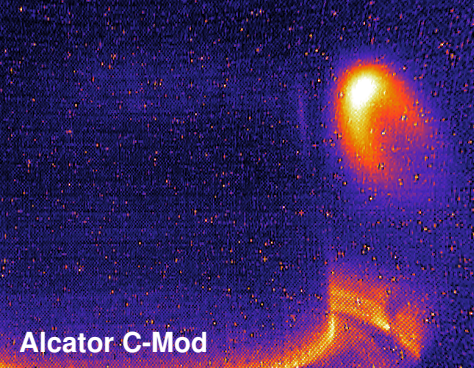
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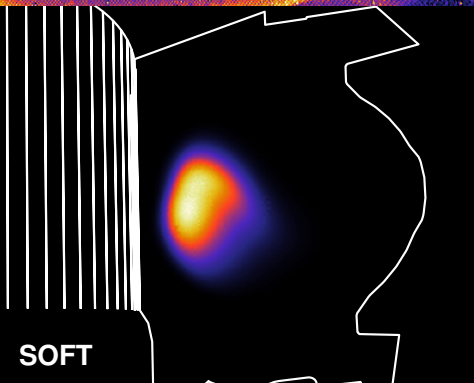
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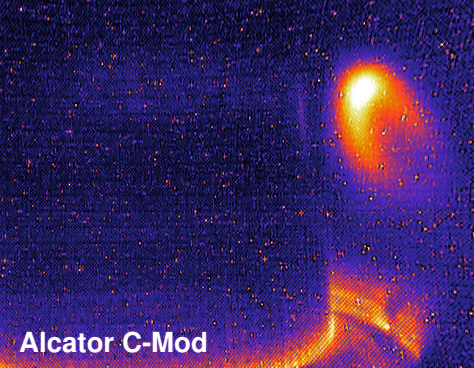
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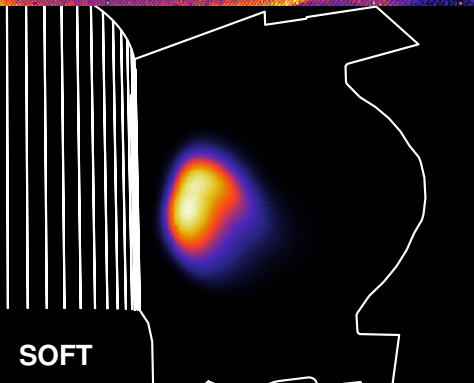
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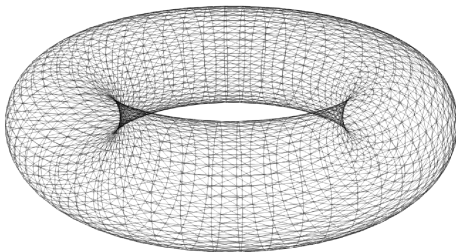
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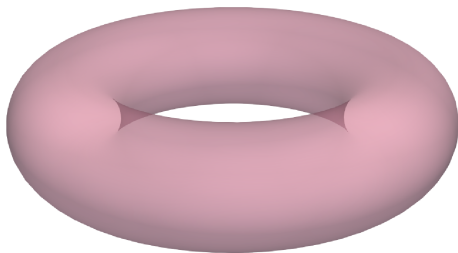
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- Integrate all emitted radiation  $d^2P/d\lambda d\Omega$
- ...with the distribution of particles  $f(\mathbf{x}, \mathbf{p})$
- ...falling in along the line-of-sight  $\mathbf{n}$
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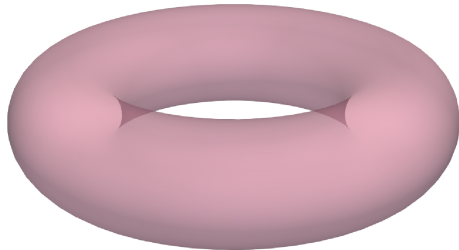
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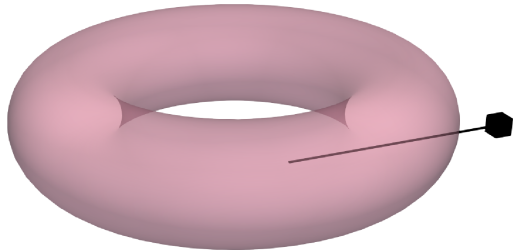
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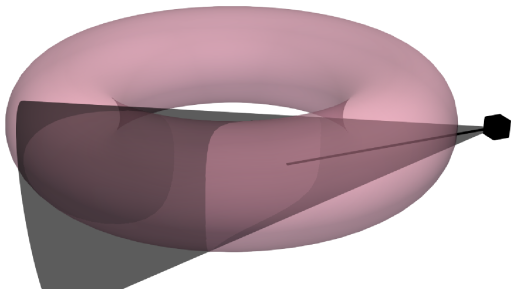
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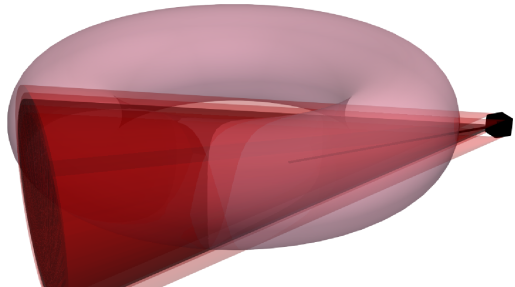
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Trick in SOFT: Use other coordinates for  $\int \dots d\mathbf{x}d\mathbf{p}$

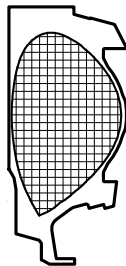
Parametrize using guiding-center orbits ( $\mu$  conservation)

$$\mathbf{x} \rightarrow \begin{cases} \rho, & \text{Initial major radius (outer midplane)} \\ \tau, & \text{Time parameter along orbit} \\ \phi, & \text{Initial toroidal angle} \end{cases}$$

$$\mathbf{p} \rightarrow \begin{cases} p_{\parallel}^{(0)}, & \text{Initial parallel momentum} \\ p_{\perp}^{(0)}, & \text{Initial perpendicular momentum} \\ \zeta, & \text{Gyro-angle (averaged out)} \end{cases}$$

Why?

- Guiding-center orbits  $\implies$  orders-of-magnitude faster computation
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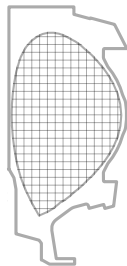
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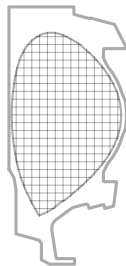
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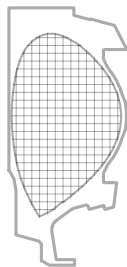
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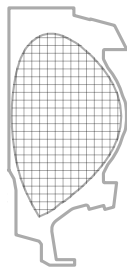
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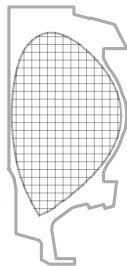
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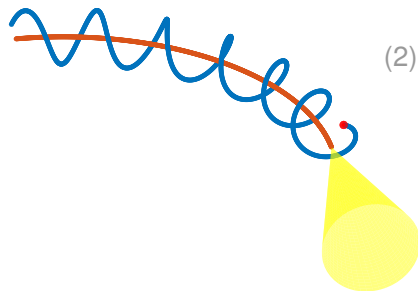
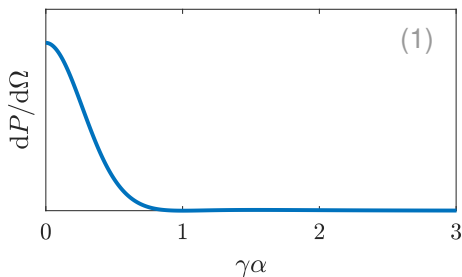
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How to handle the emission,  $d^2P/d\lambda d\Omega$ ?

Angular distribution:  $\frac{d^2P}{d\lambda d\Omega} = \dots$  from e.g. Jackson, (1)

Cone model:  $\frac{d^2P}{d\lambda d\Omega} = \frac{1}{2\pi} \frac{dP}{d\lambda} \delta(\cos \mu - \cos \theta_p)$  (2)

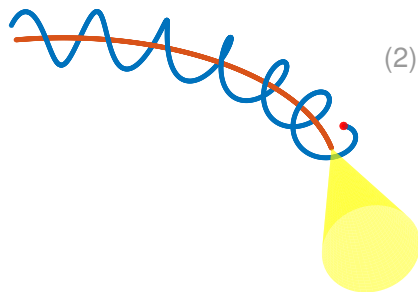
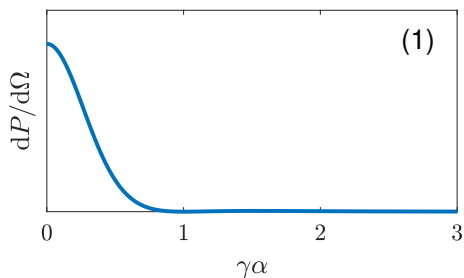


$\alpha$  = Angle between particle velocity & line-of-sight

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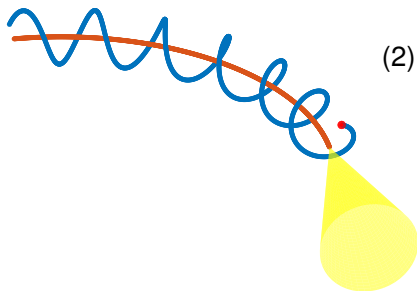
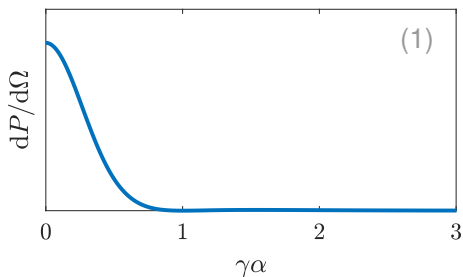


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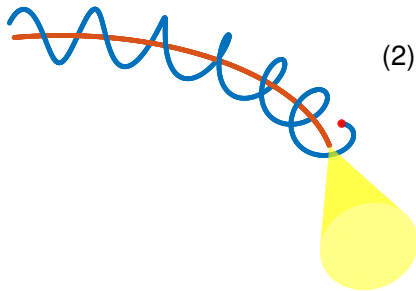
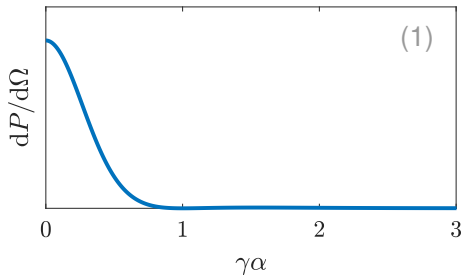
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( $\sim 10$ - $100$  times faster)



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# **Elements of a RE radiation image**

Specify momentum

$$(p, \theta_p) = (p_0, \theta_0)$$

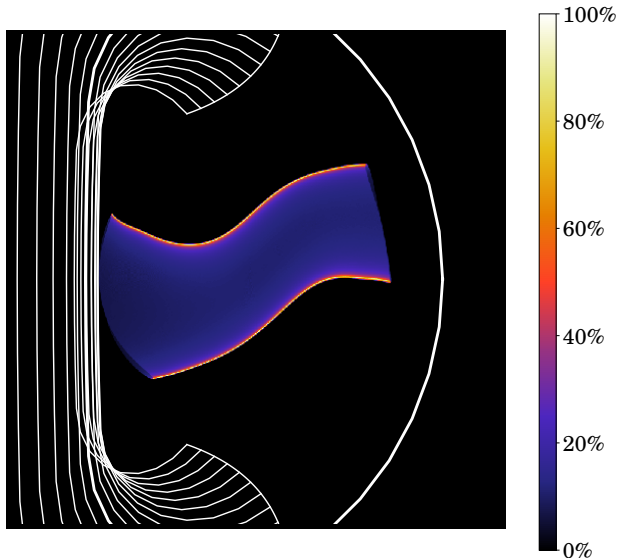
What does the corresponding image look like?



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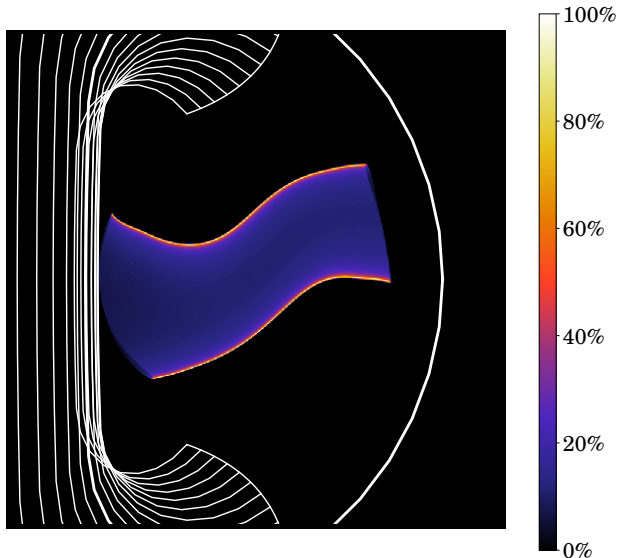


Specify momentum

$$(\rho, \theta_p) = (\rho_0, \theta_0)$$

What does the corresponding image look like?

Edges much brighter than body



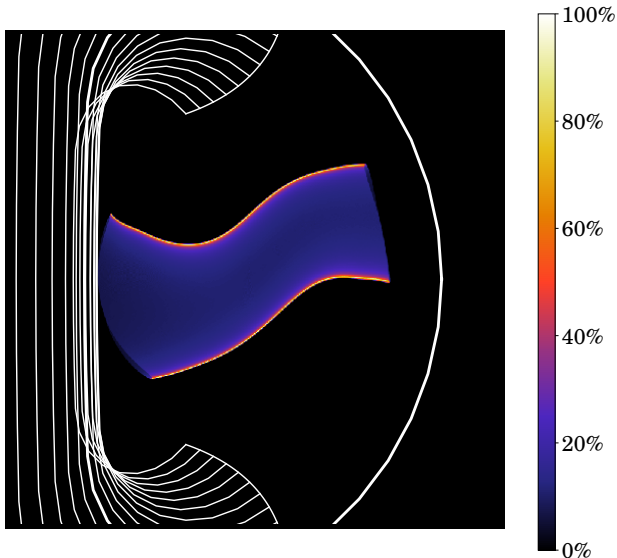
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Radiation emitted on “surface”  
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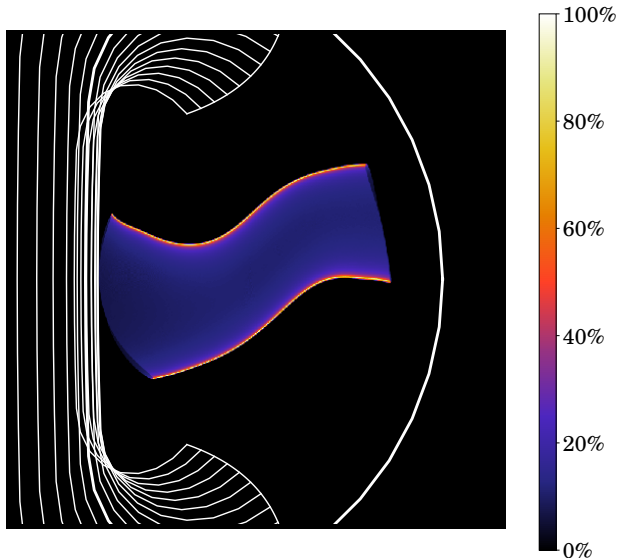
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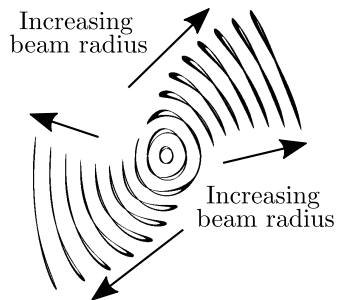
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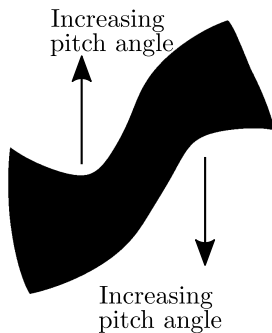
⇒ Line integration effect



Radial location



Pitch angle



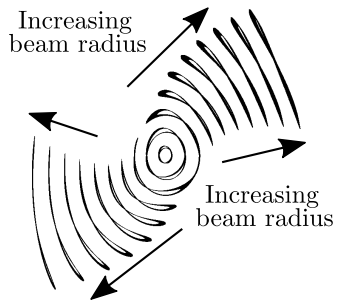
Energy



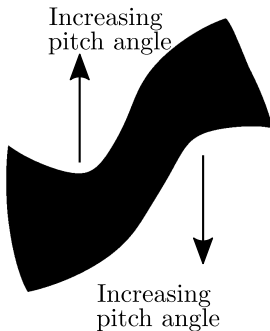
High energy:

$$P(\lambda) \propto \sqrt{B}$$

Radial location



Pitch angle

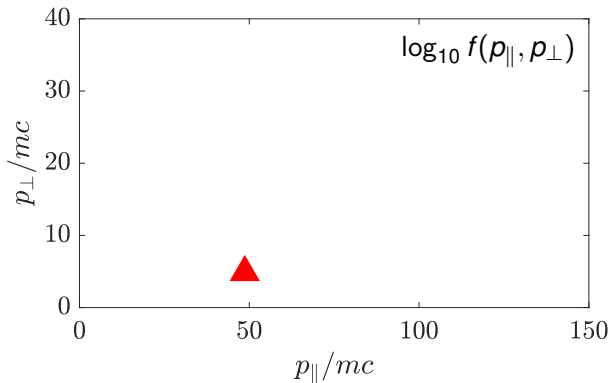
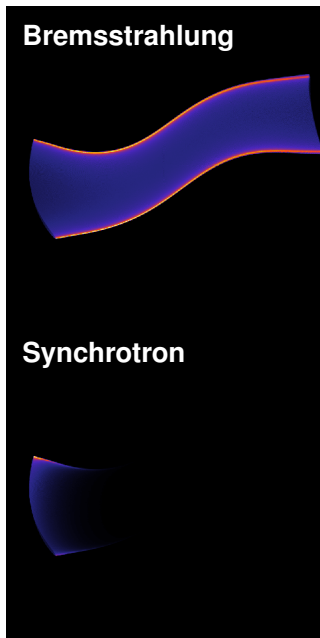


Energy

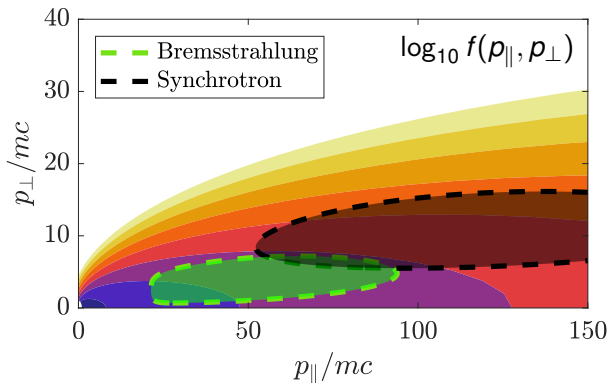
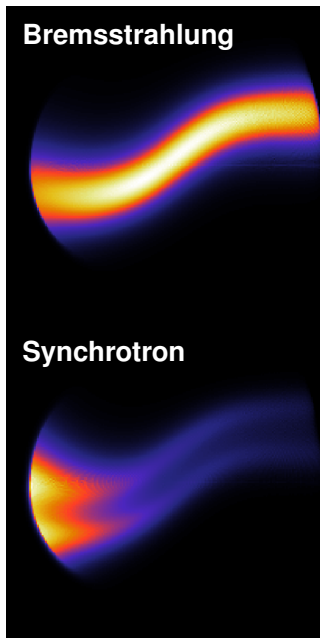


Low energy:

$$P(\lambda) \propto \exp \left[ - (B_c/B)^{1/3} \right]$$

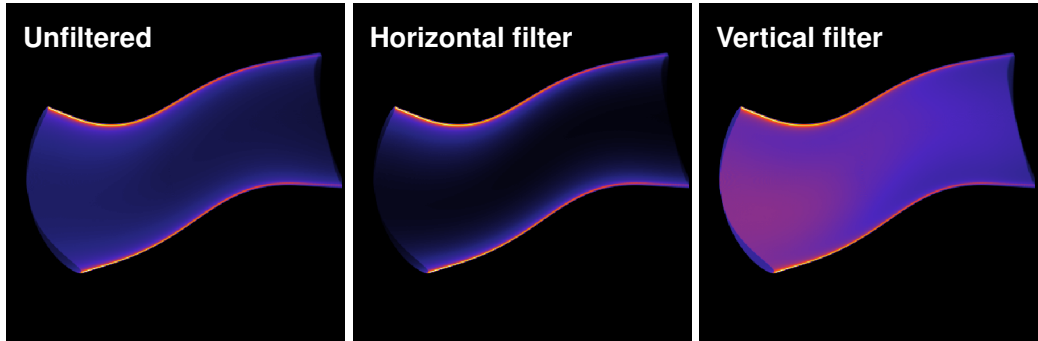


- Different generation mechanisms
  - ▶ Bremsstrahlung: Collisions
  - ▶ Synchrotron: Gyration
- Different windows into distribution

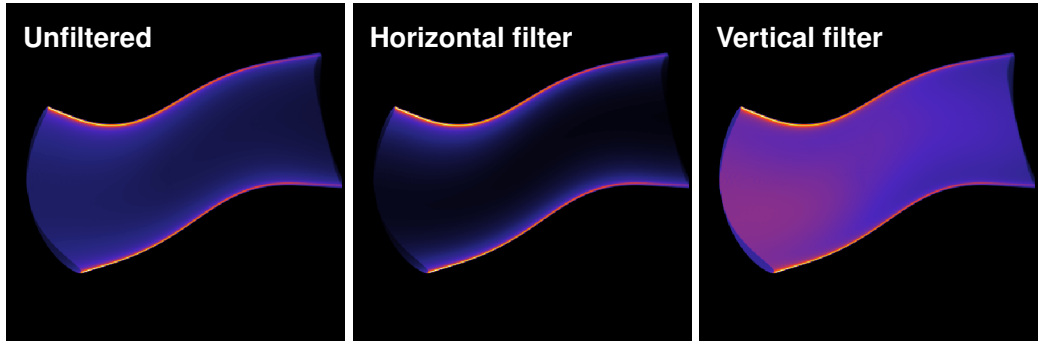


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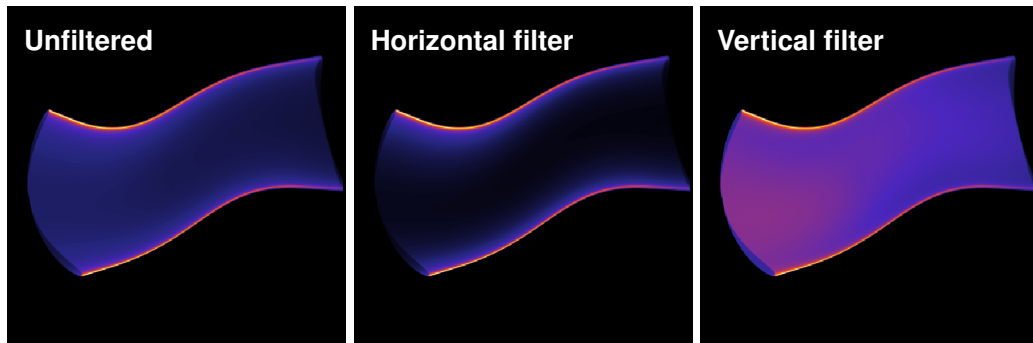




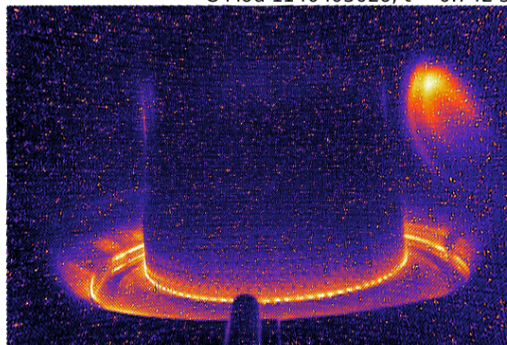
- Filters out **body** (horizontal) and **edges** (vertical)
- Views same parts of momentum-space
- Closely coupled to magnetic geometry



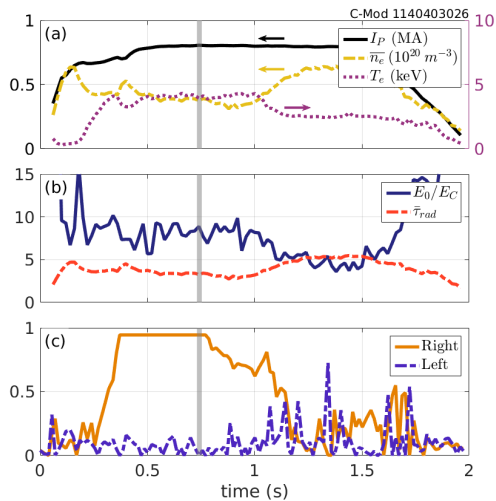
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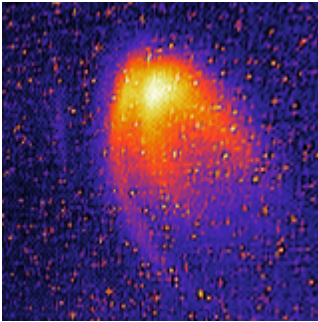


- Filters out **body** (horizontal) and **edges** (vertical)
- Views same parts of momentum-space
- Closely coupled to magnetic geometry

C-Mod 1140403026,  $t \sim 0.742$  s

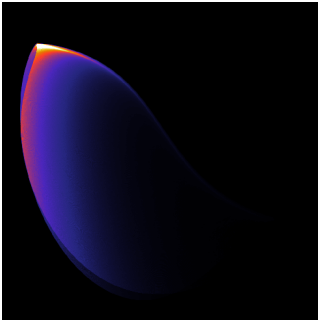
- Low-density QRE in C-Mod
- $B = 5.4$  T on-axis
- Much synchrotron in visible range

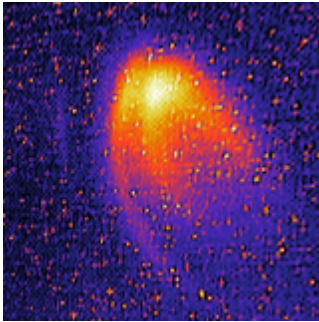




### Understanding the synchrotron spot

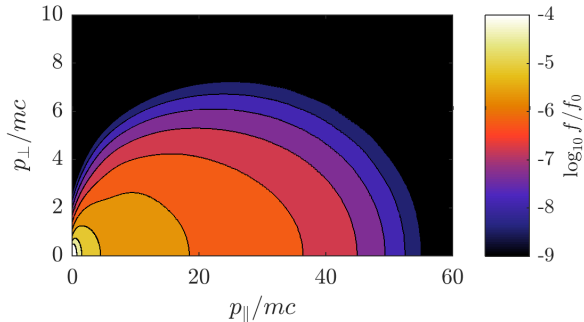
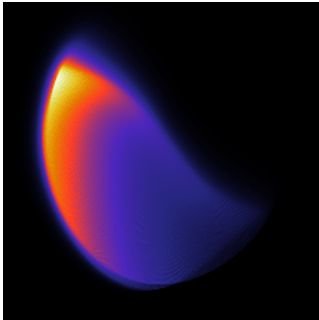
1. Single energy (15 MeV), single pitch angle (0.15 rad)
2. CODE distribution function from plasma parameters
3. + simple radial distribution
4. Radial distribution fit

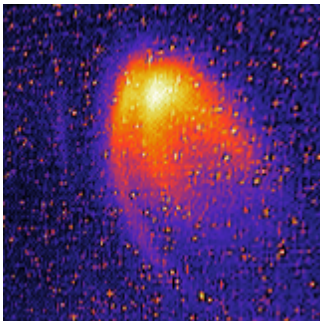




## Understanding the synchrotron spot

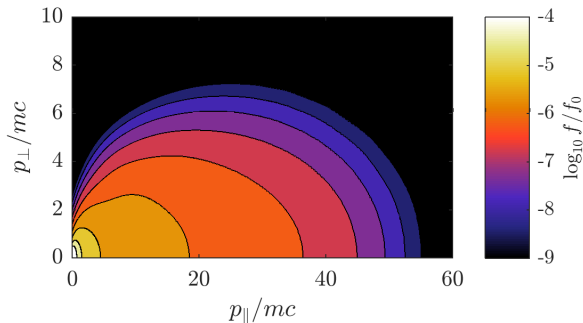
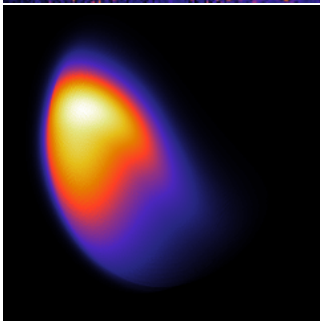
1. Single energy (15 MeV), single pitch angle (0.15 rad)
2. CODE distribution function from plasma parameters
3. + simple radial distribution
4. Radial distribution fit

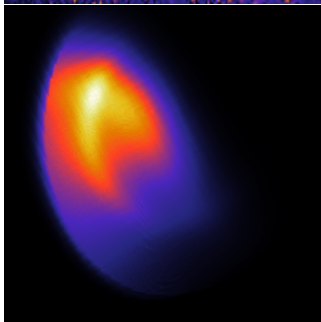
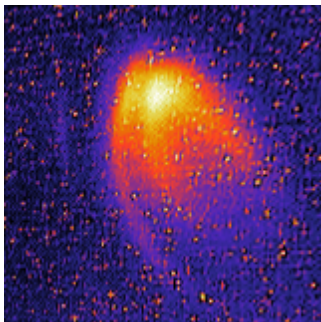




## Understanding the synchrotron spot

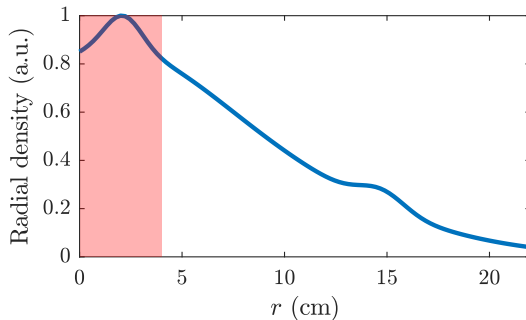
1. Single energy (15 MeV), single pitch angle (0.15 rad)
2. CODE distribution function from plasma parameters
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4. Radial distribution fit





## Understanding the synchrotron spot

1. Single energy (15 MeV), single pitch angle (0.15 rad)
2. CODE distribution function from plasma parameters
3. + simple radial distribution
4. Radial distribution fit





- SOFT simulates runaway **bremsstrahlung** & **synchrotron** radiation
- Radiation highly sensitive to both spatial & velocity parts of runaway **distribution**
- Linearly polarized SR views **same part** part of distribution as unfiltered SR
- Non-uniform radial density needed to explain C-Mod image

### Further reading:

[1] Hoppe M. et al., *SOFT: a synthetic synchrotron diagnostic*, Nucl. Fusion **58** 026032 2018

[2] Hoppe M. et al., *Interpretation of runaway electron synchrotron and bremsstrahlung images*, Nucl. Fusion **58** 082001 2018

[3] Tinguely R. A. et al., *Measurements of runaway electron synchrotron spectra at high magnetic fields in Alcator C-Mod*, Nucl. Fusion **58** 076019 2018

SOFT and other runaway electron tools are freely available at <http://ft.nephy.chalmers.se/retools>