Radiation of runaway electrons in tokamaks with fan instability

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Highlights

- Runaway electron beam is susceptible to fan instabilities, which can drive whistler waves and cause significant pitch-angle scattering of resonant electrons.
- Runaway electrons scattered by fan instabilities have different properties on wave radiation than before, including electron cyclotron waves, extraordinary electron (EXEL) waves and very low frequency whistler waves.
- Because of R-wave cutoff, the whistler waves associated with fan instabilities cannot propagate to the edge, whereas the very low frequency whistler waves ($<$ 200MHz) and extraordinary waves with $\omega \approx \omega_{pe}$ can propagate to the edge and get detected.

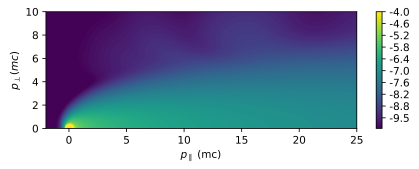
Outline

1. Motivation: Direct observations of plasma waves radiated by runaway electrons
2. Simulation model of fan instabilities
3. Simulation results of fan instability in QRE scenario
4. Radiation and wave excitation of runaway electron after fan instabilities
5. Accessibility of plasma waves emitted by runaway electrons
6. Summary

Motivation: Direct observations of plasma waves radiated by runaway electrons

Runaway electrons are susceptible to fan instabilities

- High energy runaway electron beam can be generated in tokamak experiments, including both quiescent scenarios and disruptions.
- REs can excite plasma waves, mainly whistler waves (WW), through wave-particle resonances.



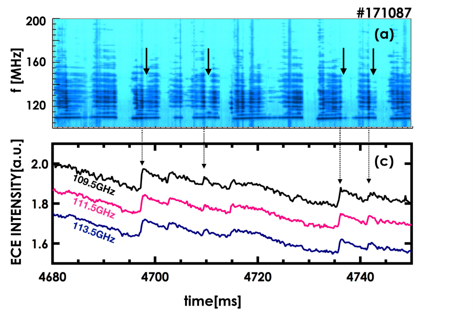
- Bump-on-tail → Cherenkov resonance ($\omega - k_\parallel v_\parallel = 0$)
- Anisotropy → Doppler resonance. ($\omega - k_\parallel v_\parallel = n\omega_{ce}/\gamma$)

- The excited modes can cause significant pitch angle scattering of resonant electrons.

V.V. Parail and O.P. Pogutse, Nucl. Fusion 18, 303 (1978).
T. Fülöp, G. Pokol, P. Helander, and M. Lisak, Physics of Plasmas 13, 062506 (2006).
G.I. Pokol, A. Kómár, A. Budai, A. Stahl, and T. Fülöp, Phys. Plasmas 21, 102503 (2014).
P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

Whistler (helicon) waves are observed in recent RE experiments on DIII-D

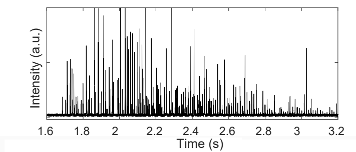
- In DIII-D flattop RE experiments, whistler with frequency range 100-200 MHz ($\omega_{ci} < \omega < \omega_{LH}$) are observed directly at plasma edge.
- Wave amplitudes show bursting behavior, which is correlated with the ECE signals.
- Discrete structure is found in the wave spectrum.
- Similar phenomena are observed in disruption experiments in DIII-D.

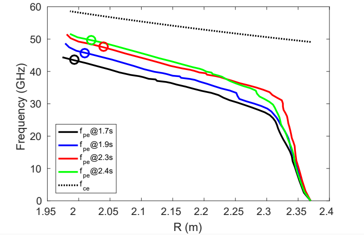


D.A. Spong et al., Phys. Rev. Lett. 120, 155002 (2018)

Intermittent RF radiation at core plasma frequency are observed in EAST tokamaks

- In recent runaway electron experiments in EAST, bursts of RF radiation with frequency close to the core plasma frequency (ω_{pe}) are observed near the edge.
- Similar phenomena have been observed in Alcator experiments.
- Amplitudes of the waves are significantly higher than thermal radiation like cyclotron emission ($T_{\mathrm{rad}} \sim 10^5$ keV)





Yong Liu et al., submitted to Nucl. Fusion.

R.F. Gandy, I.H. Hutchinson, and D.H. Yates, Phys. Rev. Lett. 54, 800 (1985).

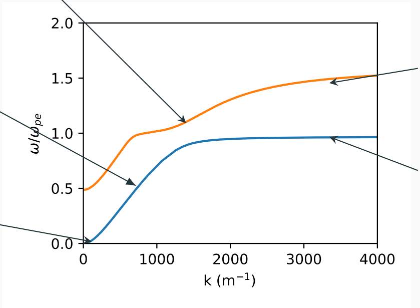
Simulation model of fan instabilities

Both whistler waves and extraordinary electron waves are included in the simulation model

- Wave frequencies are calculated using cold plasma dispersion relation, including both electron and ion cyclotron resonances and whistler wave resonance.

extraordinary electron wave

plasma wave frequency with for $k_{\parallel}/k = 0.9$



whistler wave

electron cyclotron resonance

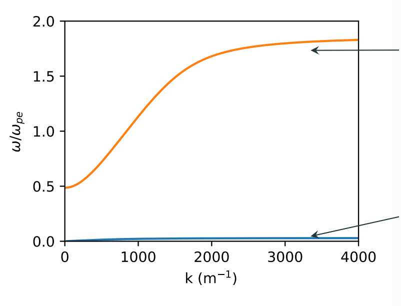
Alfven wave

whistler wave resonance cone

Both whistler waves and extraordinary electron waves are included in the simulation model

- Wave frequencies are calculated using cold plasma dispersion relation, including both electron and ion cyclotron resonances and whistler wave resonance.

plasma wave frequency with for $k_{\parallel}/k = 0.1$



upper hybrid resonance

lower hybrid resonance

RE distribution in momentum space is calculated dynamically

RE distribution evolution is calculated by solving the kinetic equation with time, in 2D momentum space (after gyro-averaging) using finite-element method.

collision operator

$$\frac{\partial f}{\partial t}+\frac{eE_{\parallel}}{mc}\left(\xi\frac{\partial f}{\partial p}+\frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi}\right)+C\left[f\right]+\frac{\partial}{\partial \mathbf{p}}\cdot\left(\mathbf{F}_{\mathrm{rad}}f\right)+D\left[f\right]=S_A\left[f\right]$$

electric field force

synchrotron radiation
reaction force

wave diffusion

RE avalanche source term

Mode energy is calculated according to linear growth rate and background fluctuations

linear growth rate

$$\frac{d\mathcal{E}(k,\omega)}{dt} = 2\Gamma(k,\omega)\mathcal{E}(k,\omega) + \mathcal{K}(k,\omega)$$

wave energy

background fluctuation

Growth of unstable modes are calculated according to RE distribution

$$\Gamma(k,\theta) = \frac{\omega_{pe}^2}{\mathcal{D}} \int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi \delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma)(p^2/\gamma)\hat{L}f$$

$$\mathcal{K}(k,\theta) = \frac{\omega_{pe}^2}{\mathcal{D}} \int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi \delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma) m v^2 f$$

$$\hat{L} = \frac{1}{p}\frac{\partial}{\partial p} - \frac{1}{p^2}\frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi}\frac{\partial}{\partial \xi} \qquad \mathcal{D} = \frac{1}{\omega}\mathbf{E}^* \cdot \frac{\partial}{\partial\omega}(\omega^2\epsilon)\cdot \mathbf{E}$$

$$Q_n = \left[\frac{n\omega_{ce}}{\gamma k_\perp v} J_n(k_\perp\rho) + E_z \xi J_n(k_\perp\rho) + iE_y\sqrt{1-\xi^2}J_n'(k_\perp\rho)\right]^2$$

ξ is the cosine of pitch angle

- For $n = 0$ (Cherenkov): Γ depends on $\partial f/\partial p_\parallel$, Landau damping & bump on tail
- For $n < 0$ (Normal Doppler): Anisotropic distribution ($\partial f/\partial\xi > 0$) stabilize the mode
- For $n > 0$ (Anomalous Doppler): Anisotropic distribution gives positive growth rate

The growth rate Γ is subtracted by the damping rate due to collisions.

P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

Diffusion of RE in momentum space is addressed using quasilinear model

$$D[f] = \frac{2e^2}{\pi\mathcal{D}} \sum_{n=-\infty}^{\infty} \int d^3\mathbf{k}\, \hat{L}\left[p_\perp \delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma)\mathcal{E}(k,\theta) Q_n p_\perp \hat{L} f\right]$$

We only take into account $n = 0$ and $n = \pm 1$, which are the dominant resonances of whistler waves.

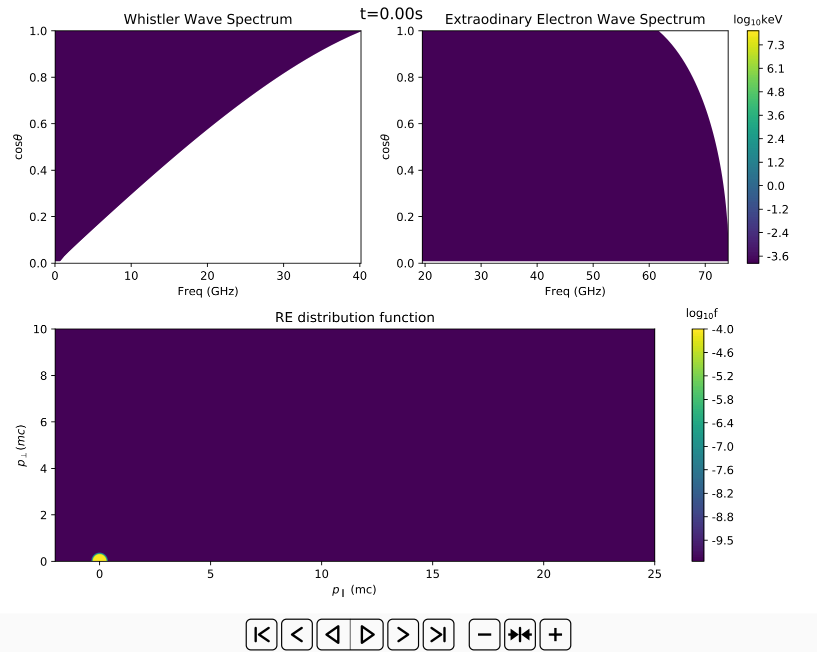
A. N. Kaufman, "Quasilinear diffusion of an axisymmetric toroidal plasma," Phys. Fluids 15, 1063 (1972).

Simulation results of fan instability in QRE scenario

RE growth in flattop phase of discharge with low electron density

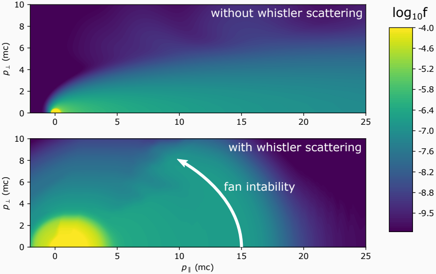
$$n_e = 2.0 \times 10^{19}\mathrm{m}^{-3} \quad B = 2T \quad T_e = 1.1\mathrm{keV} \quad E/E_{CH} = 9$$

- With low electron density and $E > E_{CH}$ (Connor-Hastie), the runaway electron population can grow through both Dreicer generation (slide-away of high-energy electrons from Maxwellian tail) and the avalanche (knock-on collisions).
- Whistler modes get excited after RE population reached certain threshold.
  - This threshold is mostly determined by the Landau damping rate.


t=0.00s
Whistler Wave Spectrum
cosθ
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
Freq (GHz)
Extraodinary Electron Wave Spectrum
cosθ
20
30
40
50
60
70
Freq (GHz)
log₁₀keV
7.3
6.1
4.8
3.6
2.4
1.2
0.0
-1.2
-2.4
-3.6
RE distribution function
$p_\perp$ (mc)
10
8
6
4
2
0
-2.4
0
5
10
15
20
25
$p_\parallel$ (mc)
log₁₀f
-4.0
-4.6
-5.2
-5.8
-6.4
-7.0
-7.6
-8.2
-8.8
-9.5

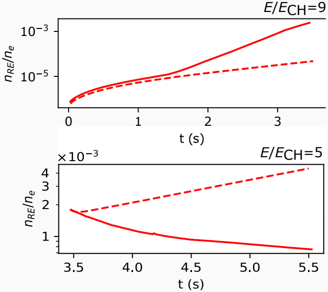
RE distribution in momentum space is significantly changed by the fan instabilities

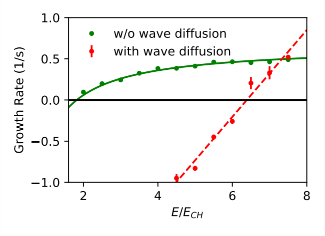
- Low frequency whistler waves (LFWW, 1GHz-10GHz) first get excited, and scatter RE in high energy regime ($15 < \gamma < 20$, fan instability).
    - Stop RE from going into higher energy regime
- High frequency whistler waves (HFWW, 10GHz-40GHz, close to resonance cone) get excited later, and scatter RE in low energy regime ($2 < \gamma < 5$).



Role of excited waves on RE avalanche and critical electric field

- The excited waves can cause both energy diffusion and pitch angle scattering of REs.
- For $E \gg E_{CH}$, energy diffusion enhances the RE avalanche growth.
    - Resonance region of excited waves overlaps with the runaway-loss separatrix, so energy diffusion causes more electrons entering the runaway region.
- For smaller E field, overlapping does not happen and pitch-angle scattering suppresses the avalanche growth.
- Competition of two effects results in a higher critical electric field than predicted by Rosenbluth-Putvinski theory, which is much closer to experimental observations.





C. Liu, E. Hirvijoki, G.-Y. Fu, D.P. Brennan, A. Bhattacharjee, and C. Paz-Soldan, Phys. Rev. Lett. 120, 265001 (2018).

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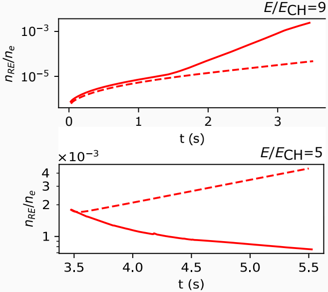

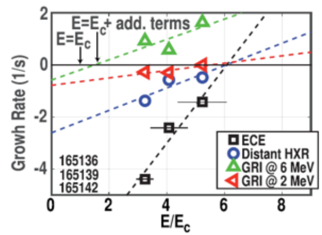

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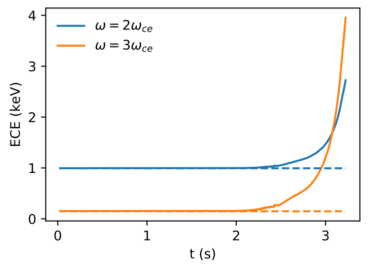
Radiation and wave excitation of runaway electron after fan instabilities

Effects of excited waves on the electron-cyclotron-emission (ECE) signals

- The pitch-angle scattering transfer electrons' momentum from $p_{\parallel}$ to $p_{\perp}$, which can enhance the power of electron-cyclotron emission.
- We develop a new ECE synthetic diagnostic tool for runaway electrons, in order to benchmark with experiments.

Results:

- ECE signals from REs start to grow abruptly after the high frequency whistler waves are excited, and can overwhelm the ECE from thermal electrons.
- Signals at higher frequencies is more enhanced than lower frequencies.



C. Liu, L. Shi, E. Hirvijoki, D.P. Brennan, A. Bhattacharjee, C. Paz-Soldan, and M.E. Austin, Nucl. Fusion 58, 096030 (2018).

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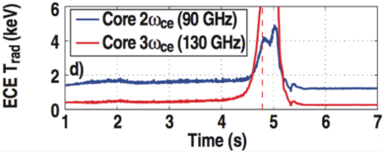
- The results match well with DIII-D ECE diagnostic results.



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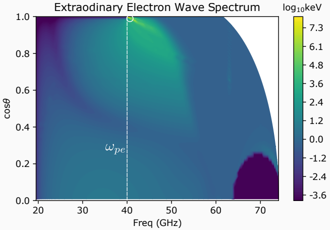
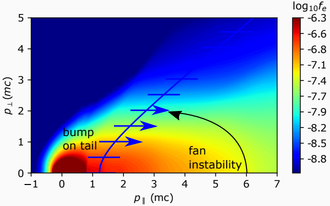
Both EXEL waves and very low frequency whistler waves are excited by bump-on-tail

- For anomalous Doppler resonance, both EXEL waves ($\omega \approx \omega_{pe}$) and very low frequency whistler waves ($\omega < 200\text{MHz}$) can only resonate with very high energy runaway electrons ($\gamma > 100$).
- For Cherenkov resonance ($\omega = k_\parallel v_\parallel$), the resonance energy can be much lower.
    - Excitation through Cherenkov resonance requires a bump-on-tail distribution.
- Due to the strong pitch-angle scattering of runaway electron by fan instabilities, electrons can cumulate at certain pitch angle, which can cause bump-on-tail instabilities.
- Wave-particle interaction can relax bump-on-tail instabilities and cause electron to lose energy.





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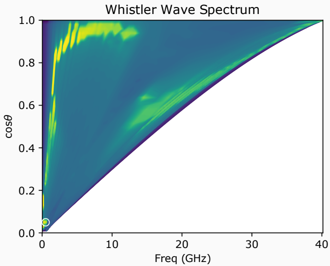
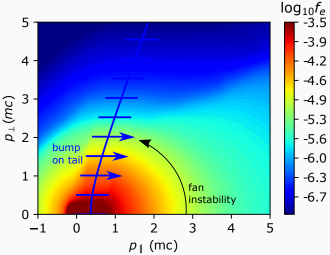
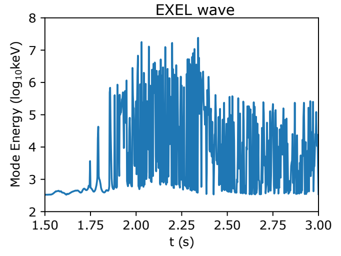
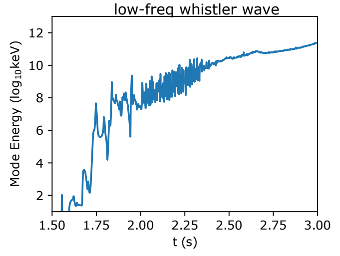




Oscillatory behavior of excited EXEL waves

- Oscillatory behavior of both EXEL waves and very low frequency whistler waves are observed in RE experiments.
- In our simulation, the evolution of wave amplitudes experience oscillations for both two waves.
    - Related to the excitation-saturation nonlinear cycle of fan instability (predator-prey model).
- For very low frequency whistler waves, the oscillation is in correlation with growth-damping cycle of ECE signals, which is consistent with experiments.
- For EXEL waves, the oscillation period is one order-of-magnitude higher than experiment.





Accessibility of plasma waves emitted by runaway electrons

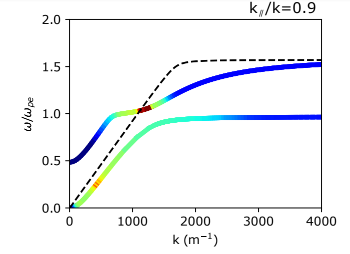
Can excited waves propagate to the edge?

The excited waves by RE have wide frequency range, but why only the very low frequency whistler waves (<200MHz) and RF waves with $\omega \approx \omega_{pe}$ are observed?

- Some of the waves generated at core (where most of REs lie) cannot propagate to the edge due to density cutoff.
- Existing diagnostic methods (magnetic probes, RF antenna, spectrometer) cannot measure the waves in certain frequency range.

The density cutoff can be estimated using $rk_\phi = const$ and $k_\perp = 0$.

- Dashed line represents cutoff dispersion, and only waves above dashed line can propagate to the edge.
- Color represents amplitudes of excited waves



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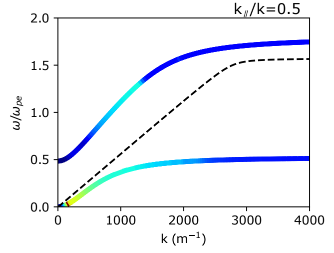

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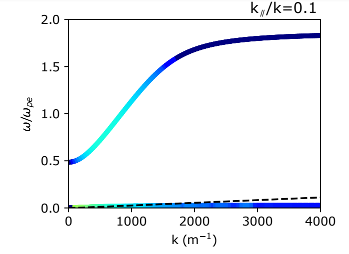

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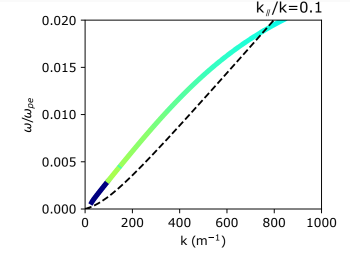
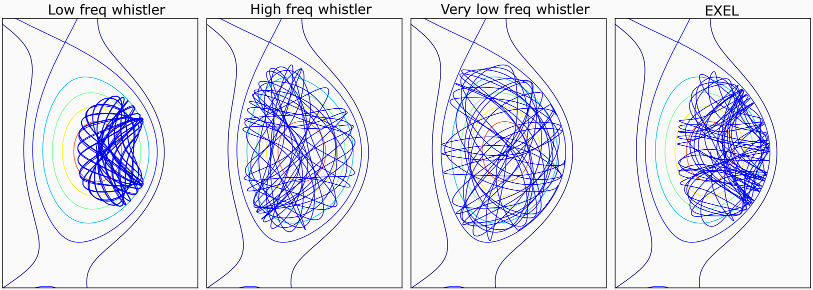


Ray-tracing calculations of excited waves



- The propagation of both the low frequency whistle waves and high frequency whistler waves are limited by the R-wave cutoff.
- Electron cyclotron waves, very low frequency whistler waves (<200MHz) and the EXEL waves can propagate to the edge on the low-field-side.

Summary

Summary

- Using a self-consistent quasilinear simulation model, we find that the RE tail generated in tokamak experiments can excite fan instabilities can cause pitch-angle scattering of resonant electrons
- Runaway electrons affected by fan instabilities can radiate electron cyclotron waves, EXEL waves and very low frequency whistler waves.
- Due to density cutoff, these radiated waves can propagate to the edge, whereas the whistler waves associated with fan instabilities cannot.
- Future work:
    - Extend distribution function calculation from 2D to 3D using bounce-average model.

Thank you

Backup slides

Backup