Theory and Simulation of Disruption Mitigation

by

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on behalf of the General Atomics Theory and Computational Sciences Group

with

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Acknowledgments

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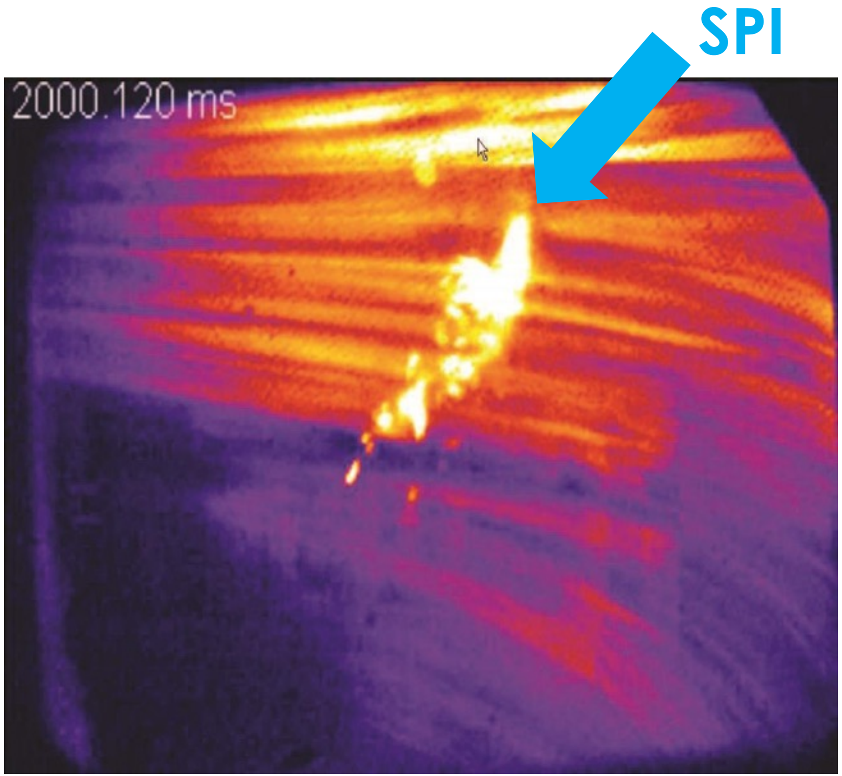
- R.W. Harvey for GENRAY simulations

Future tokamaks will require disruption mitigation

- **Disruptions result in rapid loss of stored plasma energy**
  - Thermal quench can melt of plasma-facing components
  - Current quench can produce
    - Damaging wall forces
    - Dangerous runaway electrons
- **Impurity injection can mitigate disruptions by radiating stored energy**
- **Shattered pellet injection (SPI) currently under experimental investigation on several tokamaks**



DIII-D shattered pellet injection
D. Shiraki, IAEA presentation 2016

Studying of disruption dynamics and mitigation requires multiphysics models

- **Simulations, validated against mitigation experiments, are required to project techniques to future devices**
- **Integrated model is required to capture all relevant physics**
  - Magnetohydrodynamics (MHD) for macroscopic evolution of disruption dynamics
  - Atomic physics for ionization and radiation from injected impurities
  - Drift-kinetics for phase-space evolution of runaway electron population
  - Wave-plasma interactions needed to understand runaway electron scattering due to fast waves
- **Disparate spatial and temporal scales make numerical modeling particularly challenging**

Outline

- **Code upgrades for pellet mitigation modeling (CTTS)**
  - M3D-C1: Lyons, Ferraro, & Jardin
  - NIMROD: Kim & Liu
  - Ablation models: Parks
- **Axisymmetric benchmark between M3D-C1 and NIMROD (CTTS)**
  - Lyons, Kim, Liu, Ferraro, & Jardin
- **NIMROD shattered-pellet-injection modeling (CTTS & ITER contract)**
  - Kim & Liu
- **Fast-wave mitigation of runaway electrons (SCREAM)**
  - Parks

SciDAC Centers

Code upgrades for pellet mitigation modeling

- M3D-C1: Lyons, Ferraro, & Jardin
- NIMROD: Kim & Liu
- Ablation models: Parks

M3D-C1 & NIMROD are being upgraded to address fundamental disruption mitigation physics

- **Full, nonlinear, 3D extended MHD solvers**
  - M3D-C1 uses a complete finite-element representation
  - NIMROD uses finite elements in poloidal plane and Fourier modes toroidally
- **Both have been coupled to the KPRAD[1] impurity model**
  - Low-density, coronal model based on ADPAK rate coefficients
  - Impurity charge states and electron density evolve according to ionization and recombination
  - Thermal energy lost from plasma due to ionization and radiation (line, Bremsstahlung, and recombination)
  - Subcycled much faster than typical MHD time steps

[1]D.G. Whyte, et al., Proc. of the 24th Euro. Conf. on Controlled Fusion and Plasma Physics, Berchtesgaden, Germany, 1997, Vol. 21A, p. 1137.

M3D-C1 has been recently coupled to KPRAD

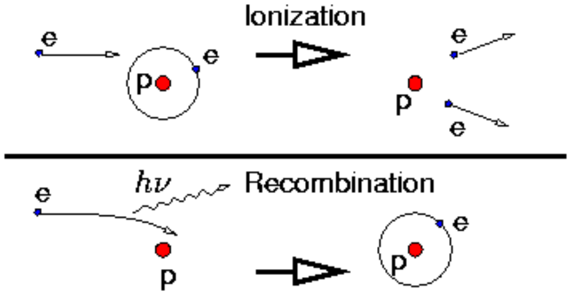
- **Four different coupling methods: [1/2] [p/T] equation(s)**
  - With single equation, radiation losses split between electrons and ions
  - With two equations, electrons lose energy and ions equilibrate
  - With temperature equation(s), dilution cooling must be explicitly included

$$n_e\left[\frac{\partial T_e}{\partial t}+\mathbf{v}\cdot\nabla T_e+\Gamma T_e\nabla\cdot\mathbf{v}\right]=(\Gamma-1)\left(\eta J^2-\nabla\cdot\mathbf{q}_e+Q_{ei}-\mathcal{P}_{rad}\right)-T_e\left(\frac{\partial n_e}{\partial t}+\mathbf{v}\cdot\nabla n_e\right)$$

$$n_{ti}\left[\frac{\partial T_i}{\partial t}+\mathbf{v}\cdot\nabla T_i+\Gamma T_i\nabla\cdot\mathbf{v}\right]=(\Gamma-1)\left(-\nabla\cdot\mathbf{q}_i-Q_{ei}-\mathbf{\Pi}:\mathbf{v}\right)-T_i\left(\frac{\partial n_{ti}}{\partial t}+\mathbf{v}\cdot\nabla n_{ti}\right)$$

- **KPRAD updated to split recombination energy**
  - Ionization converts thermal to potential energy
  - Recombination releases thermal (kinetic) and potential energy as radiation
  - Potential (~10^1-10^3 eV) greatly exceeds kinetic in cold plasma (~10^0 eV)
  - Only kinetic part subtracted from thermal energy

Kinetic energy to potential energy



Kinetic & potential to radiation

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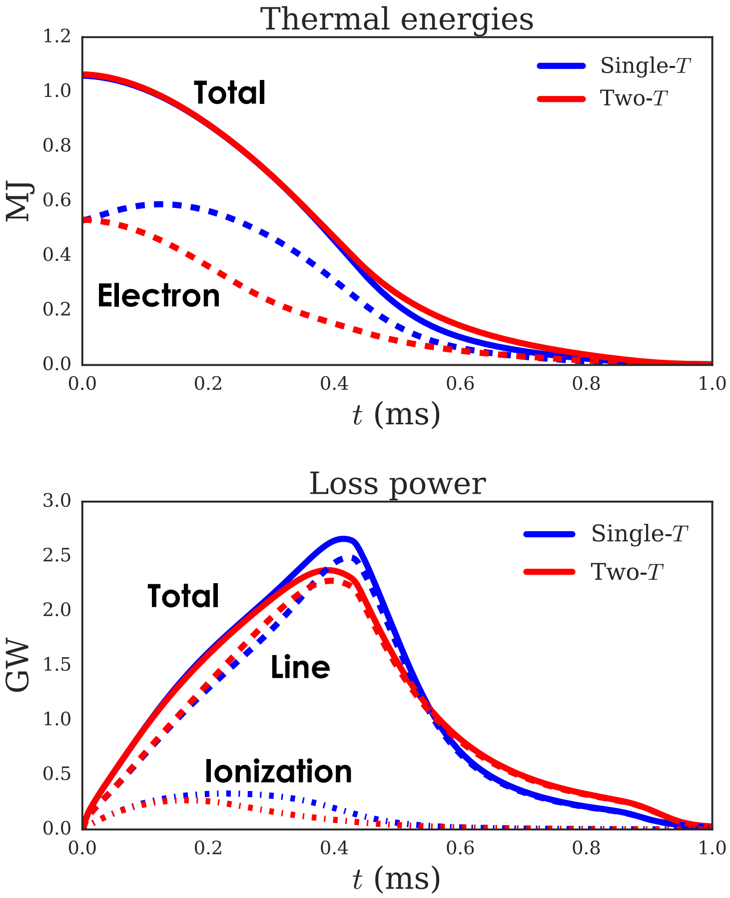
Two temperature equations leads to slower thermal quench

- **Early-time behavior**
  - Total-thermal energy and radiation identical to single temperature eq.
  - Electron thermal energy drops monotonically without early rise
- **Ionization rate less due to decreased electron temperature**
- **Longer thermal energy tail due to electron-ion equilibration**



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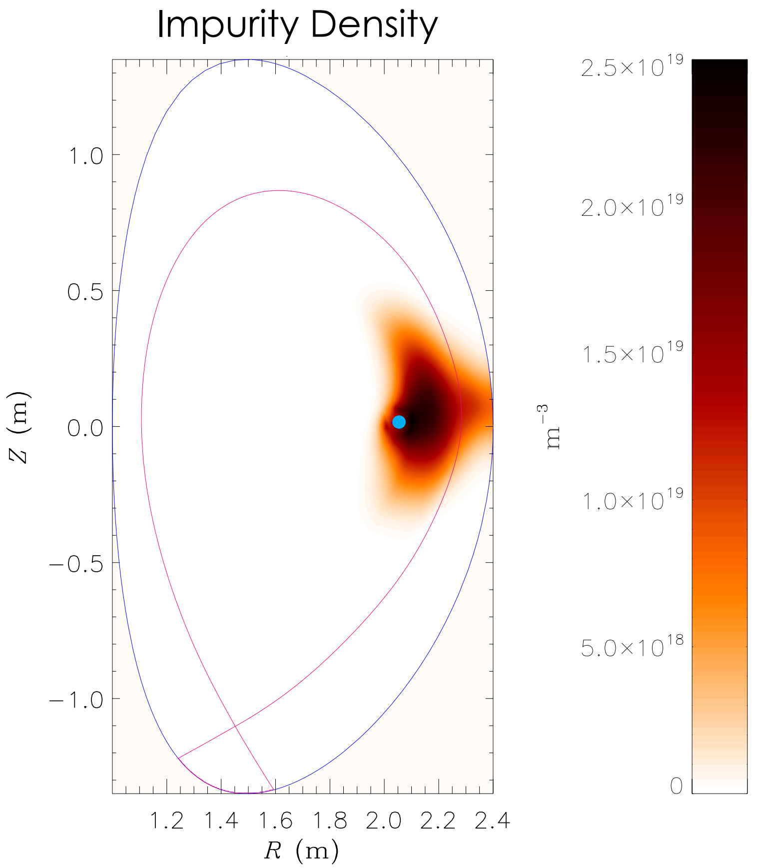
Ablation model for Ne-D2 pellets implemented in M3D-C1

- **Practical, analytic expression fit to more complex ablation model (Parks)**

$$G\,(\mathrm{g/s}) = \lambda\,(X)\left(\frac{T_e}{2000\ \mathrm{eV}}\right)^{5/3}\left(\frac{r_p}{0.2\ \mathrm{cm}}\right)^{4/3}\left(\frac{n_e}{10^{14}\ \mathrm{cm}^{-3}}\right)^{1/3}$$

  λ is fitting function, depending on molar fraction of D2, X

- **M3D-C1 implementation**
  - Advance pellet location in time
  - Calculate number of particles ablated and pellet-surface recession at each time step
  - Deposit main ion and/or impurities onto arbitrary spatial distribution (e.g. 2D or 3D Gaussian)



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NIMROD is ready to assess viability of shattered-pellet injection

- **Single-fluid, resistive MHD model uses single temperature equation**

$$n_{tot}\left(\frac{\partial T}{\partial t}+\mathbf{V}\cdot\nabla T\right)=(\Gamma-1)\left(-p\nabla\cdot\mathbf{V}-\nabla\cdot q+Q-\Pi:\nabla\mathbf{V}\right)-T\sum\frac{\Delta n_\alpha}{\Delta t}$$

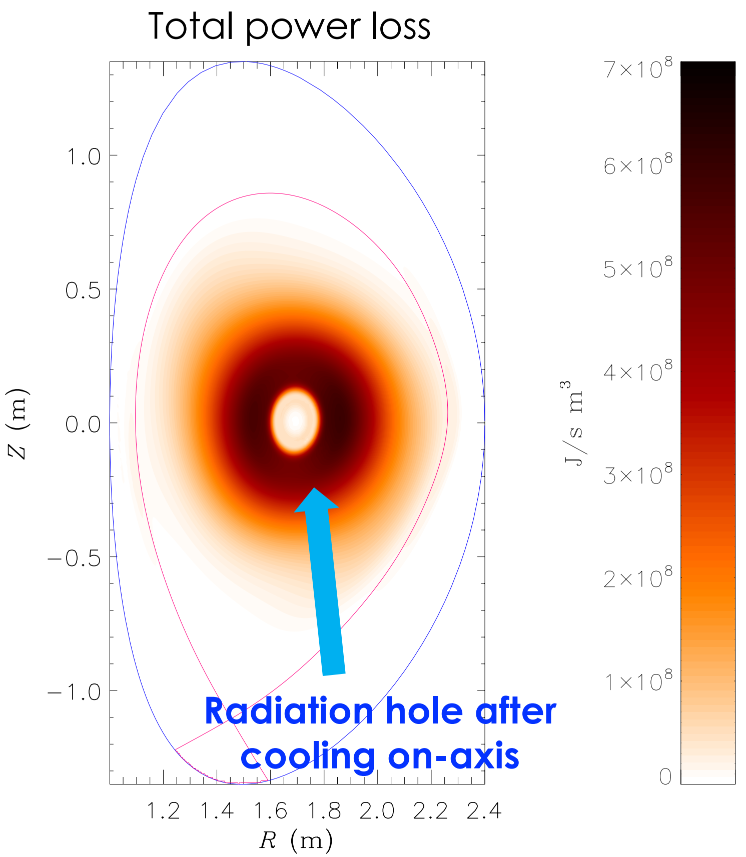
  - $n_{tot}=n_i+n_e+\sum_Z n_Z$ (impurities include neutrals)
  - Heat flux, radiation and heating, dilution cooling (ablation and electrons)
- **KPRAD radiation/ionization (same as massive gas injection [Izzo NF46 2006])**
- **Particle-in-cell (PIC) based SPI model recently added**
  - Discrete PIC marker represents subset of SPI fragments
  - Initially use simple"pencil beam"model for fragment plume - straight line trajectory, uniformly spaced identical particles
  - Easy to modify, extremely flexible
- **Ongoing study of both DIII-D and ITER SPI thermal quench**
- **Beginning SPI validation study with DIII-D experiment**

Axisymmetric benchmark between M3D-C1 and NIMROD

– Lyons, Kim, Liu, Ferraro, & Jardin

Fast impurity injection in DIII-D core used as test case

- **DIII-D shot 137611 @ 1950 ms**
- **2D, nonlinear, single-fluid**
- **Neutral argon or neon impurity deposition**
  - No impurities to start
  - Gaussian source: $\frac{dn_z}{dt} = \nu \frac{R_0}{R} \exp\left[-\frac{(R-R_0)^2 + (Z-Z_0)^2}{2\delta^2} \right]$
  - $\delta = 0.25$ m & $\nu = 10^{23}$ m^{-3} s^{-1}
- **Constant main ion density: 10^{20} m^{-3}**
- **Constant diffusivities**
  - Isotropic density, momentum, and thermal diffusivities: 10 m^2/s
  - Parallel thermal diffusivity: 10^6 m^2/s
  - Resistivity: 10^{-5} Ohm*m, 7.96 m^2/s



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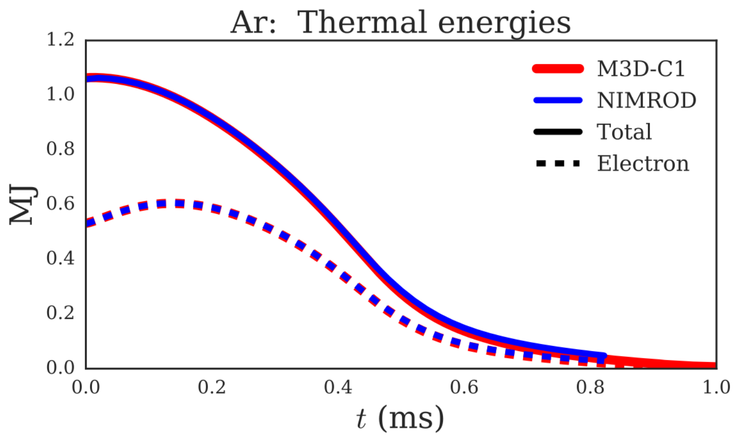
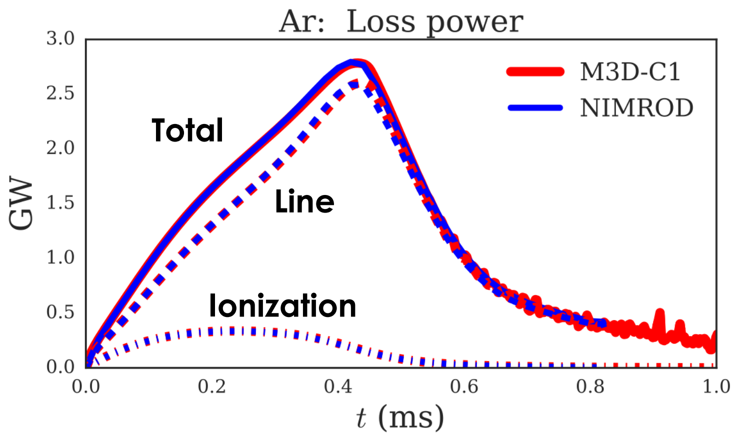
Excellent agreement for argon-injection benchmark

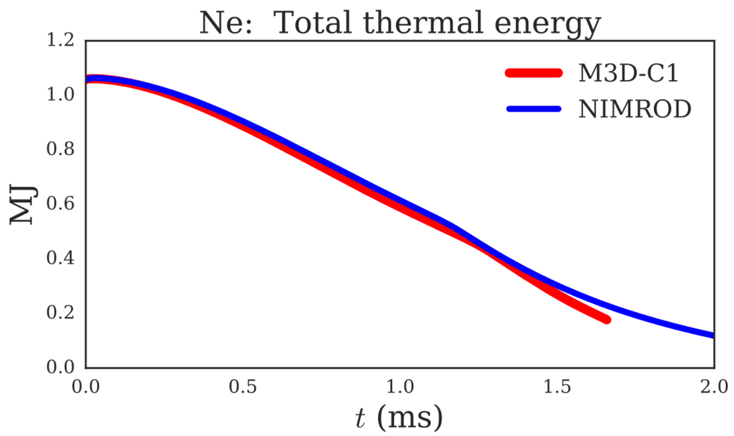
- **Codes show near-identical time evolution of**
  - Thermal energies
  - Loss power and each component
- **Thermal quench time: ~0.6 ms**
- **Peak radiation: ~0.45 ms**
- **Ohmic heating**
  - Fairly small due to constant resistivity
  - Will increase at end of thermal quench when Spitzer resistivity used: $\eta \sim T^{-3/2}$
  - Spitzer benchmark underway

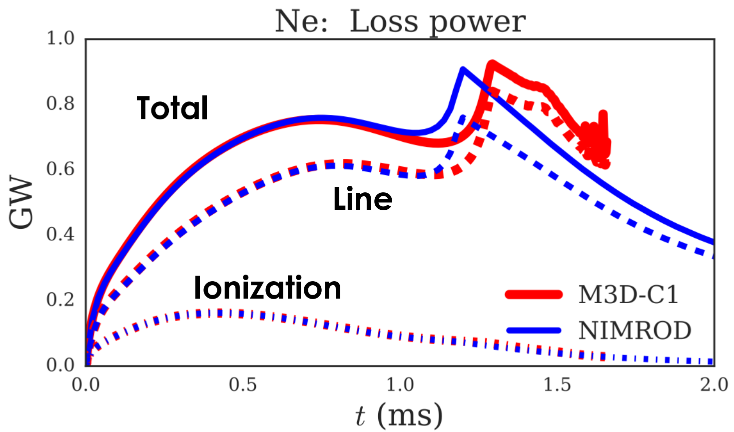




Good agreement for neon, with some late-time differences

- **Quench ~3x slower than argon (~1.8 ms)**
- **Good agreement between codes observed before 1 ms**
  - Radiation/ionization nearly identical
  - Slightly higher thermal energy seen in NIMROD
- **Late-time discrepancies under investigation**
  - Earlier radiation peaks in NIMROD
    - *Personal: possibly due to inclusion of potential energy in recombination power*
  - M3D-C1 sees steady decay toward zero, while NIMROD quench has longer tail





NIMROD shattered-pellet-injection modeling

– Kim & Liu

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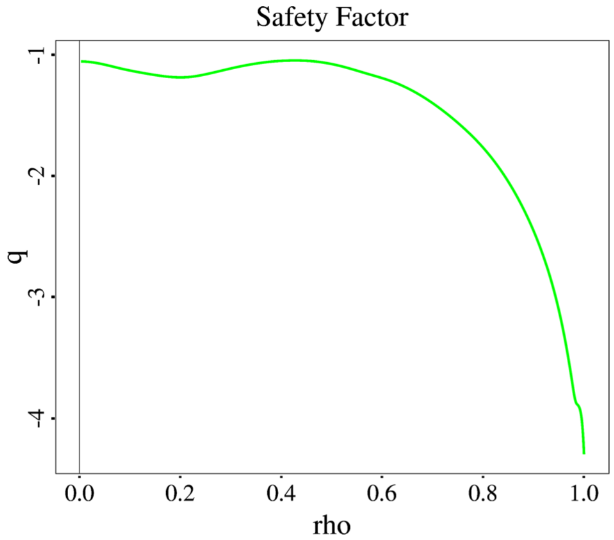
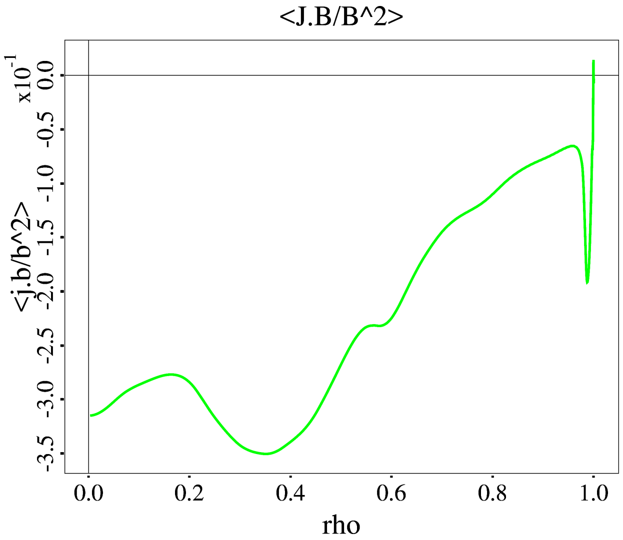
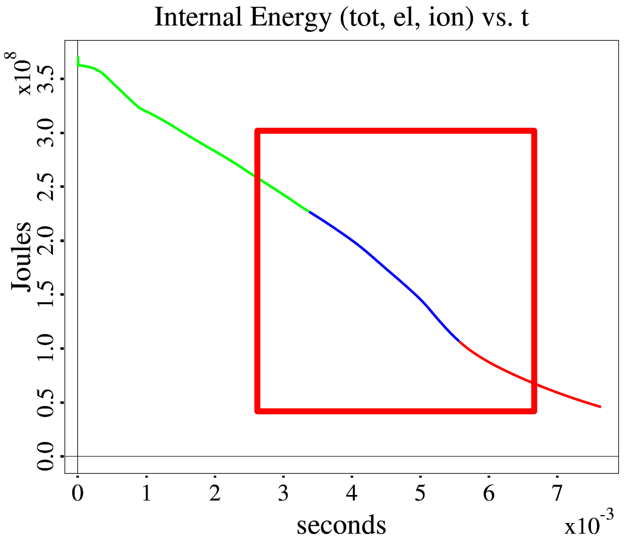
SPI simulation of 0.5 kPa·m³ Ne pellet injected into 12.5 MA ITER hybrid discharge







- 128x128 finite-element grid, toroidal modes n=0-5, $S \sim 10^6$, $Pr \sim 10^5$
- 125 fragments/25 particles, r_0=1.71 mm, v=500 m/s, Δr_{dep}=40 cm, $\Delta\varphi_{dep}=\pi$
- (48 hrs + 48 hrs + 48hrs) x 384 processors
- Note dip in internal energy between t=0.4-0.6 ms

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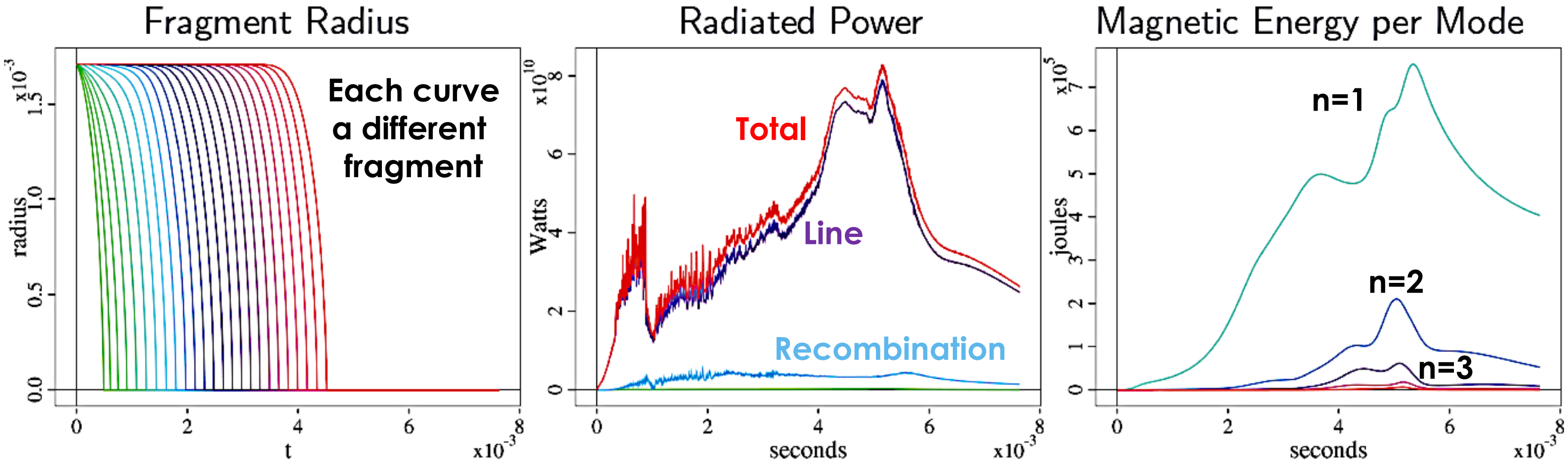
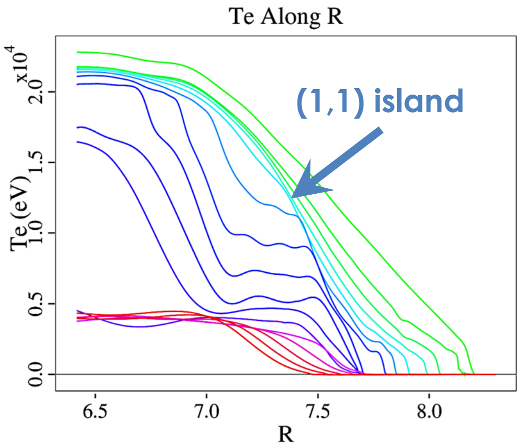
Dip coincides with maximum radiation and peak mode activity



- All fragments ablate by 4.5 ms
- 318.5 MJ of thermal energy lost (mostly line radiation)
- MHD dominated by n=1 (single injector)
  - Dip in mode energy coincides with end of ablation
  - Radiation peaks with maximum n=1 kink amplitude
- Core temperature collapses between 4.5 and 5 ms



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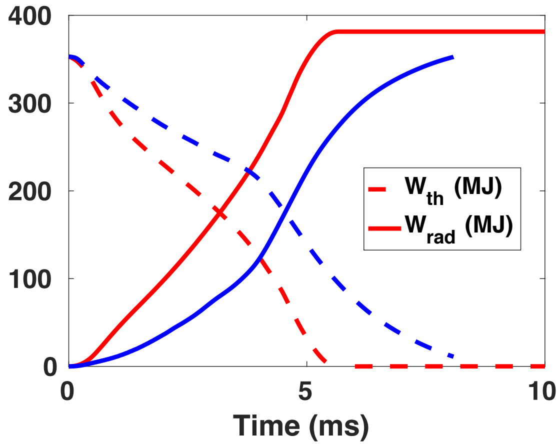
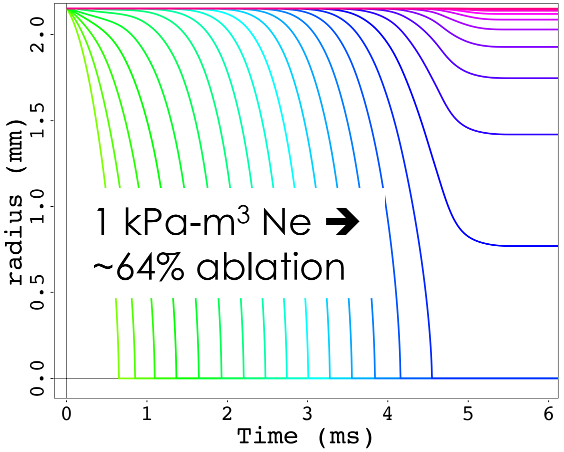
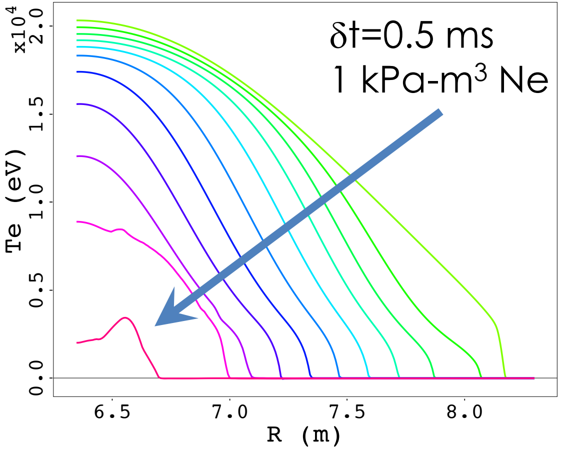
ITER 15 MA baseline scenario simulation: Double amount of pure-neon SPI reduces TQ time by ~35%



0.5 kPa-m^3

1 kPa-m^3





- **0.5 kPa-m^3 neon**
  - 8 ms incomplete TQ
  - 100% ablation of injected pellet during TQ
- **1 kPa-m^3 neon**
  - 5 ms TQ
  - ~64% ablation (total 0.64 kPa-m^3)
- **TQ time traces not very sensitive to assumed plasma resistivity & viscosity**

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Fast-wave mitigation of runaway electrons

– Parks

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Fast-wave injection have been proposed as a means to mitigate runaway electron beams

- **Antenna could be used to excite the helicon or whistler waves**
  - Cause quasi-linear scattering of the runaway electrons in the perpendicular direction
  - Enhance the synchrotron damping
- **Question: Will collisional dissipation in post-disruptive discharge significantly degrade wave energy in the core?**
- **Can be determined analytically from the perpendicular absorption coefficient $k_{\perp i}$**

Fast waves propagation through cold, post-disruptive discharge

- **Time-averaged spectral power dissipation rate:** $\mathcal{P} = \mathrm{Re}(E^* \cdot J)/2$
- **Current density in terms of dielectric tensor:** $J = -i\omega(\epsilon - \mathrm{I}) \cdot E/4\pi$
- $\mathcal{P} = \frac{\omega}{8\pi}\mathrm{Re}(E^* \cdot \epsilon_a \cdot E)$ **with anti-Hermitian part:** $\epsilon_a = \begin{pmatrix} S_{im} & 0 & 0 \\ 0 & S_{im} & 0 \\ 0 & 0 & P_{im} \end{pmatrix}$
  - $S_{im} = (\nu_{ei}/\omega)(\omega_{pe}^2/\omega_{ce}^2)$
  - $P_{im} = (\nu_{ei}/\omega)(\omega_{pe}^2/\omega^2)$

Electron-ion collision frequency is same one appearing in Spitzer's ⊥ electrical resistivity

- $\mathcal{P} = \frac{\nu_{ei}}{8\pi}\frac{\omega_{pe}^2}{\omega_{ce}^2}\left\{|E_x|^2 + |E_y|^2 + \frac{\omega_{ce}^2}{\omega^2}|E_\parallel|^2\right\}$ **(assuming** $\omega^2/\omega_{ce}^2 \ll 1$ **)**
- **Poynting flux in ⊥ (or x) direction:**

$$\mathcal{S}_\perp = \frac{c^2}{16c\pi\omega}\left\{2k_\perp(|E_y|^2 + |E_\parallel|^2) - k_\parallel(E_x E_\parallel^* + E_x^* E_\parallel)\right\}$$

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Dispersion relation and perpendicular absorption coefficient give wave accessibility condition

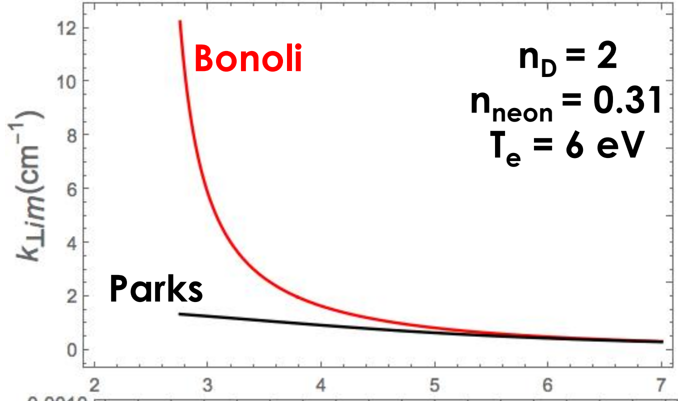
- **Conservation of energy** $\mathcal{P} = 2k_{\perp im}\mathcal{S}_{\perp}$

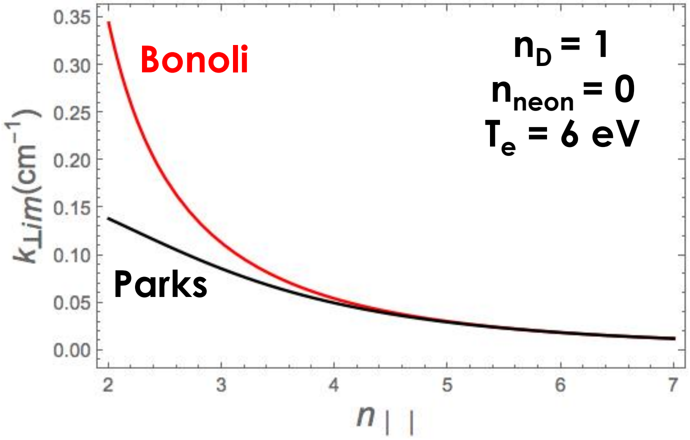
Perpendicular absorption coefficient

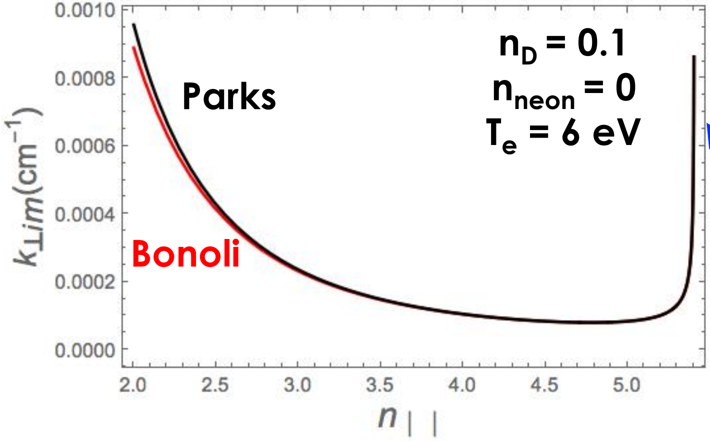
$$k_{\perp im} = \frac{\nu_{ei}}{2cn_{\perp}}\frac{\omega_{pe}^2}{\omega_{ce}^2}\left\{\frac{1+\frac{D^2}{\left(n_{\perp}^2+n_{\parallel}^2-S\right)^2}+\frac{\omega_{ce}^2 n_{\parallel}^2 n_{\perp}^2}{\omega^2(P-n_{\perp}^2)^2}}{\frac{D^2}{\left(n_{\perp}^2+n_{\parallel}^2-S\right)^2}+\frac{n_{\parallel}^2 n_{\perp}^2}{(P-n_{\perp}^2)^2}+\frac{n_{\parallel}^2}{(P-n_{\perp}^2)}}\right\}$$

- **Cold plasma dispersion relation** $\mathcal{D} \equiv \det[nn - n^2 I + \epsilon_h] = 0$ **used to express** $n_{\perp}^2(n_{\parallel}^2)$
- **Choose** $\omega_{ci} \ll \omega \ll \omega_{ce}$
  - Resonance condition for relativistic REs $\omega - k_{\parallel}v_{\parallel} - l\,{}^{\omega_{ce}}/_{\gamma} = 0$
  - DIII-D helicon antenna frequency $f = 500\text{ MHz},\ B\sim 2\text{ T}, \gamma\sim 20-50$
  - $n_{||}^{res} = \frac{\omega_{ce}}{\gamma\omega} - 1 \sim 2-5$
- **ACCESSIBILITY: for typical DIII-D post-disruption densities $n_{14} > 0.2$ the slow (lower hybrid) wave cannot propagate to core but the fast whistler/helicon wave can**

Significant corrections to Bonoli's* perpendicular absorption coefficient for high density cases at low $n_{\parallel}$



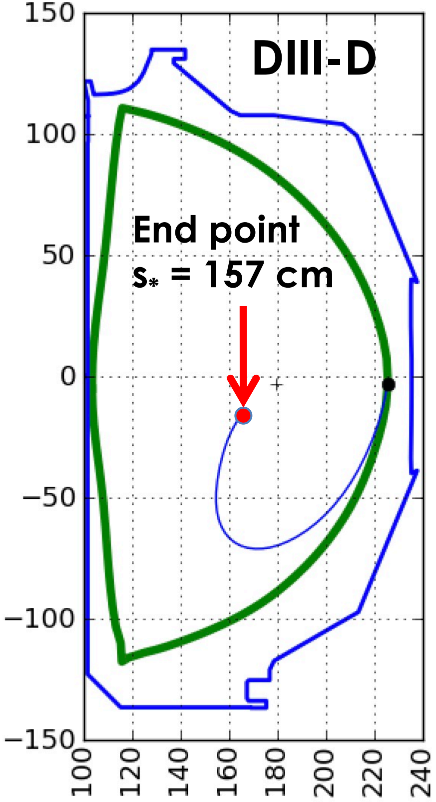




- Good agreement at high $n_{\parallel}$ & low density but **Bonoli's perturbation method** breaks down at low $n_{\parallel}$ & high density
- When $n_{\parallel}$ becomes too large we approach cutoff, $n_{\perp} \rightarrow 0$, making $k_{\perp im}$ **blow up**, as indicated in Parks formula

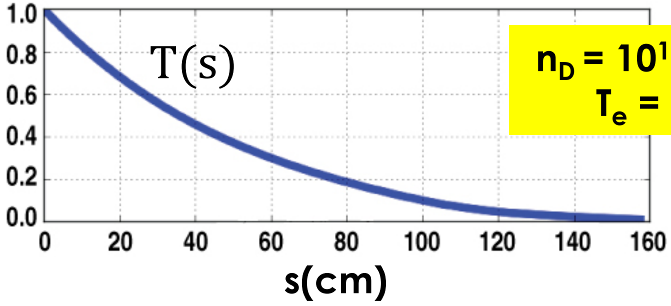
*M. Porkolab, P.T Bonoli, S.C. Chiu in 11th Topical Conf. on RF Power in Plasmas, May 17-19, 1995,Palm Springs, CA PFC/JA-95-30

Initial GENRAY simulation shows wave power is collisionally depleted to 1% at "end point" of ray path

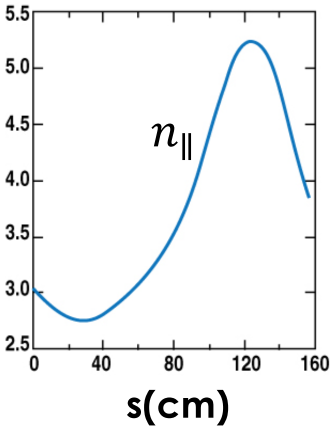


$$\frac{|E|^2(s)}{|E|^2(0)} \equiv \mathrm{T}(s) = \exp - \int_0^s 2k_{\perp im} \frac{v_{g\perp}}{\sqrt{v_{gR}^2 + v_{gZ}^2}} ds$$

Group velocities along ray path



$n_D = 10^{14}$, $n_{Ne} = 0$,
$T_e = 10$ eV



- Fishhook-shaped ray path worsens power depletion in the core
- Severe dissipation even when T_e is 10-100 eV
- Less dissipation with lower density, but worse with more impurities
- GENRAY run for high-β discharge; need to redo in post-disruptive discharge reconstruction

Conclusions

- **M3D-C1 and NIMROD are being brought to bear on disruption mitigation problems**
  - M3D-C1 radiation and ablation models have been upgraded
  - Successful, cross-code axisymmetric benchmark performed, with extensions and 3D benchmarks underway/planned
  - NIMROD performing SPI simulations for ITER
- **Collisional damping in post-disruptive plasmas makes fast-wave mitigation of runaway electrons challenging**
- **Future work**
  - Code validation with and analysis of DIII-D SPI experiments
  - Fast-wave propagation in more-realistic, post-disruptive equilibria

Additional slides

NIMROD used to simulated SPI-induced TQ for ITER-baseline and hybrid scenarios with varying impurity contents

Ne [kPa.m^3]	Ne:D2	Ip [MA]	r_frag [mm]	S ($x10^6$)	Kperp[m^2/s]	Kpara[m^2/s]	kin_vis [m^2/s]	mesh	Δt [μs]	τ_{TQ} [ms]	Burnt/ total
0.5	0:1	15	1.71	1.85	10	10^{10}	$2x10^4$	96x96	0.2	8	125/125
1	0:1	15	2.15	1.85	10^2	10^7	$5x10^3$	96x96	0.5	5	75/125
0.5	10:1	15	4.42	1.85	10	10^{10}	$2x10^4$	64x72	0.2	4.5	65/125
0.5	10:1	15	4.42	18.5	10	10^{10}	$2x10^4$	64x72	0.2	4.5	65/125
0.5	10:1	15	4.42	1.85	10	10^{10}	$2x10^2$	64x72	0.2	4.5	75/125
0.5	10:1	15	3.51	1.85	10	10^{10}	$2x10^4$	64x72	0.2	4.5	150/250
0.5	1.5:1	15	2.51	1.85	10^2	10^7	$5x10^3$	96x96	0.5	>6	125/125
0.5	0:1	12.5	1.71	1.62	10^2	10^7	$5x10^3$	96x96	0.5	>5	125/125

- Fixed plasma resistivity and thermal conductivity coefficients
- 25 PiC markers at V=500 m/s
- n=0-5

Ray path without collisional damping

Calculation by Harvey
Published by Prater

Citation to be provided upon request