Magnetic Surface Loss and Electron Runaway

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Facts That Define Issues

Loss and re-formation of magnetic surfaces

The current spike during the thermal quenches implies a large fraction of the magnetic surfaces are destroyed. If only a fraction are destroyed, a strong skin current arises on the remaining magnetic surfaces near their boundary with the destroyed surfaces

The flattening of the current profile and the loss of plasma pressure during the thermal quench removes the drive for plasma kinking. As seen in NIMROD and JOREK simulations, the plasma should quickly return to an axisymmetric state *unless the normal magnetic field to the walls can be made strongly non-axisymmetric*.

Breaking of all confining magnetic surfaces appears most likely explanation for absence of runaways when they would otherwise be expected.

Primary source of runaway electron seed

Hot-tail electrons are potentially the strongest source of a runaway seed. Electrons are lost too quickly to run away on open magnetic field lines, but energetic trapped electrons are not lost. Collisional drag eliminates hot-tail electrons in $\lesssim 10$ ms. The parallel electric field must exceed the drag force, $E_{||} > E_{ch}$, and some confining magnetic surfaces must exist before all hot tail electrons are slowed for this source to be relevant.

Existence and magnitude of relativistic-electron current I_{rel}

$$I_{rel} = I_{p0} - \ell_f I_{10}$$

$$\ell_f \equiv \log_{10}\left(\frac{\text{\# of relativistic electrons required to carry current}}{\text{\# of seed electrons}}\right)$$

$I_{10} \approx 0.92$ MA and is independent of machine size.

I_{p0} is the pre-disruption plasma current. Amplification in the number of energetic electrons is $\exp(|\Delta I_p|/I_{10})$.

Removing uncertainties about the magnitude of I_{10} may be the most important contribution that theory can make.

Relativistic current due to tritium decay

The steady beta decay of tritium gives $\ell_f \approx 7.8$, which implies $I_{rel} \approx 8$ MA when $I_{p0} = 15$ MA. To prevent this, must have (1) density sufficiently high to ensure 18.6 keV electrons cannot runaway even in skin-current regions or (2) all magnetic surfaces remain open until $I_p < \ell_f I_{10} \approx 7$ MA, which means a large fraction of the current-decay time of ≈ 150 ms.

Some Implications

Experimental studies of relativistic electrons

The formula $I_{rel} = I_{p0} - \ell_f I_{10}$ implies relatively little can be directly inferred about runaway issues on ITER at $I_{p0} = 15$ MA using experiments with $I_{p0} \sim 2$ MA.

Magnetic surface loss and re-formation

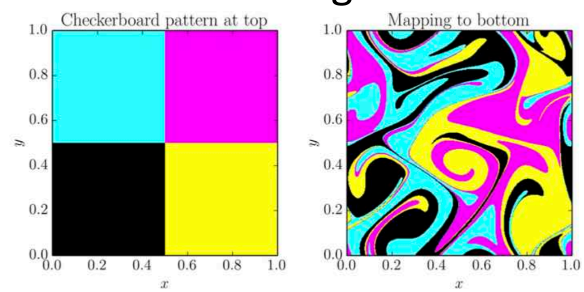
Little experimental data has been published on thermal quenches and current spikes. This information is essential to realistically assessing the danger of runaways to ITER.

Assessments require simulations that include fast magnetic reconnection, which can change current profiles many orders of magnitude faster than resistivity.

Fast Magnetic Reconnection

Basic Physics

Yi-Min Huang Figure of Ideal Evolving-*B* Lines



In an ideal 3D evolution, magnetic field lines generically become exponentially sensitive to non-ideal effects. Breaking of magnetic surfaces proceeds Alfvénically but conserves magnetic helicity $\propto \int \psi_p d\psi_t$.

$j_{||}/B$ relaxes towards a constant along B-lines,

$$\vec{B} \cdot \vec{\nabla} \frac{j_{||}}{B} = \vec{B} \cdot \vec{\nabla} \times \frac{\vec{f}}{B^2}, \text{ where } \vec{f} = \vec{j} \times \vec{B}$$

is the electromagnetic or Lorentz force. When $\vec{f} = \rho_0 d\vec{v}/dt$, relaxation of $j_{||}/B$ along the magnetic field is by Alfvén waves, $V_A = B/\sqrt{\mu_0 \rho_0}$.

For a direct numerical resolution, must resolve the longer of $\delta_\eta = \sqrt{\tau_{spike}\eta/\mu_0}$ or $\delta_{skin} = c/\omega_{pe}$. When current spike time $\tau_{spike} = 1$ ms, $T = 20$ keV, and $n_e = 10^{20}/\mathrm{m}^3$, both δ's ~ 0.5 mm. The temporal scale that must be resolved is $2\pi R_0/V_A \sim 5$ μs in ITER.

Direct simulations of fast magnetic reconnection are presently unrealistic for ITER conditions. Nevertheless, simulations must be routinely carried out to understand the implications of existing experiments on runaway issues in ITER.

Axisymmetric Representation Using Mean-Field Model

$I(\psi_t)$ is net plasma current enclosed in a region containing toroidal magnetic flux ψ_t.

$$\frac{\partial I}{\partial t} = -\frac{2}{L}\mathcal{D}[I] \quad \text{where} \quad L(\psi_t) \equiv \frac{2\psi_t}{I}\iota(\psi_t) \approx \frac{2\kappa}{1+\kappa^2}\mu_0 R_0, \quad \text{and} \quad \iota = \frac{1}{q(\psi_t)}:$$

$$\mathcal{D}[I] \equiv -\psi_t \frac{\partial}{\partial \psi_t}\left\{\mathcal{R}_\psi \frac{dI}{d\psi_t} - \frac{\partial}{\partial \psi_t}\left(\psi_t \Lambda_m \frac{\partial^2 I}{\partial \psi_t^2}\right)\right\}.$$

$\mathcal{R}_\psi$ is plasma resistivity and $\Lambda_m \sim \mu_0 V_A^2 \Psi_t^2 / N_t$ gives the helicity conserving current relaxation with N_t the number of toroidal transits it takes a B-line in the true field to cross toroidal flux Ψ_t. An axisymmetric equilibrium code is only needed for a more accurate L.

The operator $\mathcal{D}[I]$ is Hermitian, has eigenvalues, which are current decay rates γ_j, and mutually orthogonal eigenfunctions $g_j(s)$,

$$I(s,t) = \sum_j I_j g_j(s) e^{-\gamma_j t} \quad \text{where} \quad s \equiv \frac{\psi_t}{\Psi_t} \quad \text{and} \quad \int_0^1 \frac{L(s)}{s} g_j g_k ds = 0 \quad \text{unless} \quad j = k.$$

Typically $\frac{\Lambda_m}{\Psi_t \mathcal{R}_\psi} \sim 10^3$ where surfaces are broken, but $\Lambda_m = 0$ where surfaces exist.

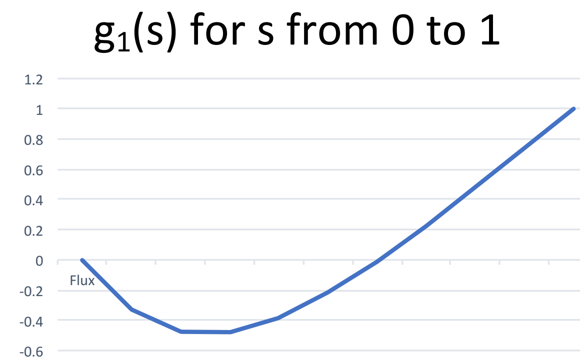
Boundary conditions are $g_j(0) = 0$, $g_j(1) = 0$, and $\mathcal{F}_{||} \equiv \psi_t \Lambda_m \frac{\partial^2 I}{\partial \psi_t^2} = 0$ at boundaries.

Eigenfunctions for Current Decay

Assume L and Λ_m are constants in space and time with $\Lambda_m >> \Psi_t \mathcal{R}_\psi$. The first two eigenvalues and eigenfunctions are

$$\gamma_0 \approx \frac{4}{\Psi_t L} \int_0^1 \mathcal{R}_\psi ds = \frac{4B_0^2}{\Psi_t^L} \int \eta d^3x \quad \text{with} \quad g_0(s) = s;$$

$$\gamma_1 \approx 56.6 \frac{\Lambda_m}{\Psi_t^2 L} \quad \text{with}$$



$$g_1(s) \approx -4.38s + 12.00s^2 - 10.31s^3 + 4.71s^4 - 1.21s^5 + 0.22s^6 - 0.03s^7 + \cdots$$

When magnetic surfaces remain in a fraction of the plasma, Λ_m is zero in those regions. Large skin currents arise on the magnetic surfaces near the boundaries with stochastic-field-line regions. **These skin currents can increase the required density to avoid runaway by an order of magnitude.**

In experiments, the magnitude of current spike is significantly reduced by resistivity. This is due to the cold plasma near the walls

$$< \eta > \equiv \frac{\int \eta d^3x}{\int d^3x} >> \eta_{central}. \tag{1}$$

Intermittent Electron Acceleration

In Cristian Sommariva's thesis, he and Eric Nardon observed intermittent electron acceleration in stochastic regions during JOREK calculations of disruptions. They found that intermittent acceleration together with rapid surface closure could lead to greatly enhanced electron runaway. Submitted to Nucl. Fusion, 2018, (arXiv: https://arxiv.org/abs/1805.05655).

Their interpretation is different, but the mean-field model predicts such an effect. The stochasticity which gives Λ_m also gives an extra term in the electron kinetic equation

$$\frac{\partial f}{\partial t}+\vec{v}_g\cdot\vec{\nabla}f-\frac{e}{2\pi R_0}\frac{\partial\psi_p}{\partial t}\frac{\partial f}{\partial\epsilon}=\mathcal{C}[f]+\frac{\partial}{\partial\psi_t}\mathcal{S}\frac{\partial f}{\partial\psi_t}\quad\text{where}\quad\frac{\Lambda_m}{L}=c_s\frac{V_A}{|v_{||}|}\mathcal{S},$$

c_s is a dimensionless constant, and $\partial\psi_p/\partial t=L\partial I/\partial t$.

The net electron acceleration given by a Λ_m relaxation is zero because $\int\psi_p d\psi_t$ is conserved, but locally $\Delta\psi_p\approx 15$ V·s can occur in the relaxation of an ITER current profile. If the relaxation is in 1 ms, the associated loop voltage is 15 keV, so in just 67 toroidal circuits an electron can pick an MeV of energy.

The required voltage for runaway $V_{ch}=2\pi R_0E_{ch}\approx 2.9$ V when $n=10^{20}/\text{m}^3$.

Situation

A successful achievement of the ITER mission is difficult to imagine when multi-month shutdowns are required on a time scale shorter than years—of order a thousand pulses. ITER must be operated as conservatively as necessary to ensure a sufficiently low probability of relativistic-electron incidents, maybe <7 MA with tritium. For a tokamak reactor, major runaway incidents must be an order of magnitude rarer.

Tritium decay induced electron runaway presents a fundamentally new danger to ITER, which will remain untested until ITER is too radioactive for major modifications.

The participants in the March 2017 ITER Workshop on Disruption Mitigation stated that they *emphatically agree that immediate decisive action must be taken to directly support research into solutions to outstanding critical issues relating to the specification and performance of the DMS* (Disruption Mitigation System). *The consensus is that significant uncertainties exist, in particular, as to whether the present baseline disruption mitigation system will offer sufficient protection to ITER from relativistic electron impacts.*

Areas of Particular Need

For runaway electrons, the worst outcome of one in a thousand shots is what is relevant, not the probable outcome.

1. Assessment of all mechanisms for runaway production with a probability of one in a thousand shots.

Skin currents, transient acceleration, and tritium decay have major implications for the mitigation strategy. *Passively forcing magnetic surfaces to remain destroyed is an almost unique strategy for avoiding runaways.*

2. Assemblage of experimental data on what happens during thermal quenches and current spikes.

For example, relative timing of quench versus spike, rise and decay times of the spike, the magnitude of the spike, changes in $\ell_i \equiv 4W_p/\mu_0 R_0 I_p^2$, and direct measurement of number of energetic electrons.

3. Speed with which a tokamak at q$_{95}$ $\approx$3 can be shutdown without disrupting. *May set required warning time for disruptions.*