Decomposing magnetic field measurements into internally and externally sourced components

6th Annual Theory and Simulation of Disruptions Workshop

R.M. Sweeney[1], E.J. Strait[2]

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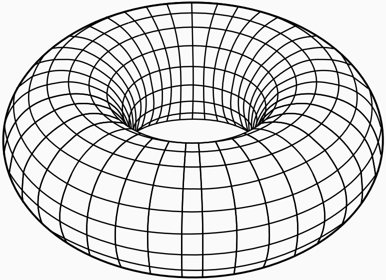
[1]ITER Organization, Route de Vinon-sur-Verdon - CS 90 046 - 13067 Saint Paul-lez-Durance Cedex, France
[2]General Atomics, PO Box 85608, San Diego, CA 92186-5608, USA

The two field components

The normal and tangential fields on a closed surface can be decomposed into (1) the component sourced by enclosed currents, and (2) the component sourced by external currents

Example: a toroidal surface



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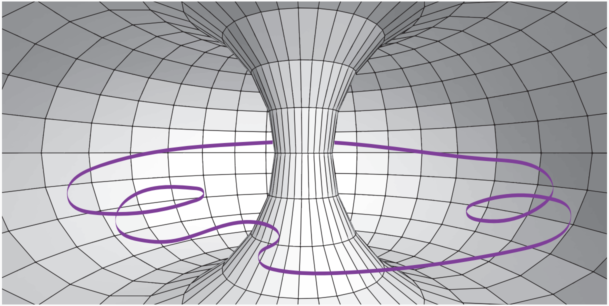

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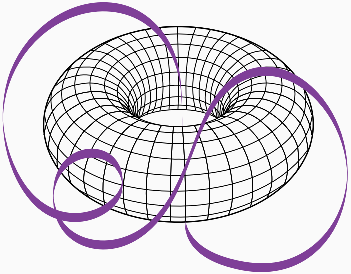


The utility of decomposing the fields

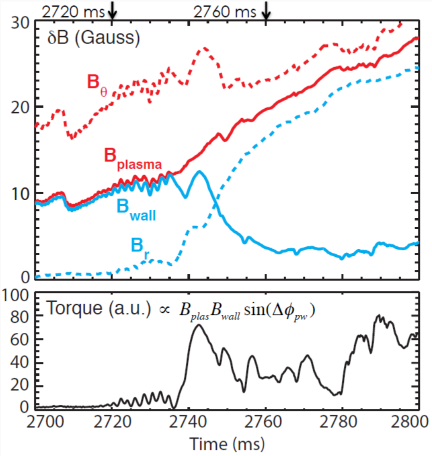
We have used this principle to decompose magnetics measurements in DIII-D; e.g. a rotating tearing mode

Test 1: analyze the locking of a $m/n = 2/1$ tearing mode

- Separate the plasma mode from the wall response
- Calculate torque between internal and external currents

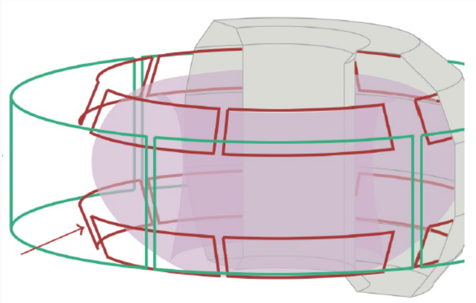
Result:

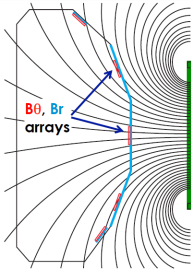
- Measure smooth mode growth through locking
- Decay of wall currents observed following locking
- Bifurcation in the wall torque observed prior to locking



We have used this principle to decompose magnetics measurements in DIII-D; e.g. an applied C-coil field in vacuum

Test 2: remove the C-coil field from a measurement of the vacuum





Result:

- Successfully recovered a vacuum measurement to within 10%

We expect this decomposition will find many applications

Ready for study on DIII-D:

1. Plasma response to C-coil fields: RWM feedback, mode entrainment
2. Measure MHD and driven wall currents separately: RWMs, rotating/locking tearing modes

Near-future studies:

1. Axisymmetric and non-axisymmetric decompositions on well diagnosed, circular cross-section devices
2. DIII-D I-coil response: ELM mitigation, RWM feedback, mode entrainment

Further in the future:

1. Extend to all well diagnosed tokamaks, including ITER
2. VDEs and halo currents: independent plasma and wall measurements
3. Time-changing equilibrium measurements: Ohmic break-down (ITER: 1 MA in wall, 100 kA in plasma), current ramp-up(-down)

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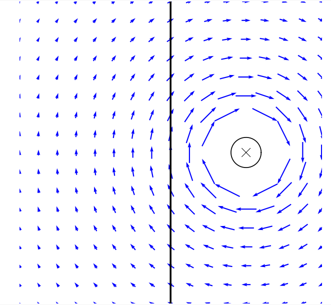
A simple model demonstrates the fundamental principle

Given the field on some closed surface, how can we determine whether the source is inside or outside?

- Let's begin with a simple situation: an infinite plane with a wire running parallel to it.



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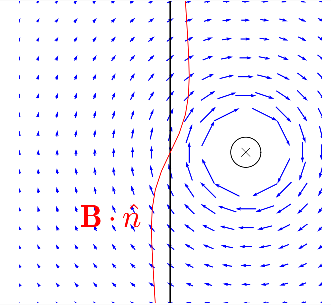

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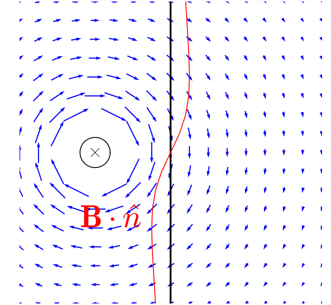

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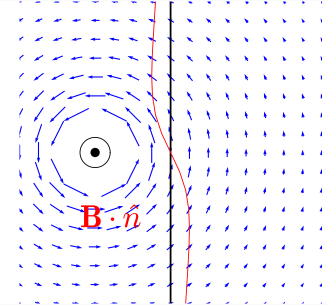

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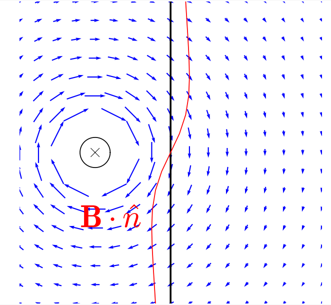

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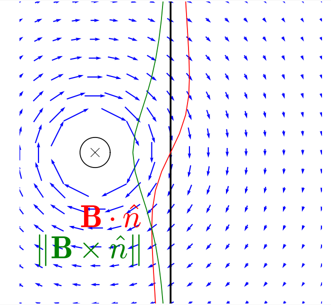

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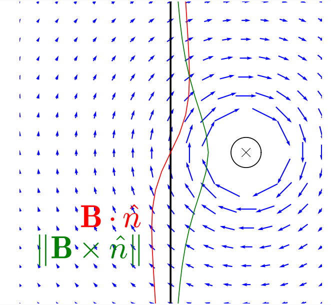
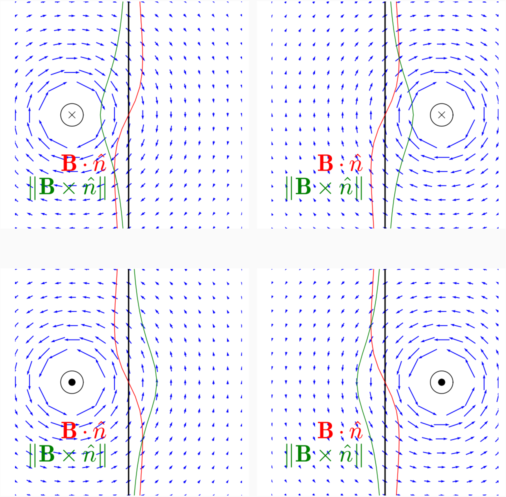


- Q: Can we determine the side the wire is on if we also include the tangential field?
- A: YES! The normal field indicates current into the screen, and to produce the tangential field, the wire must be on the right side.

Summary: in our simple example, the normal and tangential fields are sufficient to determine on which side the wire resides



The simple model is not unique; normal and tangential fields on any closed surface can distinguish internal and external sources

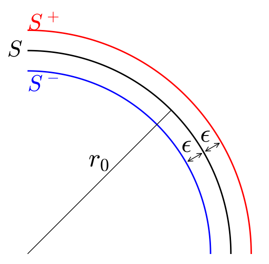
"The theory of the interaction of an external magnetic field with a plasma is greatly simplified by a mathematical theorem that defines a **unique separation of a magnetic field into the part produced by the currents in (the) region enclosed by a surface and the part produced by currents in the region external to the surface.***"*

[1] A.H. Boozer, Nucl. Fusion **55** *025001 (2015)*

Derivation of the decomposition in cylindrical geometry

A measurement surface and two adjacent virtual casings reduce the 3D problem to a 2D cylindrical surface

- The magnetic probes fixed to the inner vessel wall define a virtual surface S at radius $r = r_0$



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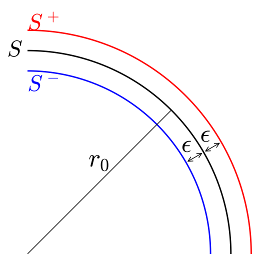
- On S, we measure the tangential and normal fields everywhere
  - Note the normal fields may be measured outside the vessel



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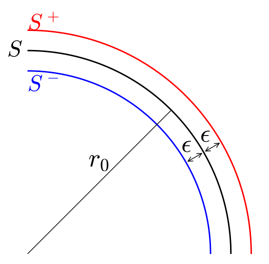


- Using the virtual casing principle [2, 3], all external currents $\mathbf{J}^{\mathrm{x}}$ are represented by surface currents $\sigma^{\mathrm{x}}_{S^+}$ on surface S^+, and all internal currents $\mathbf{J}^{\mathrm{i}}$ by surface currents $\sigma^{\mathrm{i}}_{S^-}$ on surface S^-
  - These currents satisfy $\nabla \cdot \sigma^{\mathrm{x}}_{S^+} = \nabla \cdot \sigma^{\mathrm{i}}_{S^-} = 0$

Only vacuum fields are required, for which there are exact solutions in cylindrical geometry

For a sheet current source at $r = b$, we solve $\nabla \cdot \mathbf{B} = 0$ in vacuum (i.e. curl free) and find,

		External $r < b$	$r = b$	*Internal* $r > b$
$B_r(\mathbf{r}; b) = \sum_j B_j e^{i\chi_j}$	$\times$	$\frac{I'_m(k_j r)}{I'_m(k_j b)}$	1	$\frac{K'_m(k_j r)}{K'_m(k_j b)}$
$B_\theta(\mathbf{r}; b) = \sum_j \frac{im}{k_j r} B_j e^{i\chi_j}$	$\times$	$\frac{I_m(k_j r)}{I'_m(k_j b)}$	0	$-\left\lvert \frac{K_m(k_j r)}{K'_m(k_j b)} \right\rvert$
$B_\phi(\mathbf{r}; b) = \sum_j -iB_j e^{i\chi_j}$	$\times$	$\frac{I_m(k_j r)}{I'_m(k_j b)}$	0	$-\left\lvert \frac{K_m(k_j r)}{K'_m(k_j b)} \right\rvert$

where $\chi_j \equiv m\theta - n\phi$, $k_j = n/R$, $I_m(x)$ and $K_m(x)$ are the modified Bessel functions of the first and second kind, and the prime denotes the derivative with respect to their argument.

The measured field components provide two equations, and the vacuum fields allow a reduction to two unknowns

The measured field $\mathbf{B}$ is a superposition of the internal field $\mathbf{B}^{\mathrm{i}}$ and the external field $\mathbf{B}^{\mathrm{x}}$

$$B_r = B_r^{\mathrm{i}} + B_r^{\mathrm{x}}$$
$$B_\theta = B_\theta^{\mathrm{i}} + B_\theta^{\mathrm{x}}$$

Considering a single set of field harmonics j, we can write

$$B_{jr} = B_{jr}^{\mathrm{i}} + B_{jr}^{\mathrm{x}}$$
$$B_{j\theta} = B_{j\theta}^{\mathrm{i}} + B_{j\theta}^{\mathrm{x}}$$

Finally, using the vacuum fields on the previous slide, we can rewrite the normal component of the field in terms of one of the tangential components of the field

$$B_{jr} = B_{jr}^{\mathrm{i}}(B_{j\theta}^{\mathrm{i}}) + B_{jr}^{\mathrm{x}}(B_{j\theta}^{\mathrm{x}})$$
$$B_{j\theta} = B_{j\theta}^{\mathrm{i}} + B_{j\theta}^{\mathrm{x}}$$

Result: each component of the internal and external fields is written in terms of the total fields and geometric constants!

Internal fields:

	$r_0 k_j/m \leq 1$	$r_0 k_j/m > 1$
$B^{\mathrm{i}}_{jr} =$	$\frac{\gamma^{\mathrm{x}}_j B_{jr} + iB_{j\theta}}{\gamma^{\mathrm{x}}_j + \gamma^{\mathrm{i}}_j}$	$\frac{\Gamma^{\mathrm{x}}_j B_{jr} - iB_{j\phi}}{\Gamma^{\mathrm{x}}_j + \Gamma^{\mathrm{i}}_j}$
$B^{\mathrm{i}}_{j\theta} =$	$\frac{\gamma^{\mathrm{i}}_j(-i\gamma^{\mathrm{x}}_j B_{jr} + B_{j\theta})}{\gamma^{\mathrm{x}}_j + \gamma^{\mathrm{i}}_j}$	
$B^{\mathrm{i}}_{j\phi} =$		$\frac{\Gamma^{\mathrm{i}}_j(i\Gamma^{\mathrm{x}}_j B_{jr} + B_{j\phi})}{\Gamma^{\mathrm{x}}_j + \Gamma^{\mathrm{i}}_j}$

External fields:

	$r_0 k_j/m \leq 1$	$r_0 k_j/m > 1$
$B^{\mathrm{x}}_{jr} =$	$\frac{\gamma^{\mathrm{i}}_j B_{jr} - iB_{j\theta}}{\gamma^{\mathrm{x}}_j + \gamma^{\mathrm{i}}_j}$	$\frac{\Gamma^{\mathrm{i}}_j B_{jr} + iB_{j\phi}}{\Gamma^{\mathrm{x}}_j + \Gamma^{\mathrm{i}}_j}$
$B^{\mathrm{x}}_{j\theta} =$	$\frac{\gamma^{\mathrm{x}}_j(i\gamma^{\mathrm{i}}_j B_{jr} + B_{j\theta})}{\gamma^{\mathrm{x}}_j + \gamma^{\mathrm{i}}_j}$	
$B^{\mathrm{x}}_{j\phi} =$		$\frac{\Gamma^{\mathrm{x}}_j(-i\Gamma^{\mathrm{i}}_j B_{jr} + B_{j\phi})}{\Gamma^{\mathrm{x}}_j + \Gamma^{\mathrm{i}}_j}$

where $\Gamma^{\mathrm{x}}_j = \frac{r_0 k_j}{m}\gamma^{\mathrm{x}}_j = \left|\frac{I_m(k_j r_0)}{I'_m(k_j r_0)}\right|$ and $\Gamma^{\mathrm{i}}_j = \frac{r_0 k_j}{m}\gamma^{\mathrm{i}}_j = \left|\frac{K_m(k_j r_0)}{K'_m(k_j r_0)}\right|$.

In the tokamak limit, the decomposition simplifies

The tokamak limit is given by,

$$\frac{r_0}{m} \ll \frac{R}{n},$$

i.e. the poloidal wavelength is much smaller than the toroidal wavelength.

Internal fields:

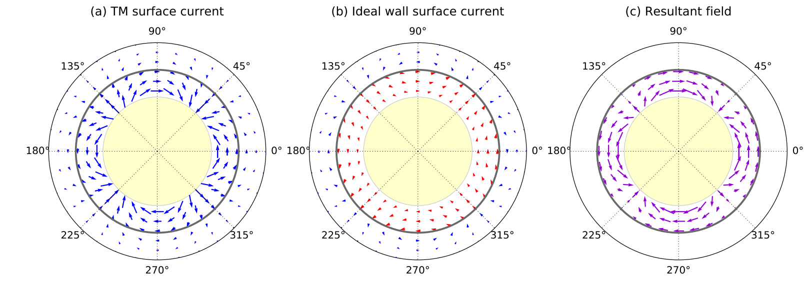
$$\boxed{\begin{aligned} B^{\mathrm{i}}_{jr} &= \frac{B_{jr} + i\frac{m}{|m|}B_{j\theta}}{2} \\ B^{\mathrm{i}}_{j\theta} &= \frac{-i\frac{m}{|m|}B_{jr} + B_{j\theta}}{2} \end{aligned}}$$

External fields:

$$\boxed{\begin{aligned} B^{\mathrm{x}}_{jr} &= \frac{B_{jr} - i\frac{m}{|m|}B_{j\theta}}{2} \\ B^{\mathrm{x}}_{j\theta} &= \frac{i\frac{m}{|m|}B_{jr} + B_{j\theta}}{2} \end{aligned}}$$

- These are the expressions used for the DIII-D applications
- Fields in the RFP require the full form on the previous slide

Tokamak limit example: fast rotating $m/n = 2/1$ tearing mode



Internal fields at r_0:

$$B^{\mathrm{i}}_{21r} = 0 + i$$
$$B^{\mathrm{i}}_{21\theta} = 1 + 0i$$

External fields at r_0:

$$B^{\mathrm{x}}_{21r} = 0 - i$$
$$B^{\mathrm{x}}_{21\theta} = 1 + 0i$$

Measured field at r_0:

$$B_{21r} = 0 + 0i$$
$$B_{21\theta} = 2 + 0i$$

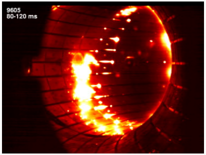
$$B^{\mathrm{i}}_{21r} = \frac{B_{21r} + iB_{21\theta}}{2} = i \qquad B^{\mathrm{x}}_{21r} = \frac{B_{21r} - iB_{21\theta}}{2} = -i$$

$$B^{\mathrm{i}}_{21\theta} = \frac{-iB_{21r} + B_{21\theta}}{2} = 1 \qquad B^{\mathrm{x}}_{21\theta} = \frac{iB_{21r} + B_{21\theta}}{2} = 1$$

Present devices and outlook for ITER

The cylindrical model is ready to be used on large aspect-ratio, circular cross-section devices now

RFX-mod



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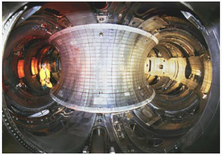
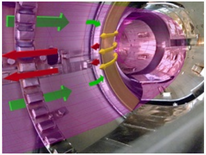
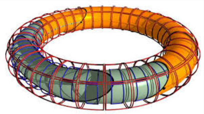


HBT-EP

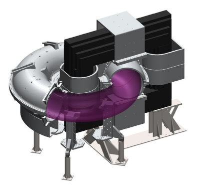


- magnetic diagnostics?

EXTRAP-T2R

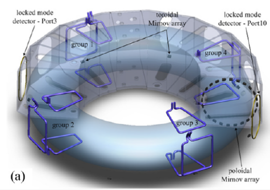


MST



- magnetic diagnostics?

J-TEXT



- needs more B_r sensors

This decomposition is expected to improve plasma control, particularly in ITER

ITER applications:

- Ohmic break-down where 1 MA wall currents and 100 kA plasma currents are expected
  - Decomposition will isolate the wall currents from the plasma currents
- Equilibrium reconstructions, particularly in the following cases:
  - Plasma current ramp-up and ramp-down (i.e. remove vessel currents)
  - Discharges with ELM mitigation by 3D fields

The ITER magnetic diagnostic includes:

- 24×6 (# poloidal$\times$#toroidal) poloidal field probes
- Additional 207 poloidal field probes distributed outside divertor
- 20×6 and 9×9 arrays of saddle loops
- Additional 60 radial field sensors (local measurement)

Conclusion: the ITER magnetic diagnostic is well suited for this decomposition

Conclusion

Summary

1. Magnetic fields can be decomposed on a closed-surface into the components sourced by internal and external currents
2. In tokamaks, this often permits the separation of plasma measurements from external sources which include:
   - Vessel eddy currents
   - Applied 3D fields
   - Axisymmetric control fields
3. Expected disruption applications include:
   - Rotating/locking MHD modes
   - VDEs
   - Halo currents
   - Calculating torques between the plasma and the wall

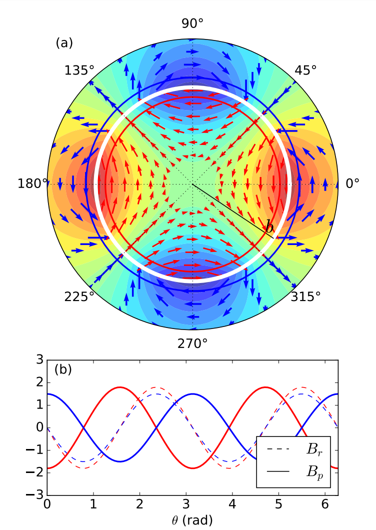
Summary (cont.)

1. The decomposition is solved exactly for the cylindrical, circular cross-section vessel
   - Work is ongoing to generalize to arbitrary vessel shapes
2. The cylindrical decomposition is demonstrated on two DIII-D discharges with surprising success:
   2.1 Separation of tearing mode fields from vessel eddy currents during locking
   2.2 Removal of C-coil fields to within 10% from a vacuum measurement
3. Plasma control, particularly in ITER, is expected to benefit from this decomposition
   - Control systems provided with independent measurements of plasma and external currents

Questions?

Backup slides: cylindrical current sheet

- Current sheet (white) carrying $m/n = 2/1$ current
- Normal (radial) field is continuous across the sheet
- The tangential (poloidal) field inverts across the sheet
- The relationship between the normal and tangential fields determines which side of the current sheet the observer is on



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