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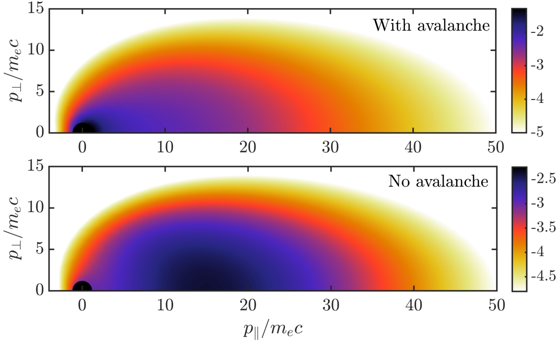
Self-consistent modelling of electron runaway during tokamak disruptions, or: Can gas injection mitigate runaway electrons?

Ola Embréus

Linnea Hesslow, Mathias Hoppe, Oskar Vallhagen, Lucas Unnerfelt, Tünde Fülöp

Runaway momentum dynamics has matured significantly over the past decade:

$$\frac{\partial f}{\partial t} + \underbrace{E_\parallel \frac{\partial f}{\partial p_\parallel}}_{\text{acceleration}} + \underbrace{\frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F}_{\mathrm{RR}} f)}_{\text{radiation-reaction}} = \underbrace{C_e[f]}_{\text{collisions}} + \underbrace{C_{\text{knock-on}}[f]}_{\text{avalanche}} + \underbrace{C_{\text{brems}}[f]}_{\text{bremsstrahlung}}$$



CODE is our Fokker-Planck tool solving the kinetic equation for $f(t, p, \theta)$.

`http://ft.nephy.chalmers.se/retools/`

Computation time dramatically increased $\Rightarrow$ can we revisit the traditional RE fluid?

$$\begin{cases} j_{RE} & = -en_{RE}\langle v_{||}\rangle \\ n_{RE} & = \int \mathrm{d}\mathbf{p}\, f_{RE} \\ \langle v_{||}\rangle & = \frac{1}{n_{RE}}\int \mathrm{d}\mathbf{p}\, v_{||} f_{RE} \end{cases}$$

$$\frac{\mathrm{d}j_{RE}}{\mathrm{d}t} = -e\langle v_{||}\rangle\frac{\mathrm{d}n_{RE}}{\mathrm{d}t} - en_{RE}\frac{\mathrm{d}\langle v_{||}\rangle}{\mathrm{d}t} \equiv \gamma_{seed} + j_{RE}\left[\Gamma_{ava} + \frac{\mathrm{d}}{\mathrm{d}t}\ln\langle v_{||}\rangle\right].$$

The above are only definitions. Reduced kinetic models powerful in quasi-steady state

$$\gamma_{seed}(t;\, E,\, Z,\, ...) \approx \gamma_{seed}(E(t),\, Z(t),\, ...)$$
$$\Gamma_{ava}(t;\, E,\, Z,\, ...) \approx \Gamma_{ava}(E(t),\, Z(t),\, ...)$$
$$\langle v_{||}\rangle(t;\, E,\, Z,\, ...) \approx \langle v_{||}\rangle(E(t),\, Z(t),\, ...)$$

In quasi-steady state conditions:
Runaway fluid models are as accurate as the kinetic model used to determine Γ and $\langle v_{||} \rangle$.

Open RE generation theory questions:

- How does gas injection affect RE generation?
- Is hot-tail generation amenable to an accurate reduced description?
- When are reduced kinetic models applicable?

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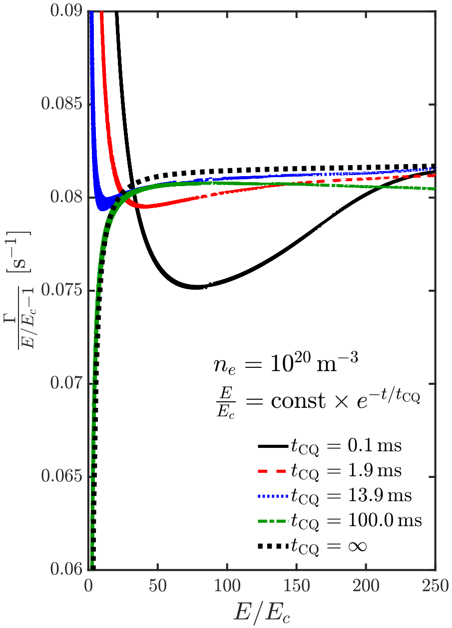


Avalanche: Generalization of RP calculation [L Hesslow *et al.*, NF **59** (2019 084004]

$$\Gamma_{\text{ava}} = 4\pi n_e^{\text{tot}} r_0^2 c \frac{(m_e c)^2}{p_\star^2},$$

$$p_\star^2 = \frac{\sqrt{\nu_s(p_\star)\nu_D(p_\star)p_\star^6/\gamma_\star^3}}{eE},$$

ν_s and ν_D arbitrary slowing-down and deflection collision frequencies.

Asymptotic matching, valid also when $E \sim E_c^{\text{eff}}$ or $n_Z \ll n_D$:

$$\Gamma_{\text{ava}} = \frac{e}{m_e c \ln\Lambda} \frac{n_e^{\text{tot}}}{n_e} \frac{E - E_c^{\text{eff}}}{\sqrt{4 + \bar{\nu}_s \bar{\nu}_D(p_\star)}} \qquad (14)$$

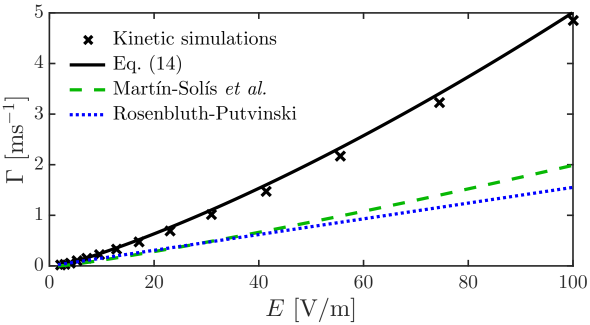


Figure: Steady state avalanche growth rate with $n_D = 10^{20}\,\text{m}^{-3}$, singly ionized argon with $n_Z = n_D$ at temperature $T = 10\,\text{eV}$.

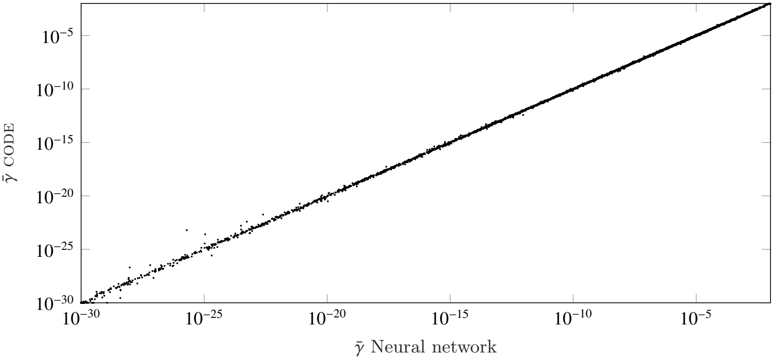
Atomic and kinetic model in [L Hesslow *et al.*, JPP **84** (2018) 905840605]

Dreicer generation: Typically negligible in ITER, sometimes important for present-day devices.

- Exponentially sensitive to many parameters
- Difficult to fit data to simple analytic formulas
- Neural networks trained on large database of kinetic simulations accounting for cold impurities
- Significantly faster to evaluate than kinetic simulations

Figure: Comparison of normalized Dreicer growth rate $\bar{\gamma}$ (arb. units).

(disclaimer: all conditions for the validity of the model are not strictly satisfied during post-disruption conditions)



GO: 1D fluid model for RE generation during disruptions (infinite aspect ratio)

Electric field and REs:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right) = \mu_0\frac{\partial}{\partial t}(\sigma E + j_{\mathrm{RE}})$$

$$\frac{\partial j_{\mathrm{RE}}}{\partial t} = ec\left[\left(\frac{\partial n_{\mathrm{RE}}}{\partial t}\right)^{\text{generation}} + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{\mathrm{RE}}\frac{\partial n_{\mathrm{RE}}}{\partial r}\right)\right]$$

Electron energy:
$$\frac{3}{2}\frac{\partial n_e T_e}{\partial t} = P_\Omega - P_{\mathrm{rad}} - P_{\mathrm{ionize}} + P_c^e + \frac{3n_e}{2r}\frac{\partial}{\partial r}\left(r\,\chi_e\frac{\partial T_e}{\partial r}\right)$$

Ion energies:
$$\frac{3}{2}\frac{\partial n_i T_i}{\partial t} = P_c^i + \frac{3n_i}{2r}\frac{\partial}{\partial r}\left(r\,\chi_i\frac{\partial T_i}{\partial r}\right)$$

$$P_c^j = \sum_k \frac{3n_j}{2\tau_{jk}}(T_k - T_j) \quad \text{(collisional energy transfer)}$$

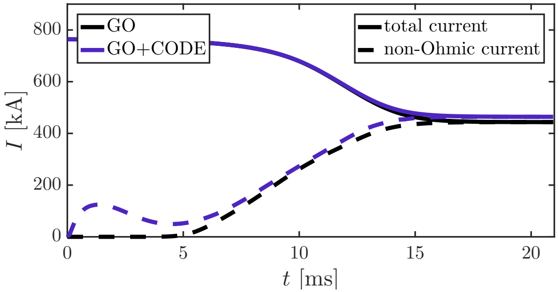
$$P_{\mathrm{rad}} = P_{\mathrm{brems}} + \sum_j P_{\mathrm{line},j},\ \text{ADAS: } P_{\mathrm{line},j} = n_e n_j L_j(n_e, T_e)$$

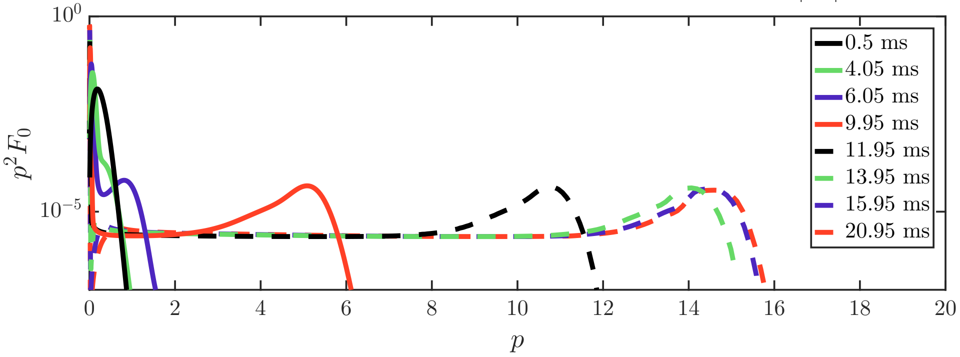
Charge states resolved:
$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = n_e\Big[I_{i-1}n_{i-1} - (I_i + R_i)n_i + R_{i+1}n_{i+1}\Big]$$

Main limitations:

- No impurity transport $\leftrightarrow$ static impurity density profiles
- Intact flux surfaces $\leftrightarrow$ ad-hoc diffusion coefficients
  (possible generalization: hyperresistivity and improved RE transport model)
- Steady-state growth rates $\leftrightarrow$ crude hot-tail model

- Result from coupled GO+CODE simulations: prescribed T_e evolution $T_e = T_{\text{final}} + (T_{\text{initial}} - T_{\text{final}})e^{-t/t_{\text{TQ}}}$.
- Scenario based on ASDEX ($I_p \approx 800\,\text{kA}$, $T_{\text{initial}} \approx 10\,\text{keV}$ and $n_e \approx 3 \times 10^{19}\,\text{m}^{-3}$ on-axis, $T_{\text{final}} = 5\,\text{eV}$, $t_{\text{TQ}} = 2\,\text{ms}$)





How does RE generation during the CQ depend on injected gas quantities?
T_e and n_e constant (post-TQ), $j_{\mathrm{RE}} \ll j_{\mathrm{tot}}$ and no radial transport:

$$j_{\mathrm{RE}}(r) = ecn_{\mathrm{seed}}(r)10^{N_{\mathrm{ava}}(r)},$$

$$N_{\mathrm{ava}} \approx \ln(10)t_{\mathrm{CQ}} \int_{E_c^{\mathrm{eff}}}^{E_{\mathrm{initial}}} \frac{\Gamma_{\mathrm{ava}}(E)}{E}\mathrm{d}E,$$

$$E(t,\, r) \approx E_{\mathrm{initial}}(r)e^{-t/t_{\mathrm{CQ}}}.$$

(RP model: $N_{\mathrm{ava}} \approx 10^{16}$ in ITER)

Figure: Avalanche gain during ITER-like CQ with constant background parameters.

In reality, the RE current saturates when it approaches the ohmic current in magnitude!

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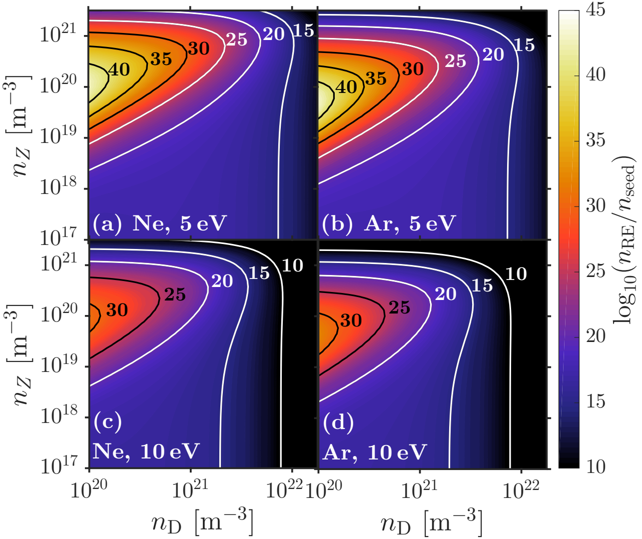

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Test calculation with GO: non-negligible RE current and self-consistent E field.
$n_Z = n_D = 10^{20}\,\mathrm{m}^{-3}$; $T_e = 5\,\mathrm{eV}$; $n_{\mathrm{seed}} = 10^3\,\mathrm{m}^{-3}$.

- With Rosenbluth-Putvinski avalanche model: $I_{\mathrm{RE}} \approx 1\,\mathrm{MA}$
- New growth rate with impurity corrections: $I_{\mathrm{RE}} \approx 7\,\mathrm{MA}$ (both for Ar and Ne)

Next steps:

- Self-consistent temperature evolution
- Physically-based seed generation (tritium & Compton short-term, hot-tail in pipeline)
- Modelling of past and future experiments

Note: challenging to validate our model on today's experiments!

- Simple RE fluid models provide insights into efficacy of material injection for RE mitigation
  - *According to our atomic-physics model, injection exacerbates the RE problem in ITER*
  - Ongoing: validate model with existing experiments
- EUROfusion pilot project: 6 participating institutes, 10 ppy total until the end of 2020 (extension possible).
- Open workshop (REM-8) on runaway modelling in Gothenburg (Sweden), January 13-17, 2020