Fast wave instabilities excited by runaway electrons and mitigation of runaway electron current

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Outline

1. Experimental observation of fast wave in DIII-D disruption experiments

2. Frequencies and mode structures of fast waves in DIII-D

3. Resonance of runaway electrons and fast wave

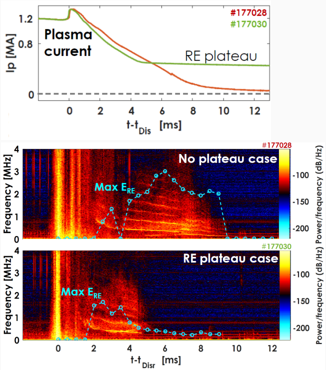
4. Collisional damping of fast wave in cold plasma

5. Diffusion of REs due to excited fast waves

Experimental observation of fast wave in DIII-D disruption experiments

Direct observation of kinetic instabilities in the current-quench phase

- In DIII-D disruption experiments, low-frequency (a few MHz) kinetic instabilities are identified during current quench using ultrafast magnetic pick-up coils.
    - When low-frequency modes are strongly excited, the RE plateau will not build up.
    - Increase Ar density reduces the number of high-energy REs, suppress instabilities, help RE plateau survive.
- Modes in this frequency range are usually attributed to the magnetoacoustic cyclotron instability (MCI), which results from the resonant interaction of energetic ions with fast Alfvén waves. Here the resonance can happen between waves and REs.



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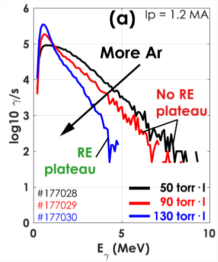
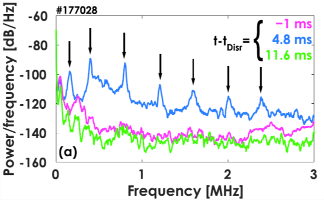
K.G. McClements et al., Nucl. Fusion 55, 043013 (2015).

Mode excitation and plateau dissipation have thresholds on RE energy

- Using Gamma Ray Imager (GRI) to model RE energy spectrum, it is shown that excitation of modes and dissipation of RE plateau depend on the existence of high-energy REs.
  - Max $E_{RE} > 2.5 - 3$ MeV is required for the mode excitation.
  - RE plateau formation fails when max $E_{RE} > 6$ MeV.



- The modes spectrum shows discrete structures, with frequencies 0.1-2.4MHz with a spacing of 400kHz.
  - The frequencies are of the same order of Ar cyclotron frequency. ($\Omega_{Ar} \approx 0.8 \sim 1.6$MHz)
  - The discrete frequencies decrease during current-quench.

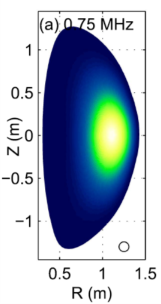


Frequencies and mode structures of fast waves in DIII-D

Compressional Alfvén eigenmodes in spherical tokamaks

- Fast magnetosonic wave is a candidate explanation for the observed instabilities.
  - Frequencies observed is in the Alfven frequency range.
  - Shear Alfven waves (slow wave) like TAE and GAE require $\omega < \omega_{ci}$, but here the frequencies can be higher than Ω_{Ar}.
- Previous studies of CAE focus on cases in spherical tokamaks (ST), which can be excited by energetic ions in plasma.
  - Due to the inhomogeneity of R^2 and v_A^2, CAE in ST is strongly localized near the edge of low-field-side, and confined inside plasma.

$$\left(\nabla^2_{\mathrm{pol}} - \frac{n^2}{R^2} + \frac{\omega^2}{v_A^2}\right) B_\phi = 0$$



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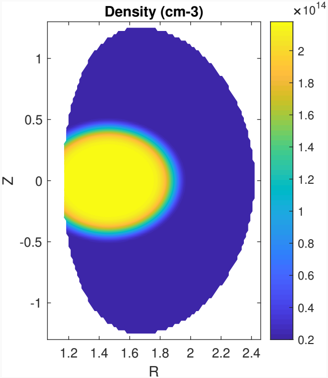
Calculating mode structures in disruptive plasma

- Assume plasma density outside last closed flux surface is non-zero, the mode can have a more global structure and can extend to boundaries.
- Here we use CAE code to calculate the mode structure of fast wave in DIII-D.
    - This code is a simplified eigenmode solver, which only solve a scalar equation (Helmholtz equation) instead of 3D field.
    - Assuming conducting wall boundary condition ($E_{||} = 0$), B_ϕ at the boundary should satisfy $\partial B_\phi / \partial n = 0$ (Neumann boundary condition).



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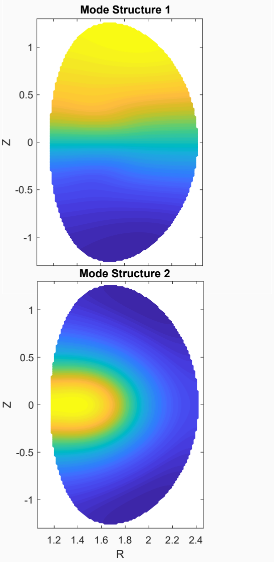
Frequencies of fast wave eigenmodes consistent with experiments

- With Ar MGI, plasma is mostly composed of Ar and electrons.
  - For $n_e = 2 \times 10^{20}\mathrm{m}^{-3}$, $n_{edge} = 2 \times 10^{19}\mathrm{m}^{-3}$, $B_0 = 2.1\mathrm{T}$, $Z_{eff} = 2$, $n_\phi = 1$ $\omega_1 = 0.43MHz$, $\omega_2 = 0.54MHz$.
  - These results depends sensitively on n_e, n_{edge} and Z_{eff}.
  - During current quench, T_e drops from 5eV to 2eV, thus Z_{eff} of Ar drops from +2 to +1, which results in a decrease of v_A and eigenmode frequencies.
- Modes have strong B_ϕ components at edge.
- Higher frequency modes correspond to $n_\phi > 1$.



Resonance of runaway electrons and fast wave

Fast wave can have resonances with transit and precession motion of high-energy electrons

In order to transfer energy to fast waves, runaway electrons must have resonances with the modes.

- $\omega_{ce} \approx$ 58GHz$\gg \omega$, so Doppler resonance ($\omega = n\omega_{ce}$) is unlikely.
- $\omega/(1/R) \approx 0.01c$, so a runaway electron ($v \sim c$) satisfying Cherenkov resonance is almost certainly a trapped particle.
- Transit and bounce frequencies of relativistic electrons ($\sim$ 13MHz) are too large compared to ω.
  - For passing electrons with small $v_{\|}$ and near a rational surface, $\omega = n\omega_{\phi} - m\omega_{\theta}$ can be satisfied.
- Precession frequency of trapped runaway electrons is about 0.3MHz, so the resonance condition $\omega = n\omega_d$ can be satisfied.
  - This mechanism was used to explain the excitation of beta-induced Alfvén eigenmodes (BAE) driven by energetic electrons.
  - Unlike transit and bounce frequencies, precession frequency is proportional to the relativistic factor γ.

HL-2A team, W. Chen, X.T. Ding, Q.W. Yang, Y. Liu, X.Q. Ji, Y.P. Zhang, J. Zhou, G.L. Yuan, H.J. Sun, W. Li, Y. Zhou, Y. Huang, J.Q. Dong, B.B. Feng, X.M. Song, Z.B. Shi, Z.T. Liu, X.Y. Song, L.C. Li, X.R. Duan, and Y. Liu, Phys. Rev. Lett. 105, 185004 (2010).

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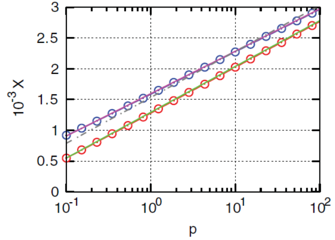
Effects of partially screened nuclei on electron collisions

- Pitch-angle scattering is weak for high-energy electrons in classical theories, as scattering coefficients drops as $1/p^2$.
- With partially-ionized high-Z impurities, the slowing-down and pitch-angle scattering of REs in high energy regime is significantly enhanced due to partially-screening.
    - High-energy electron can penetrate into electron cloud and get closer to the nuclei, interacting with bounded electrons and naked nuclei.
    - Slowing-down enhanced by factor of Z, while scattering is enhanced by Z^2.



- Calculating of effective lnΛ for pitch-angle scattering using Thomas-Fermi model and density function theory (DFT) agrees well.

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L. Hesslow, O. Embréus, A. Stahl, T.C. DuBois, G. Papp, S.L. Newton, and T. Fülöp, Phys. Rev. Lett. 118, 255001 (2017).

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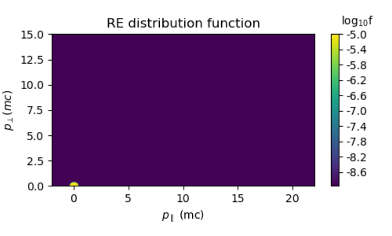
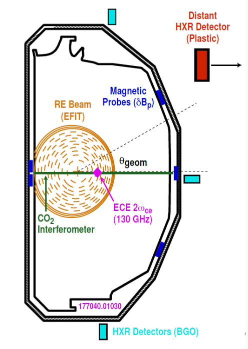
Scattering of hot-tail generated RE beam

- In this DIII-D disruption, most of REs are generated by hot-tail generation during thermal quench.
- During current equench as REs tail is dragged to higher energy, the enhanced pitch-angle scattering from Ar can scatter electrons into large pitch angles, and even become trapped electrons.
    - For both kinds of electrons, distribution function satisfies $\partial f / \partial p_{\parallel} > 0$.



Estimation of q at RE beam location

- During current quench in DIII-D, plasma current (thermal electrons+RE) drops, but the RE beam (limited by last closed flux surface) can be peaked and lead to kink instability.
  - For $I = 0.5MA$ and beam area 0.1 m^2, the current density j gives $q = 1.08$ at beam edge.



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C. Paz-Soldan et al., Plasma Phys. Control. Fusion (2019)

Fast wave can be driven by resonant REs due to gradients in momentum space and radial direction

$$\gamma_L = \frac{4\pi^2 e^2}{\mathcal{E}} \int \frac{|\langle G\rangle|^2}{\omega} \delta(\omega - n\omega_\phi + m\omega_\theta) \left(\omega \frac{\partial}{\partial E} + n \frac{\partial}{\partial P_\phi}\right)_\mu f d^3\mathbf{p}$$

$$G = \mathbf{E} \cdot \mathbf{v}_d + E_y v_\perp J_1(k_\perp \rho), \qquad P_\phi = p_\parallel R - \psi$$

$$(\omega \frac{\partial}{\partial E} + n \frac{\partial}{\partial P_\phi})_\mu = \frac{\omega}{v_\parallel} \left(\frac{\partial}{\partial p_\parallel}\right)_{p_\perp, \psi} + \left(\frac{\omega R}{v_\parallel} - n\right) \left(\frac{\partial}{\partial \psi}\right)_{p_\perp, p_\parallel}$$

- Ware pinch of trapped electrons lead to a peaked profile.
- For trapped electrons, $\omega < (n/R) v_\parallel$, so a peaked profile of RE leads to a positive growth rate.

K. Ghantous, N.N. Gorelenkov, H.L. Berk, W.W. Heidbrink, and M.A. Van Zeeland, Phys. Plasmas 19, 092511 (2012).
E. Nilsson, J. Decker, N.J. Fisch, and Y. Peysson, J. Plasma Phys. 81, 475810403 (2015).

Collisional damping of fast wave in cold plasma

Collisional damping of fast wave can be modeled using friction forces between electrons and ions

- In most literature about Alfvén eigenmodes in tokamaks, damping comes from kinetic effects such as Landau damping or collisions of resonant particles.
  - In post-disruptions, the kinetic effect is insignificant due to small T_e.
- For electron-driven modes like whistler modes, collisional damping rate can be calculated by adding a friction term in equation of motion of electrons (replace m_e with $m_e(\omega + i\nu_{ei})/\omega$ in the dielectric tensor)
  - This calculation is based on the assumption that ions are fixed when electrons are collision with them.
  - However, in Alfvén waves ions and electrons are mostly moving together since $\omega < \omega_{ci} \ll \omega_{ce}$.

- A more rigorous calculation of the collisional damping rate due to ν_{ei} can be done by adding resistivity and viscosity into multi-fluid equations.
  - Resistivity $\sim n/\tau \sim T^{-3/2}$, whereas viscosity $\sim nT\tau \sim T^{5/2}$, so for low temperature plasma the contribution from viscosity can be ignored.
  - Resistivity can be modeled using friction forces between electrons and ions.

Two-fluid equations with friction forces

$$-i\omega V_{ix} = eZ_iE_x/m_i + \omega_{ci}V_{iy} + \nu_{ie}(V_{ex} - V_{ix})$$
$$-i\omega V_{iy} = eZ_iE_y/m_i - \omega_{ci}V_{ix} + \nu_{ie}(V_{ey} - V_{iy})$$
$$-i\omega V_{ex} = -eE_x/m_e - \omega_{ce}V_{ey} + \nu_{ei}(V_{ix} - V_{ex})$$
$$-i\omega V_{ey} = -eE_y/m_e + \omega_{ce}V_{ex} + \nu_{ei}(V_{iy} - V_{ey})$$

$$\begin{pmatrix} V_{ix} \\ V_{iy} \\ V_{ex} \\ V_{ey} \end{pmatrix} = \begin{pmatrix} i\omega - \nu_{ie} & \omega_{ci} & \nu_{ie} & 0 \\ -\omega_{ci} & i\omega - \nu_{ie} & 0 & \nu_{ie} \\ \nu_{ei} & 0 & i\omega - \nu_{ei} & -\omega_{ce} \\ 0 & \nu_{ei} & \omega_{ce} & i\omega - \nu_{ei} \end{pmatrix}^{-1} \begin{pmatrix} -eZ_iE_x/m_i \\ -eZ_iE_y/m_i \\ eE_x/m_e \\ eE_y/m_e \end{pmatrix}$$

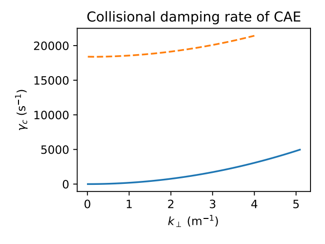
- ν_{ei} and ν_{ie} are slowing-down collision frequency.
    - Conservation of momentum requires $n_e m_e \nu_{ei} = n_i m_i \nu_{ie}$
- With $\nu_{ei} = \nu_{ie} = 0$, the equations just give the standard plasma dielectric tensor.
- The full inverse matrix becomes very complicated, but can be solved numerically.

Collisional damping rate of fast waves for low-temperature plasmas

$$\text{Collision Rate} \qquad \nu = \left(F_0 + F_1\omega^2 + O(\omega^4)\right)\nu_{ei} + O(\nu_{ei}^2)$$

$n_e = 2 \times 10^{20}\mathrm{m}^{-3}$, $T_e = 5\mathrm{eV}$, Ar with $Z_{eff} = 2$

- Using conventional method, $F_0 \neq 0$ and the damping rate is non-zero at $\omega \to 0$. However, with the new method, $F_0 = 0$ and damping rate $\nu \sim \nu_{ei} k^2$
- For the first eigenmode, $\gamma_c/\omega \approx 10^{-3}$.
- As ω gets larger and becomes close to ω_{ci}, the collisional damping rate increase.
    - As ions become less magnetized, the motion of electrons and ions have larger separation.

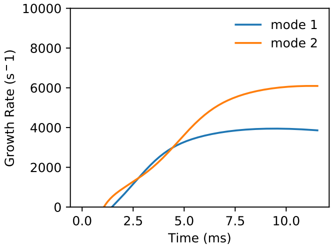


Diffusion of REs due to excited fast waves

Linear growth rate of fast wave in disruptions

- We solve the bounce-averaged kinetic equation of runaway electrons to obtain the evolution of f.
  - $n_e = 2 \times 10^{20} \mathrm{m}^{-3}$, $T_e = 5\mathrm{eV}$, $B = 2.1$ T and $E = 3\mathrm{V/m}(\approx 20E_{CH})$
  - Plasma is composed by pure Ar^{2+} and electrons.
  - Simulation is done for one flux surface, $r = 0.15\mathrm{m}$, $R = 1.67\mathrm{m}$, $q = 1.08$
  - Seed RE from hot-tail generation $n_{RE}/n_e \approx 5 \times 10^{-4}$

- The linear growth rate can be calculated from distribution function
  - Radial gradient of trapped RE is calculated by assuming $\partial f/\partial r = -f/r_0$, where $r_0 = 0.4\mathrm{m}$.
- For the first two eigenmodes, the growth rate given by RE can become larger than collisional damping.



Amplitudes of the wave required for fast diffusion of RE current

- As discussed above, the fast waves can be excited by radial gradient of resonant REs. Thus the excited waves can cause radial diffusion of resonant REs (similar to energetic ions diffused by AEs).
- For non-resonant electrons, the diffusion can also happen due to decorrelation of particles and waves

$$D = \left[\frac{v_\perp}{k_{||}v_{||} - \omega}\right]^2 \frac{1}{\Delta t}$$

  where $v_\perp = v_{||}\delta B_\perp/B$. Δt is the correlation time, here we choose Δt=2.5μs (gaps in frequency spectrum).
- We find that diffusion RE current in 15ms requires $\delta B_\perp = 3 \times 10^{-4}$T.
  - It will be helpful to use the measured ICE power at edge and mode structure to calculate the $\delta B_\perp$ at core, to identify whether the fast wave is the cause of RE diffusion.

Summary

- Analyzing the frequencies and mode structures of fast waves in DIII-D indicates that it can explain the instabilities observed in post-disruption.
- Runaway electrons can have resonance with fast waves and transfer energy by gradient of RE distribution function in both momentum space and radial directions.
- Calculations based on two-fluid models show that damping due to electron-ion collision for CAE is relatively weak compared to whistler waves.
- With large amplitude fast wave excited, all runaway electrons can be diffused radially in a short time.
- Future work:
    - Calculating the radial distribution of REs including quasilinear diffusion, and calculate the mode amplitude self-consistently.

Thank you