Simulation of MHD instabilities with runaway electron current using M3D-C^1

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Outline

- Introduction to M3D-C^1
- Basic equations of runaway electrons (RE) in M3D-C^1
- Simulation of linear kink mode with RE
- Summary and future work

1.Introduction to M3D-C^1

3D Extended MHD Equations in M3D-C[1]

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \cdot D_n \nabla n + S_n$$ Density equation

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_{\perp} \cdot \frac{1}{R^2}\nabla\Phi = -\nabla_{\perp} \cdot \frac{1}{R^2}\mathbf{E}$$ Field equation

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla\mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_i + \mathbf{S}_m$$ Momentum equation

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \bullet \mathbf{\Pi}_e\right) - \frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \bullet \nabla \mathbf{V}_e\right) + \mathbf{S}_{CD}$$ Generalized Ohm's law

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla \bullet \left(p_e \mathbf{V}\right)\right] = -p_e \nabla \bullet \mathbf{V} + \frac{\mathbf{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n + \mathbf{R}_c\right] + \nabla\left(\frac{\mathbf{J}}{ne}\right) : \mathbf{\Pi}_e - \nabla \bullet \mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t} + \nabla \bullet \left(p_i \mathbf{V}\right)\right] = -p_i \nabla \bullet \mathbf{V} - \mathbf{\Pi}_i : \nabla \mathbf{V} - \nabla \bullet \mathbf{q}_i - Q_\Delta + S_{iE}$$ Pressure equations

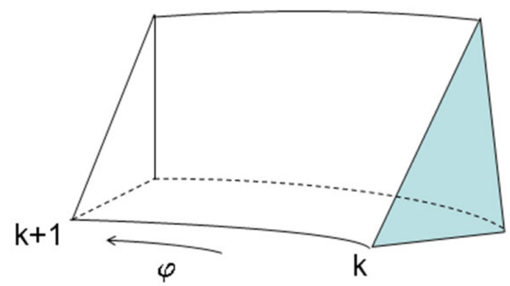
$$\mathbf{R}_c = \eta ne \mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu\left[\nabla \mathbf{V} + \nabla \mathbf{V}^{\dagger}\right] - 2(\mu_c - \mu)(\nabla \bullet \mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV} \qquad \mathbf{q}_{e,i} = -\kappa_{e,i}\nabla T_{e,i} - \kappa_{\parallel}\nabla_{\parallel} T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B}/B^2)\nabla \bullet \left[\lambda_h \nabla\left(\mathbf{J} \bullet \mathbf{B}/B^2\right)\right], \qquad Q_\Delta = 3m_e(p_i - p_e)/\left(M_i \tau_e\right)$$

Blue terms are 2-fluid terms. Also, now have impurity and pellet models for disruption mitigation. **NOT reduced MHD.**

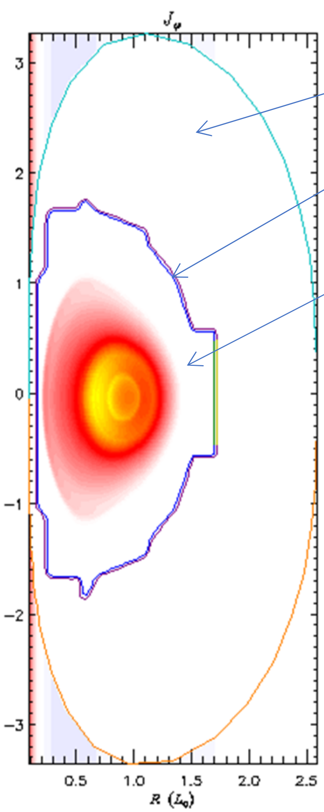
3D finite elements algorithm in M3D-C^1

- M3D-*C^1* uses high-order curved triangular prism elements
- Within each triangular prism, there is a polynomial in (R,φ,Z) with 72 coefficients
- The solution *and 1st derivatives* are constrained to be continuous from one element to the next.
- Thus, there is much more resolution than for the same number of linear elements
- Error ~ h^5



Also, implicit time-stepping allows for very long time simulations

Mesh adaption in M3D-C[1]



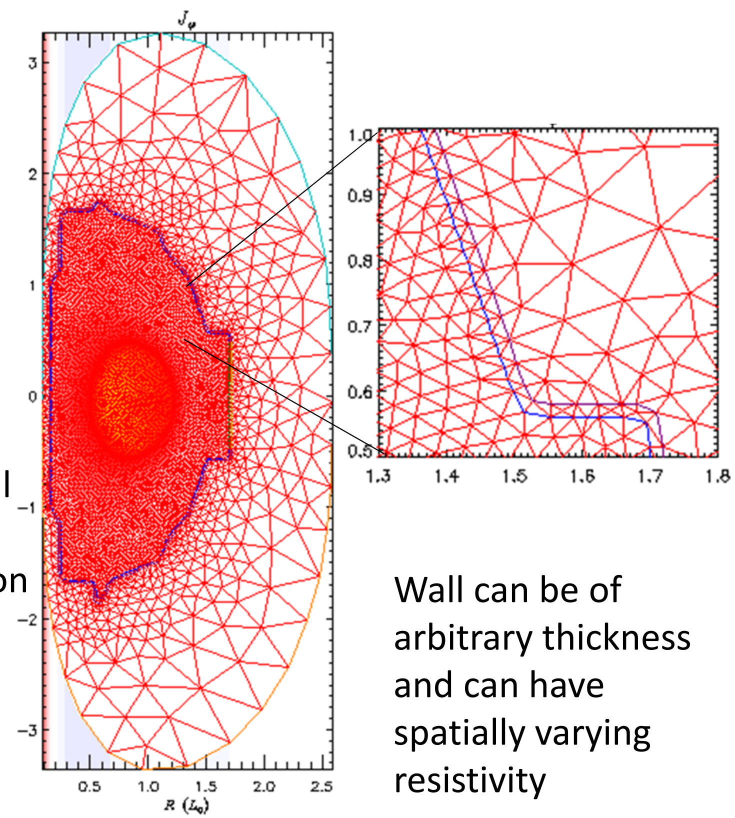
Vacuum (**J**=0)

RW (**E** = η_W **J**)

Plasma (X-MHD)

BC:
- **v**, p, n set at inner wall
- **B** set at outer (ideal) wall

- No boundary conditions on **B** or **J** at the resistive wall
- (halo) Current can flow into and out of the wall



Wall can be of arbitrary thickness and can have spatially varying resistivity

*Ferraro, et al. ,Phys Plasma**23** 056114 (2015)

2. Basic equations of runaway electrons (RE)

RE terms in 3D Extended MHD Equations

- In our model, the runaways move practically at the speed of c parallel to the magnetic field. The speed c is much higher than Alfven speed.

- Runaway electron is coupled to bulk plasmas through the runaway current in generalized Ohm's law.

- We also use parallel diffusion equation to resolve toroidal diffusivity on 3D simulation.

$$\frac{\partial n_{RE}}{\partial t} + \nabla \bullet (n_{RE} c \frac{\mathbf{B}}{B}) = S_{RE}$$ RE density equation

$$\frac{\partial n_{RE}}{\partial t} + \nabla \bullet (\mathbf{B} \frac{D_{RE}}{B^2} \mathbf{B} \bullet \nabla) = S_{RE}$$ RE parallel diffusion equation

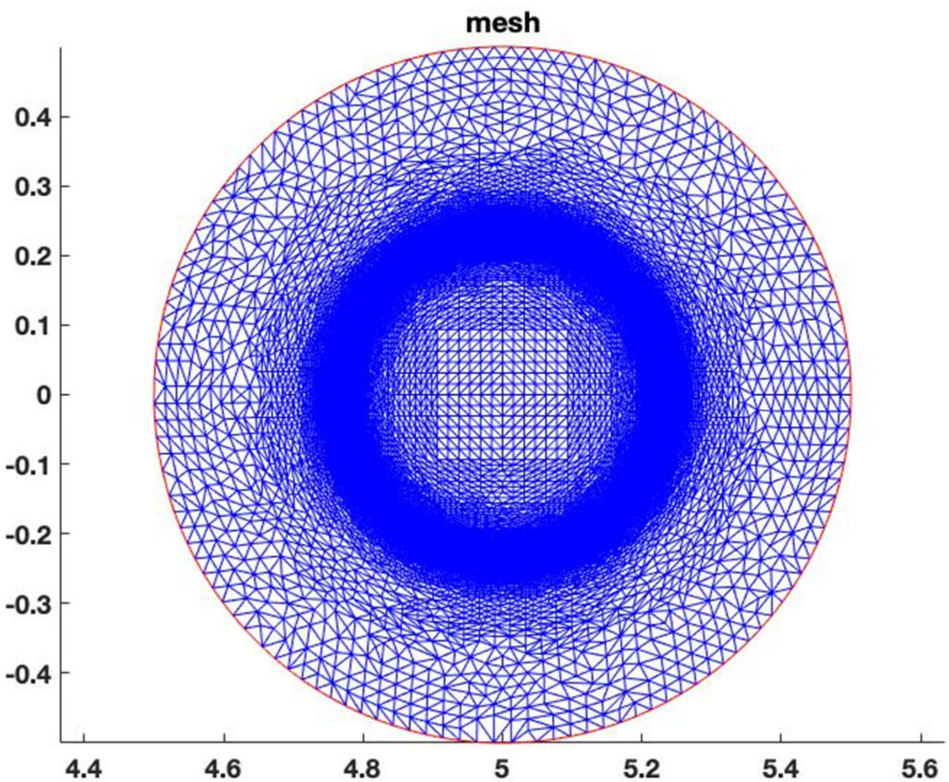
$$\mathbf{J}_{RE} = -e n_{RE} c \frac{\mathbf{B}}{B}$$ RE current assumption

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c - \mathbf{R}_{RE} - \nabla \bullet \Pi_e\right) + \mathbf{S}_{CD}$$ Single fluid Ohm's law with RE

$\mathbf{R}_{RE} = \eta ne \mathbf{J}_{RE}, D_{RE} = n_{re} \tau_{re}^{-1}$ Red terms are additional runaway electron terms.

3. Simulation of linear kink mode with RE

Mesh and basic parameters in simulation



- Parameters of equilibrium

$$\beta_0 = 1.0 \times 10^{-2}$$

$$q_0 = 0.9$$

$$q = q_0 \left[1.0 + \left(\frac{r_{norm}^2}{2} \right) \right], r_{norm} = \frac{r}{a}$$

$$a = 0.5m$$

$$B_0 = 4.2T$$

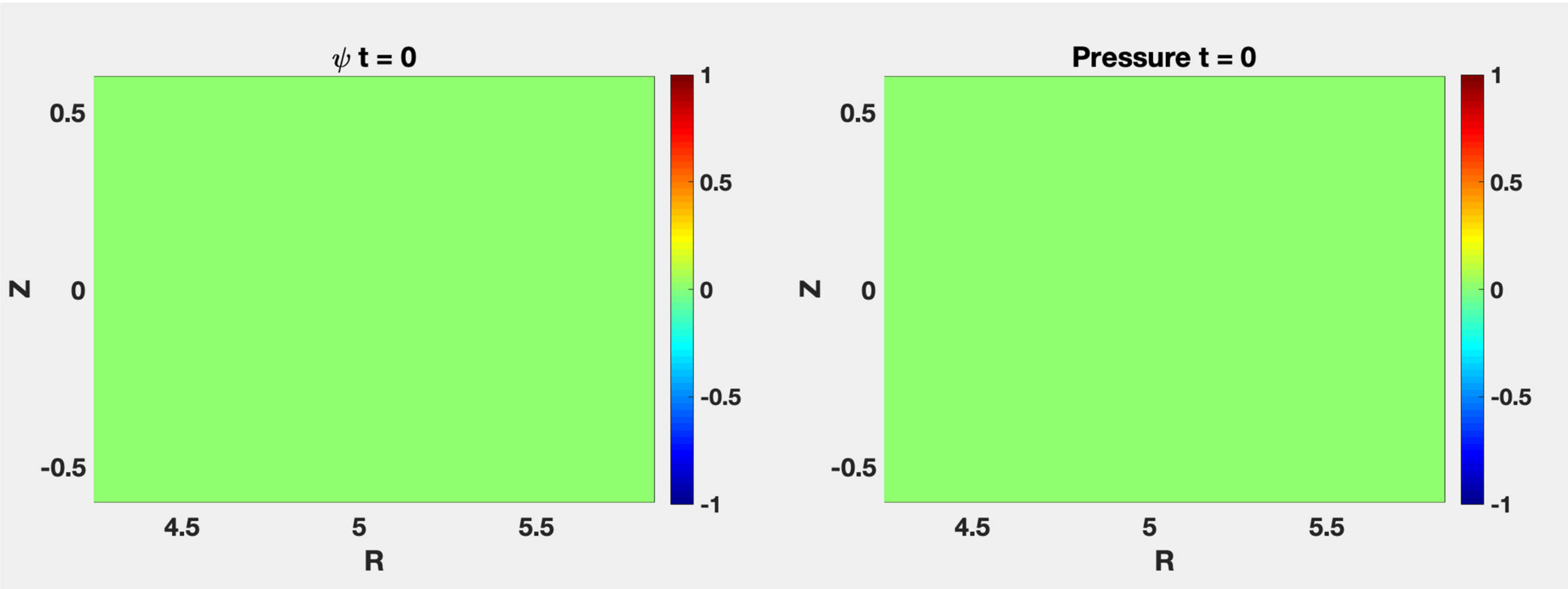
$$\eta = 1.0 \times 10^{-5}$$

$$n_0 = 1.0 \times 10^{20} m^{-3}$$

$$c = 240 v_A$$

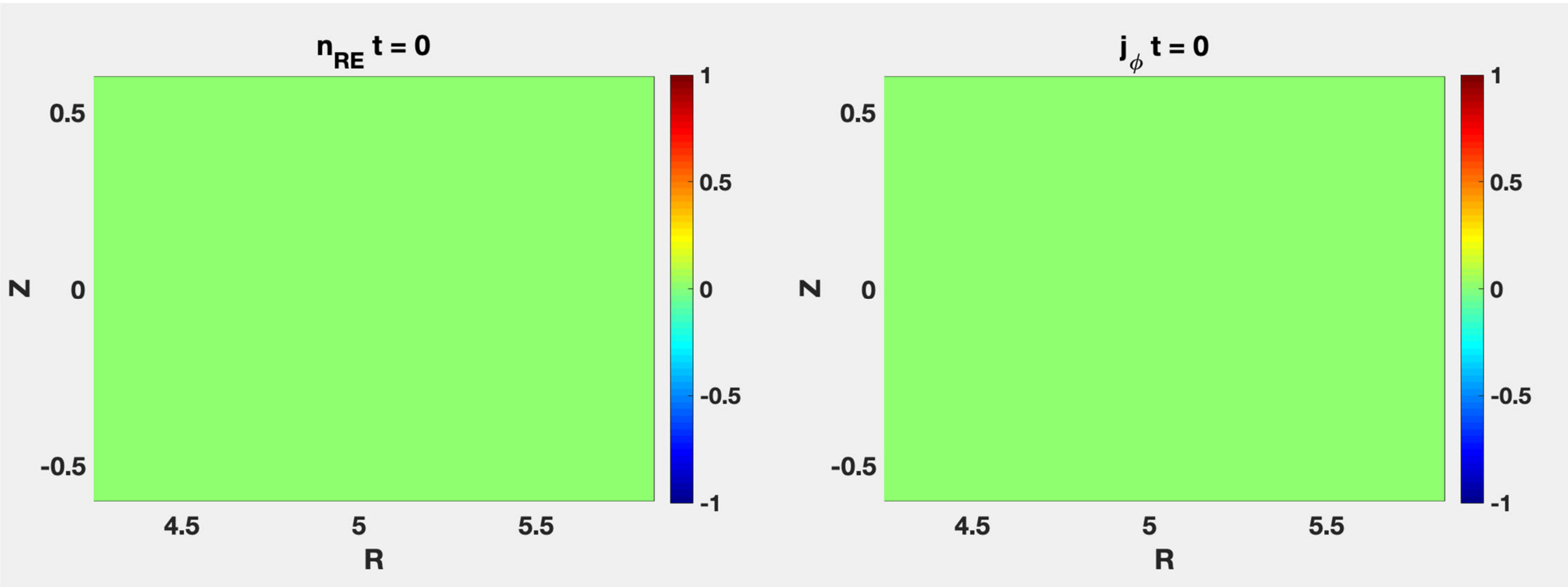
- In our simulations, we use an adaptive mesh which has increased resolution near the q = 1 rational surface.

2D results of kink mode with RE



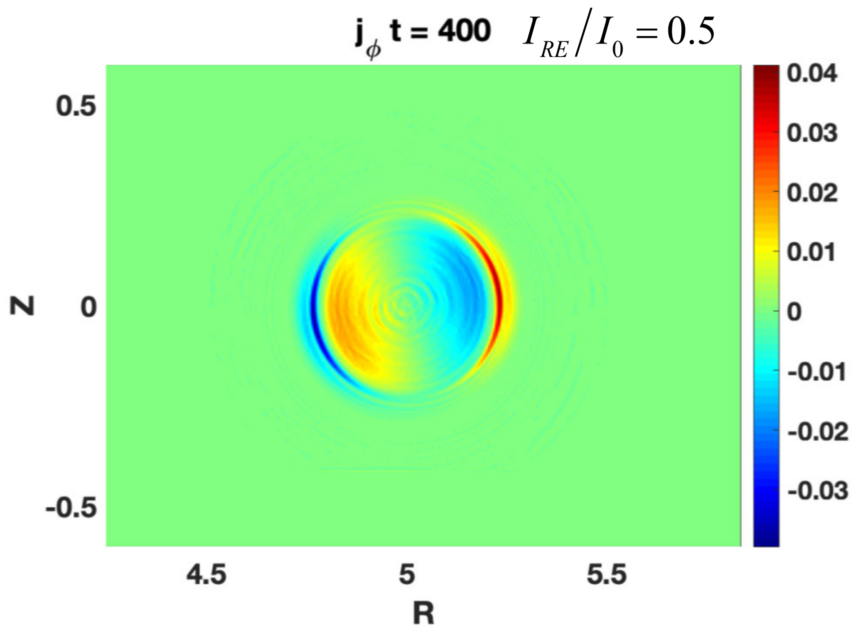
- The magnetic field and pressure perturbations are similar with the m=1, n=1 kink mode with out RE

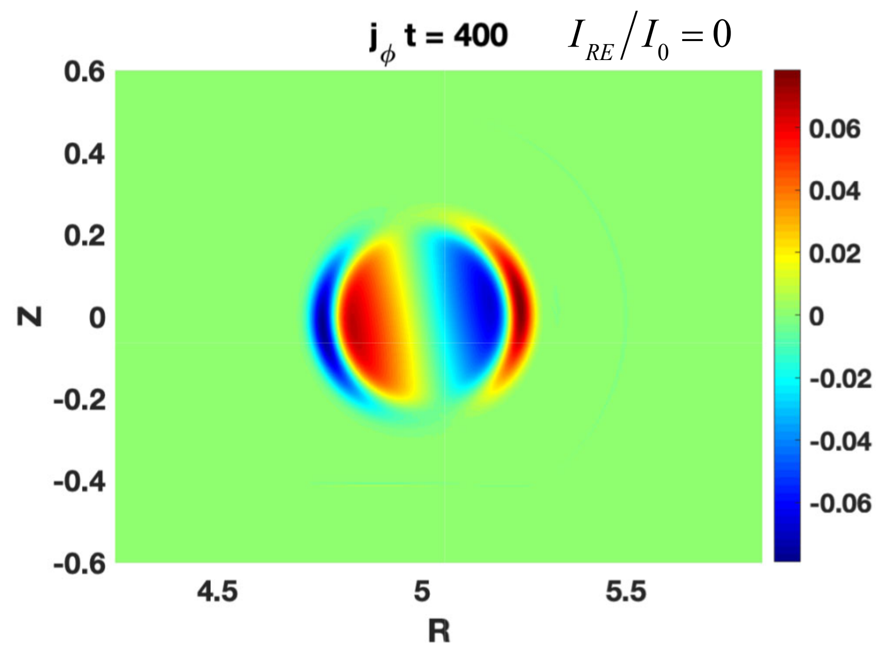
2D results of kink mode with RE



- The perturbed runaway density and toroidal current are peaked around the q=1 rational surface and the perturbed toroidal current is proportional to the perturbed RE density. $\delta J_\phi \sim \delta J_{RE} = -e\delta n_{RE} c$

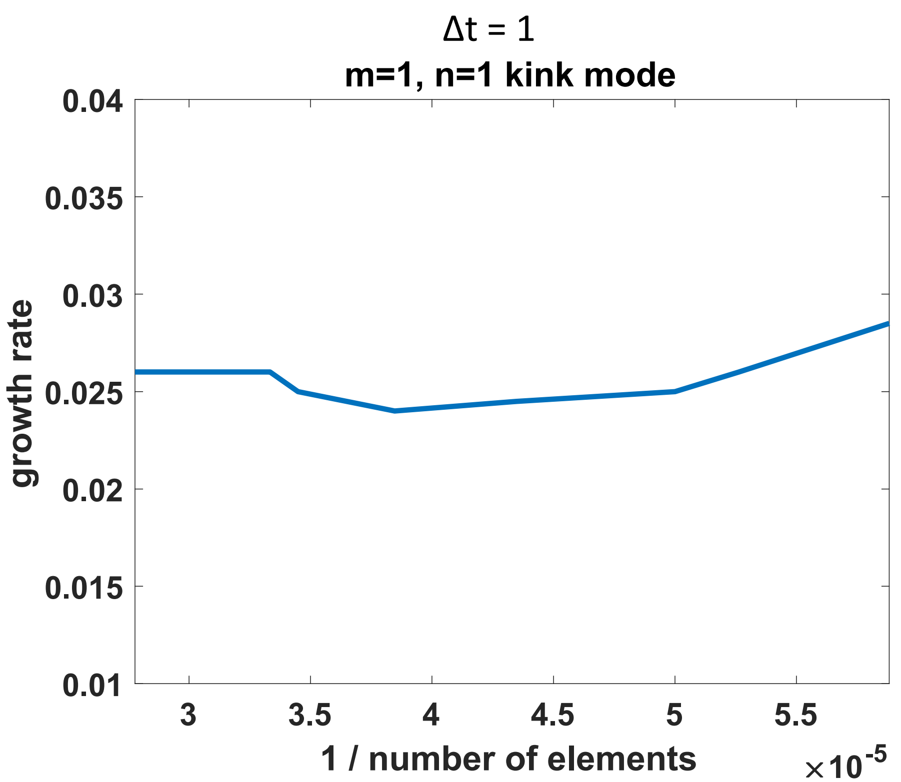
Perturbed toroidal current with and with out runaways

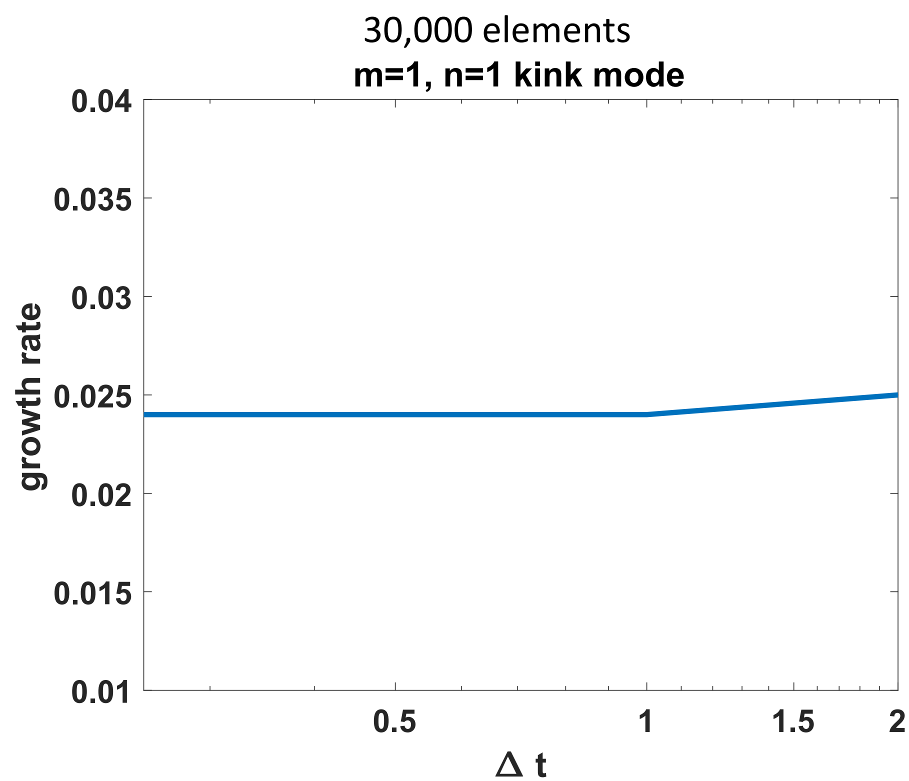




- The perturbed runaway current dominates the perturbed toroidal current.
- The current around the rational surface became more peaked and lower than without runaways.

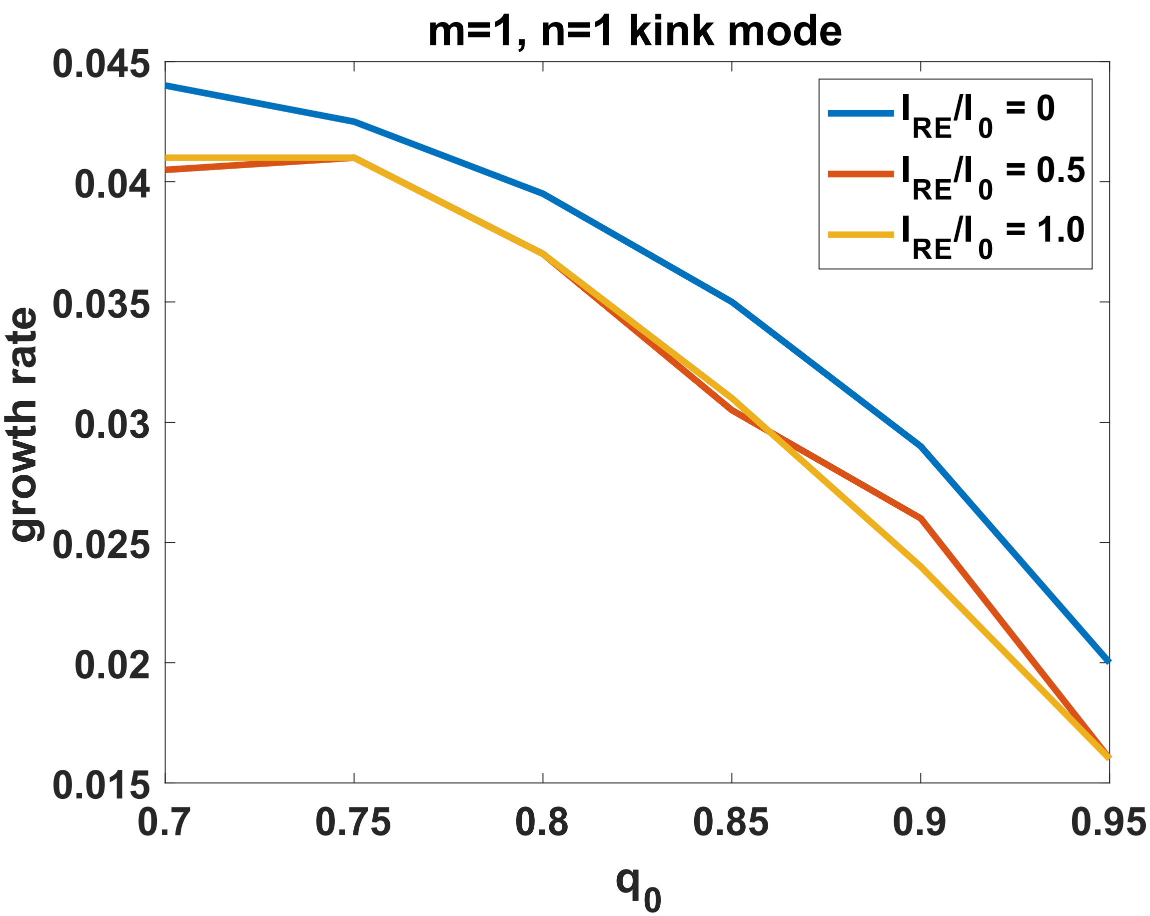
Numerical convergence study





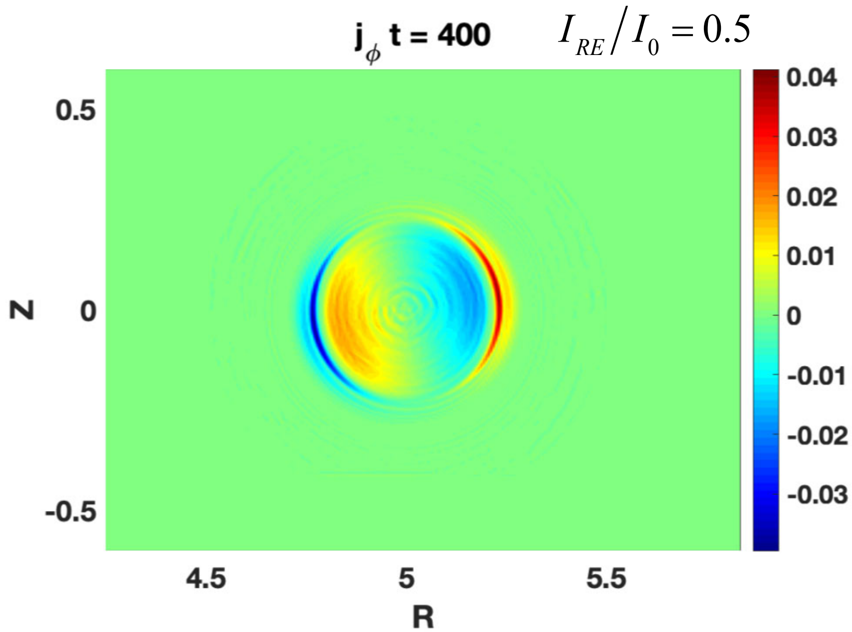
- The numerical convergency is pretty good when the number of elements more than 30,000.
- The numerical convergency is perfect when $\Delta t < 2$.

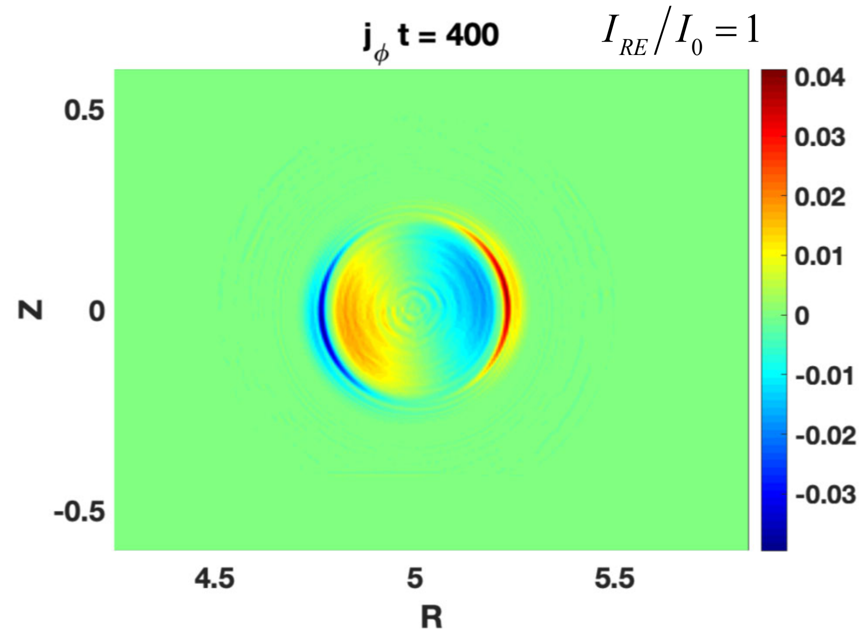
Linear growth rate of kink mode with different q profile



- Runaways reduced the growth rate of kink mode for different q profiles.
- The equilibrium runaway current ratio does not change the kink mode growth rate very much.

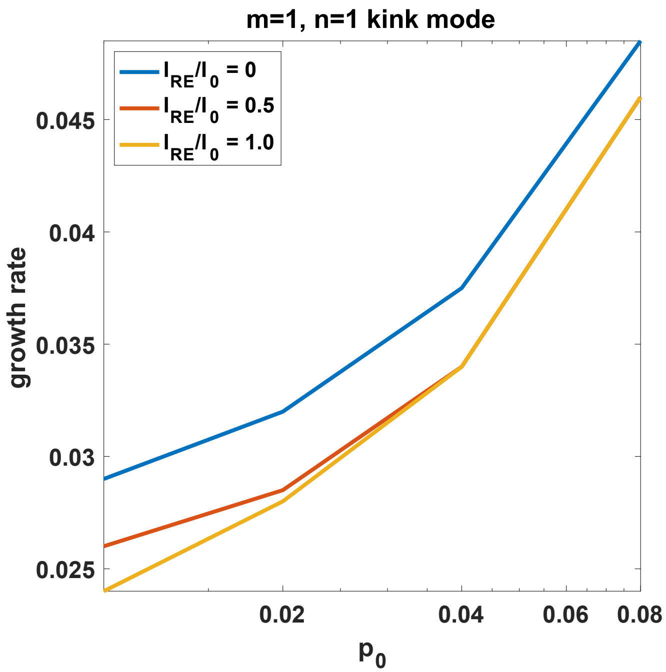
Toroidal current perturbation with different runaway current ratio

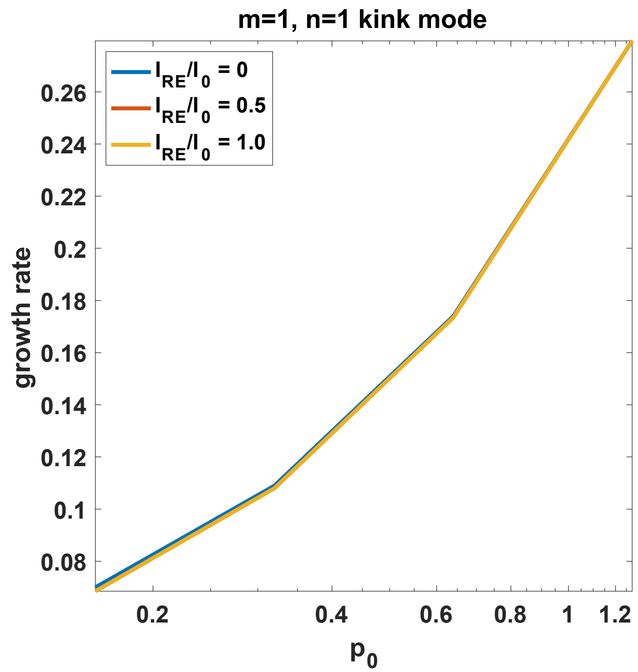




- The toroidal current perturbation does not change with the increasing of the equilibrium runaway current ratio.
- That is the reason of the runaway current ratio does not change the growth rate very much.

Linear growth rate of kink mode with different pressure

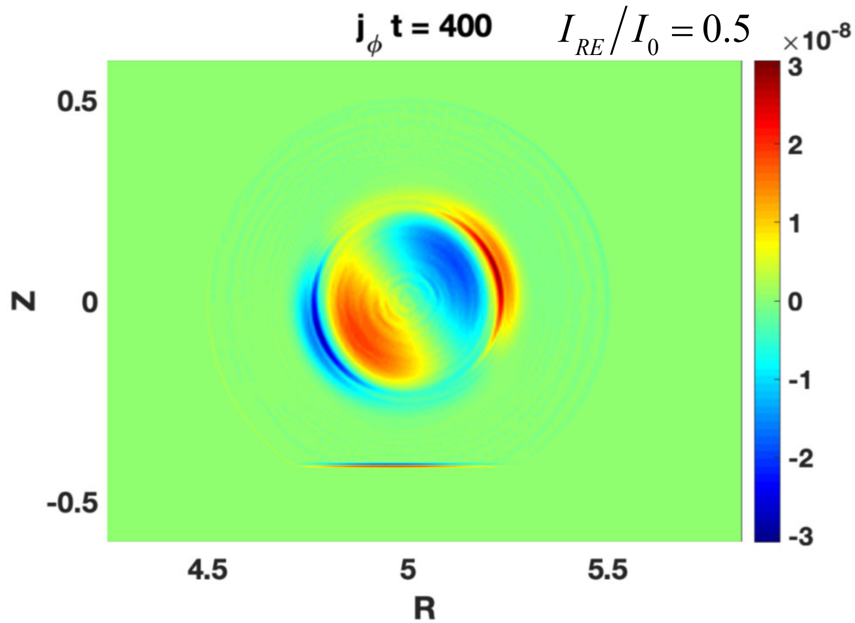


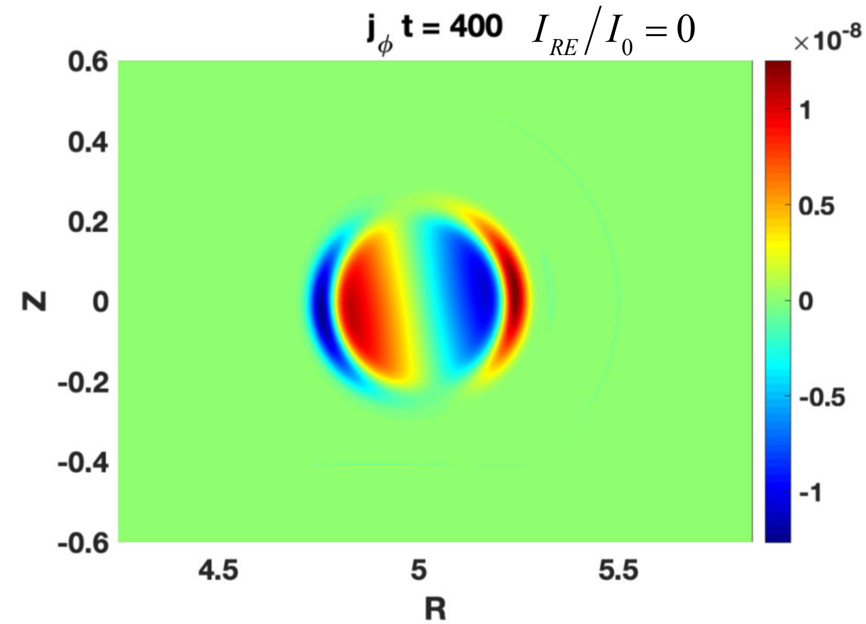


- In lower pressure cases, runaways can affect the mode and the growth rate becomes smaller.
- In higher pressure cases, the runaways will not affect the mode and the growth rate are almost the same with and without equilibrium runaway current component.

- Runaways will affect the mode's growth rate but this effect is not related to the equilibrium runaway current ratio.

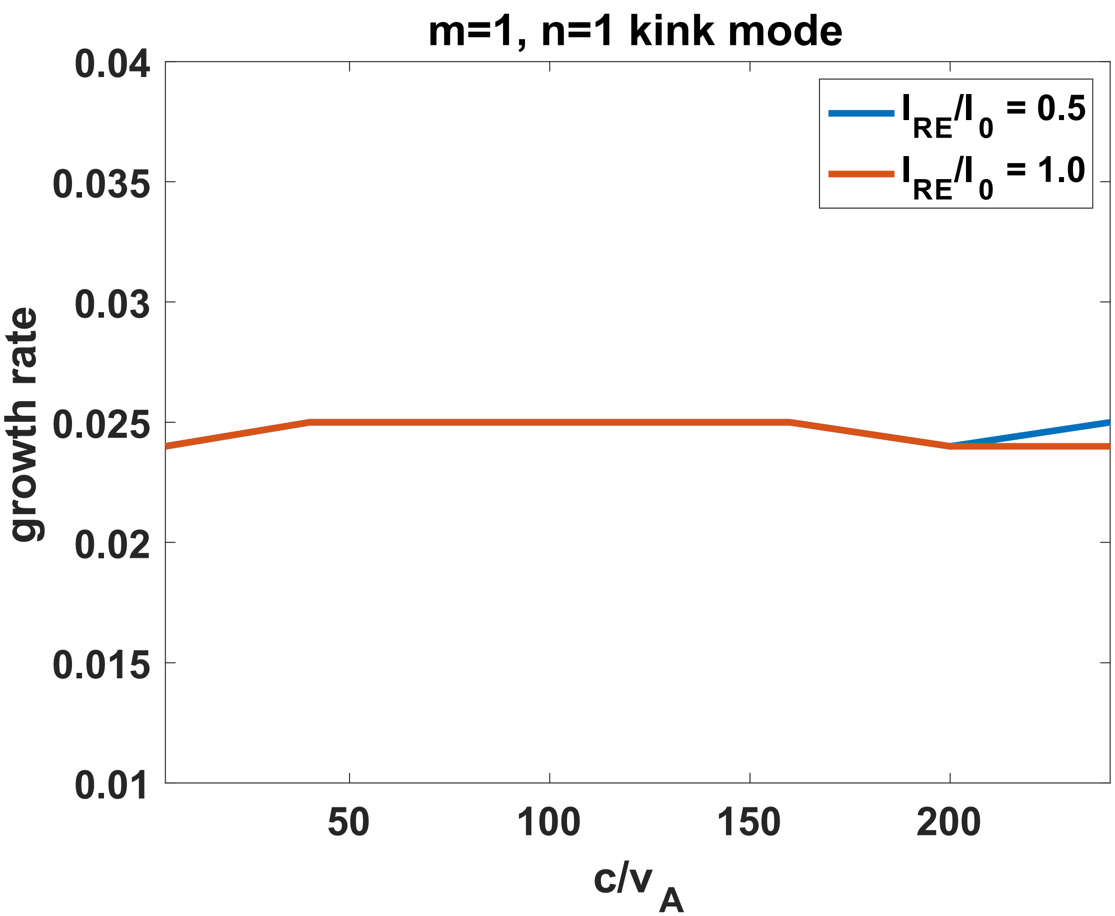
Toroidal current perturbation with different pressure





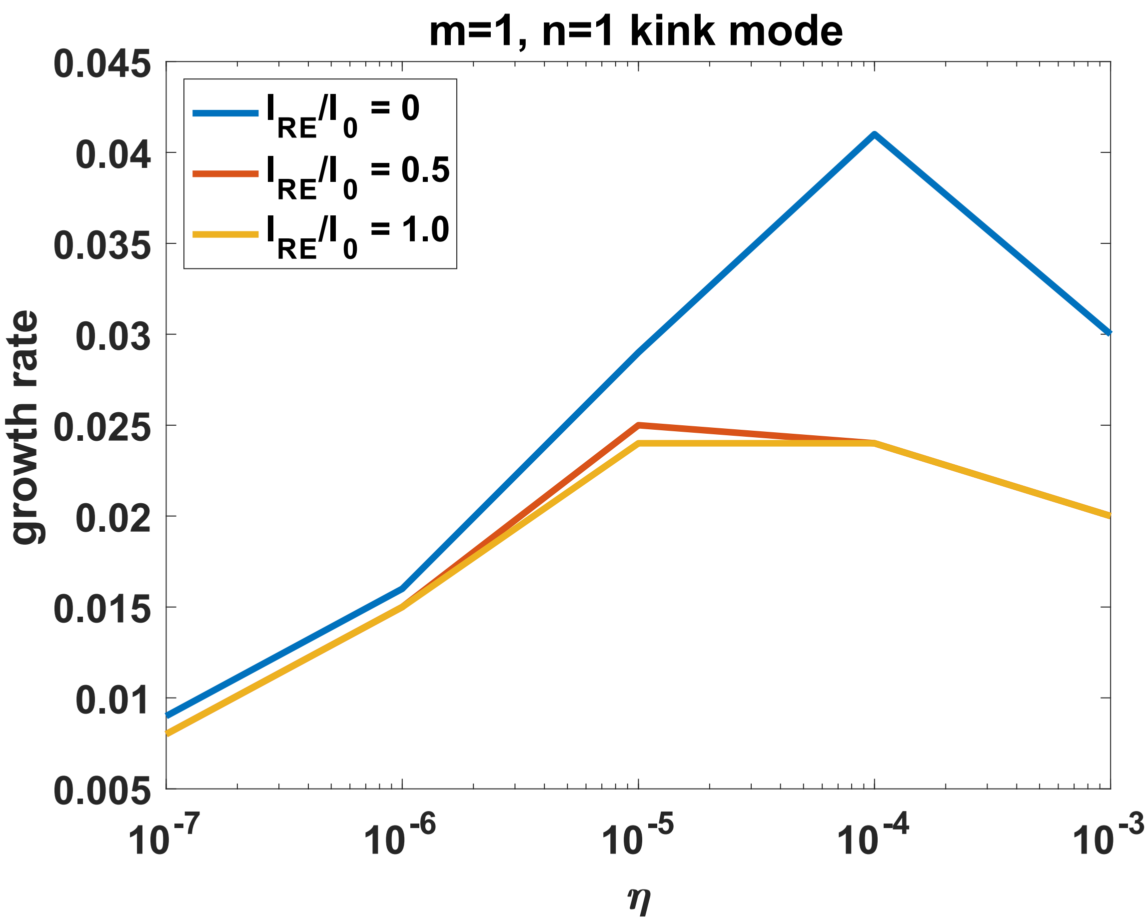
- In higher pressure cases, the runaway current's effect has been weakened and the structure of toroidal current perturbation becomes similar to kink mode without runaways.
- That is the reason of the growth rate are almost the same in higher pressure cases.

Linear growth rate of kink mode with different runaway velocity



- The velocity of runaway electrons does not affect the kink mode's growth rate.

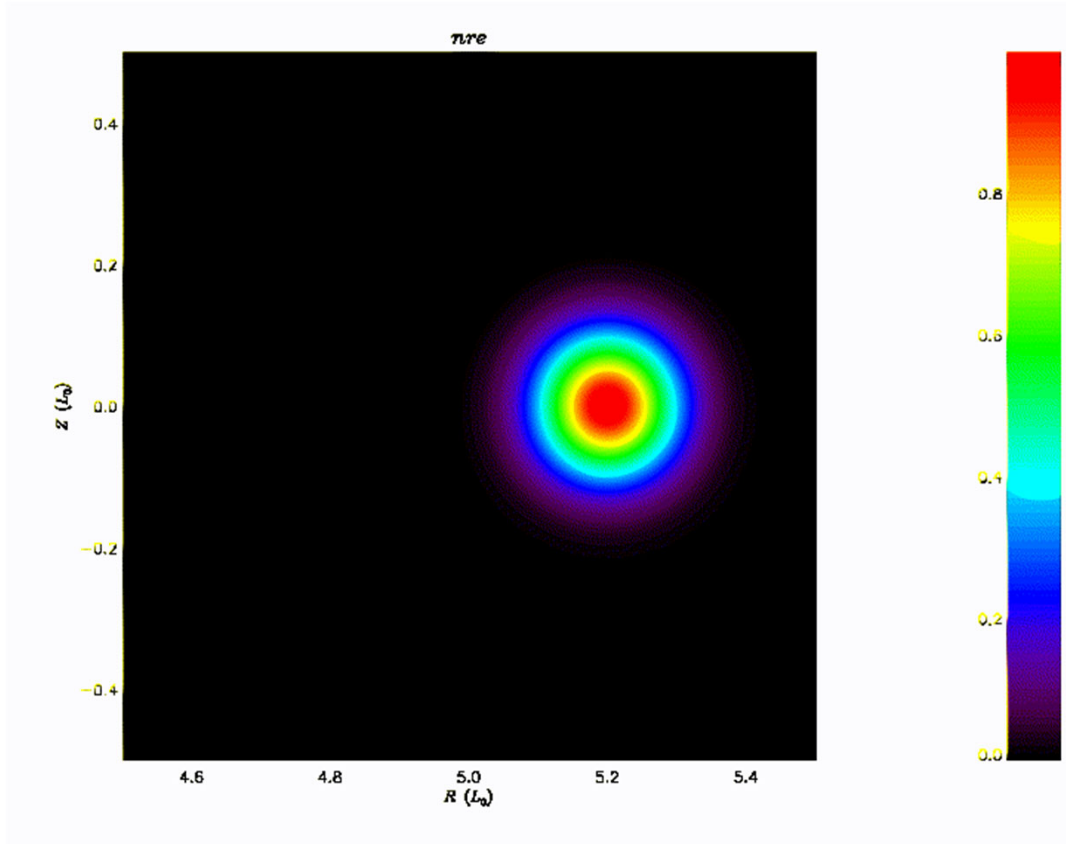
Linear growth rate of kink mode with different resistivity



- The damping effect of runaways on the kink mode growth rate is stronger at higher resistivity region.
- Increasing the equilibrium runaway current ratio does not change the kink mode growth rate.

2D nonlinear convection of RE

- Circular cross section of ϕ=0
- q=1.1 everywhere
- RE density is initialized like a gaussian localized in one side.
- Fully implicit time advance (backward Euler).
- The runaway electron density moves along the magnetic field and also has a parallel diffusion, so that it become uniform at every magnetic surfaces.



4. Summary and future works

- The perturbed toroidal current of the kink mode will be peaked around the q=1 rational surface by the influence of runaways, and that is proportional to the perturbed runaway electron density.

- The growth rate of the kink mode with RE is convergence at elements over 30,000 and $\Delta t < 2$.

- Runaways could make the kink mode's growth rate smaller at low pressure region with different q profile. This damping effect become stronger when the resistivity increase. The runaway current ratio and runaway velocity do not have any significant effect on m=1, n=1 kink modes.

- We will extend this work to nonlinear and 3D as well as m=2, n=1 kink mode in the future. We are also investigating a more implicit time advance, coupling the RE equation with the magnetic field equation. Also investigating the difference between the convection and diffusion advance of the RE density.