Energy balance during pellet assimilation

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Motivation

- Injection of shattered pellets is a critical part of the envisaged ITER disruption mitigation system.

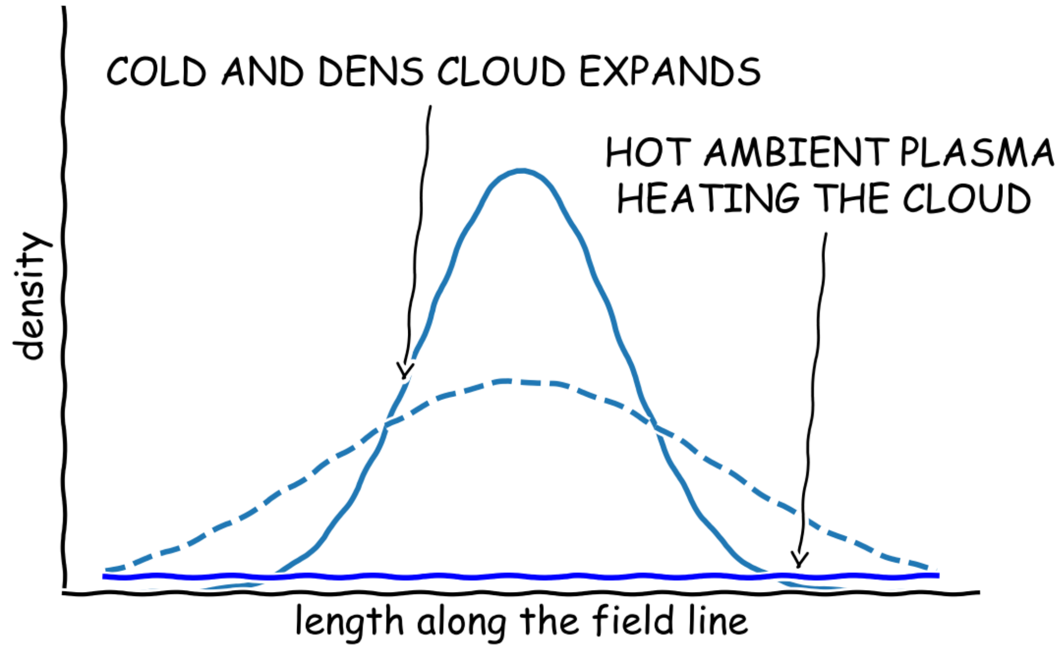
- Rapid deposition of a large amount of material is expected to result in a controlled cooling of the entire plasma. Unlike in the case of uniform gas injection, a considerable transfer of thermal energy from plasma electrons to the injected ions accompanies a localised material injection, due to ambipolar parallel expansion of the pellet produced plasmoid.

- The present work quantifies this energy transfer.

- Not considered: self-consistent ablation process, plasmoid drifts

Pellet cloud formation

- Consider a fast* pellet which crosses a field line
- Only a thin layer (~ mean free path) is heated and evaporated by the plasma heat flux
- This evaporated over-dense cloud initially expands with the ion sound speed in three dimensions. 3D expansion stops when the cloud is ionised and its hydrodynamic pressure becomes lower than the magnetic pressure
- Then the cloud expands along the field line



*A case of slow pellets is considered in [Arnold, A.M., Aleynikov, P., Helander, P., Self-similar expansion of a plasmoid supplied by pellet ablation, Accepted to PPCF (2021)]

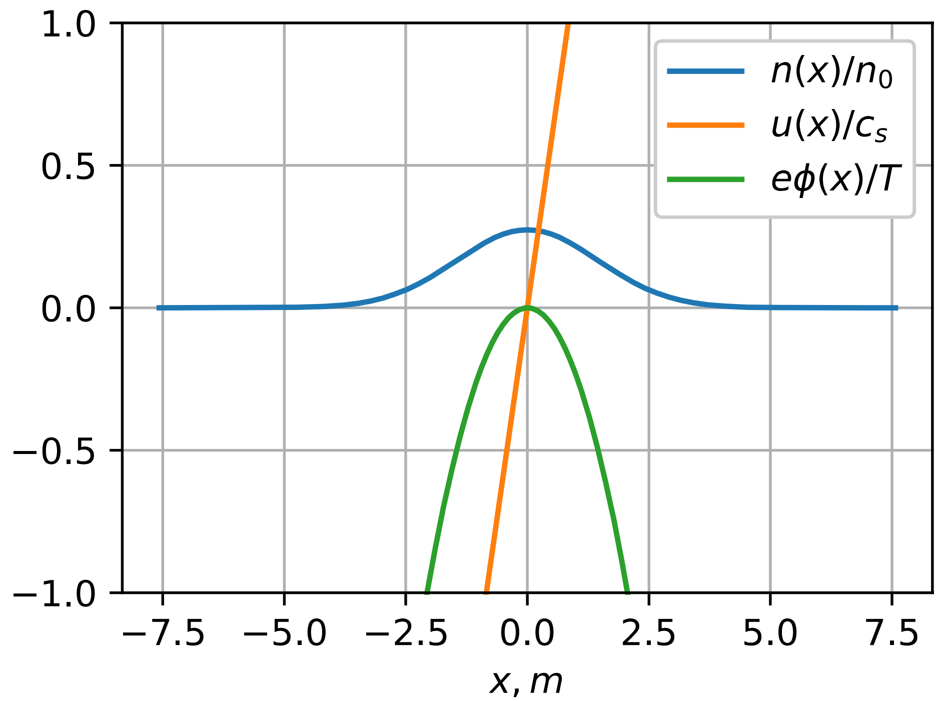
Expansion of a heated plasma into vacuum

- In the simplest case of cold plasmoid ions and constantly heated electrons the expansion is governed by the hydrodynamic equations:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0,$$

$$m_i\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -T(t)\frac{\partial \ln n}{\partial x},$$

$$\frac{d}{dt}\int_{-\infty}^{\infty}\left(\frac{3nT}{2} + \frac{m_i n u^2}{2}\right)dx = \int_{-\infty}^{\infty} Q(t) n\, dx$$



Self-similar solution

- with a solution [1]:

$$n(x,t) = n_0\sqrt{\frac{3m_i}{8\pi\tau t^3}}\ \exp\left(-\frac{3m_i x^2}{8\tau t^3}\right),$$

$$u(x,t) = \frac{3x}{2t},$$

$$T(t) = t\tau, \quad \tau = \frac{1}{3n_0}\int_{-\infty}^{\infty} nQ\, dx\,.$$

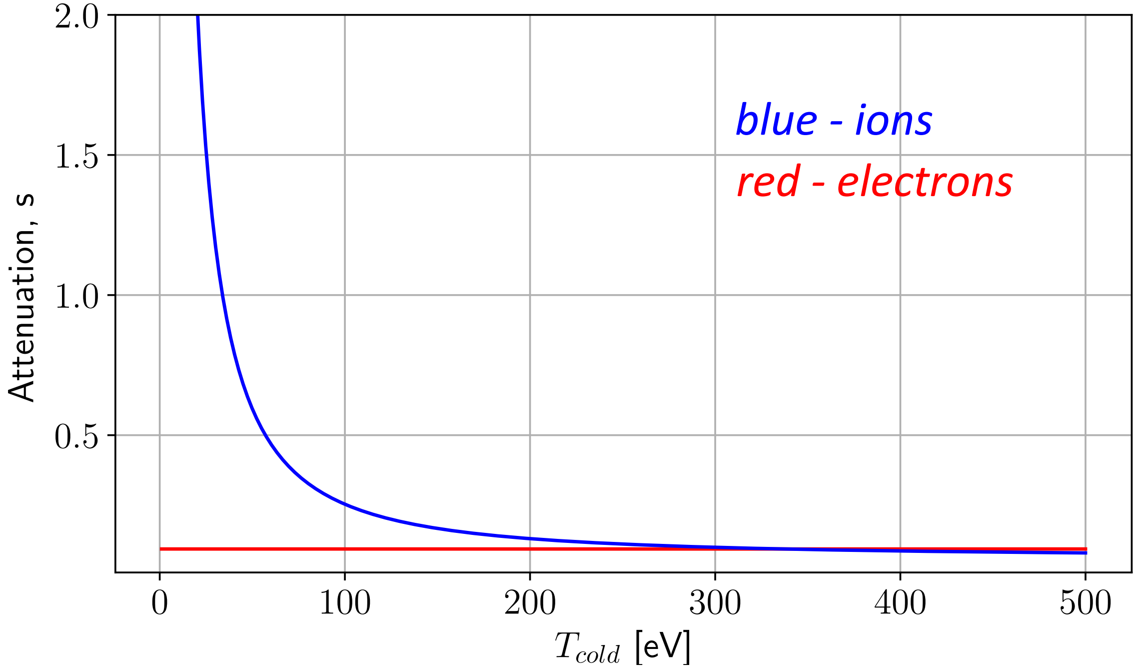
Half of the energy transmitted to the plasmoid by the ambient plasma is in the kinetic energy of the plasmoid ions

[1] Aleynikov, P., Breizman, B., Helander, P., Turkin, Y. 2019 Plasma ion heating by cryogenic pellet injection, Journal of Plasma Physics, **85**, 905850105.

Attenuation of the ambient plasma

- The cold plasmoid opacity is different for the ambient hot plasma electrons and ions.
- Because the stopping power of the hot ions on cold electrons is very high, the cold plasmoid is not transparent for the ambient ions before it is heated.
- Modelling shows that plasmoid pressure quickly becomes higher than the ambient (in under 1 μs). As plasmoid expands, its pressure starts to decrease and becomes comparable to the ambient pressure.
- The plasmoid becomes transparent to the ambient ions when it reaches 100eV (within a few μs).

Attenuation $s = \int_{-L/2}^{L/2} \frac{1}{\lambda} dx$ of 5 keV plasma in $2 \cdot 10^{22} m^{-2}$ plasmoid.



More complete model for early stage

- In order to capture the early stage of expansion accurately we developed a fluid + kinetic Lagrangian code [A. Runov, P. Aleynikov, A. M. Arnold, B. N. Breizman, and P. Helander, 2021 Modelling of parallel dynamics of a pellet produced plasmoid, Accepted to JPP]
- In the model the plasmoid is treated with the Braginskii equations (two temperatures)
- Slowing down of the incident ambient particles within the plasmoid is treated with a kinetic equation

$$v_{||}\frac{\partial f}{\partial x} = -f\nu^s,$$

where ν^s is the slowing-down frequency (Eq. (18.5) from [Trubnikov 1965]).

- Kinetic momentum and energy sources in Braginskii equations are:

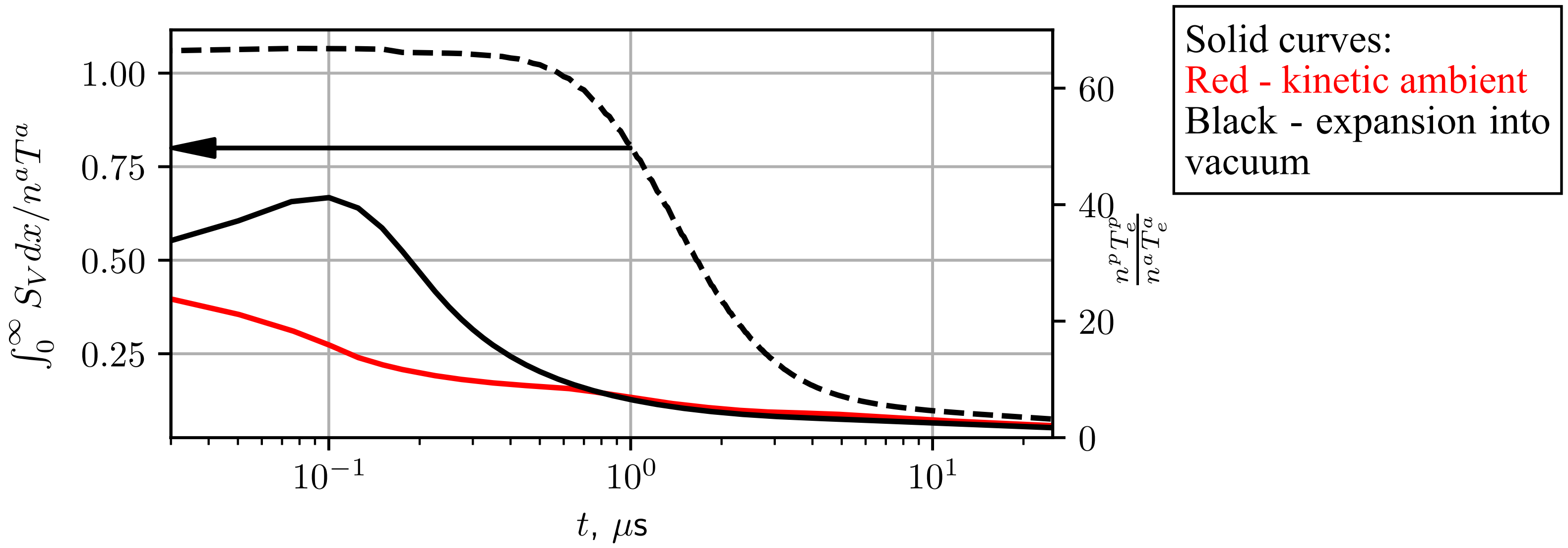
$$S_V = m_e \int \left(f_e\nu_s^{ee} + f_e\nu_s^{ei}\right) v_{||}d^3v + m_i \int \left(f_i\nu_s^{ii} + f_i\nu_s^{ie}\right) v_{||}d^3v$$

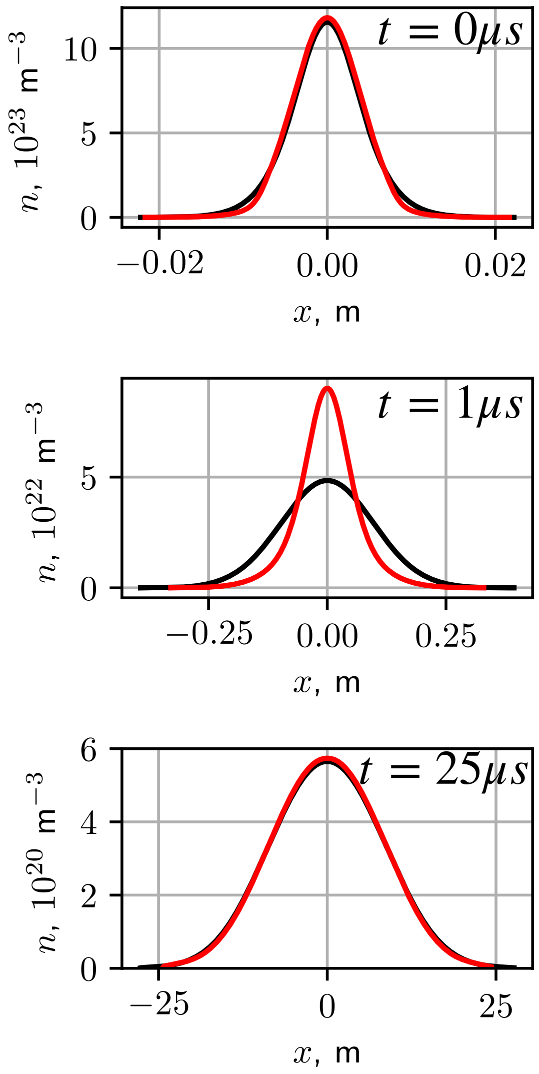
$$S_T^e = \frac{1}{2}\int \left(m_e f_e\nu_s^{ee} + m_i f_i\nu_s^{ie}\right) v^2 d^3v$$

$$S_T^i = \frac{1}{2}\int \left(m_i f_i\nu_s^{ii} + m_e f_e\nu_s^{ei}\right) v^2 d^3v$$

Compare complete and simplified models

Evolution of the integrated momentum source $\int_0^\infty S_V dx$ normilized to the ambient hydrodynamic pressure $n^a T^a$ (dashed curve, left axis). Ratio of the plasmoid electron pressure to the ambient electron pressure (solid curves, right axis).





- Initially, when the plasmoid is not transparent to the ions, the total momentum source is approximately equal to the ambient ion hydrodynamic pressure.
- As the plasmoid is heated, the ion mean free path increases and the friction forces from the left and the right ambient fluxes cancel each other.
- At later stages of expansion the solutions of complete and simplified modes agree very well. Detailed study is in [Runov et al. 2021]

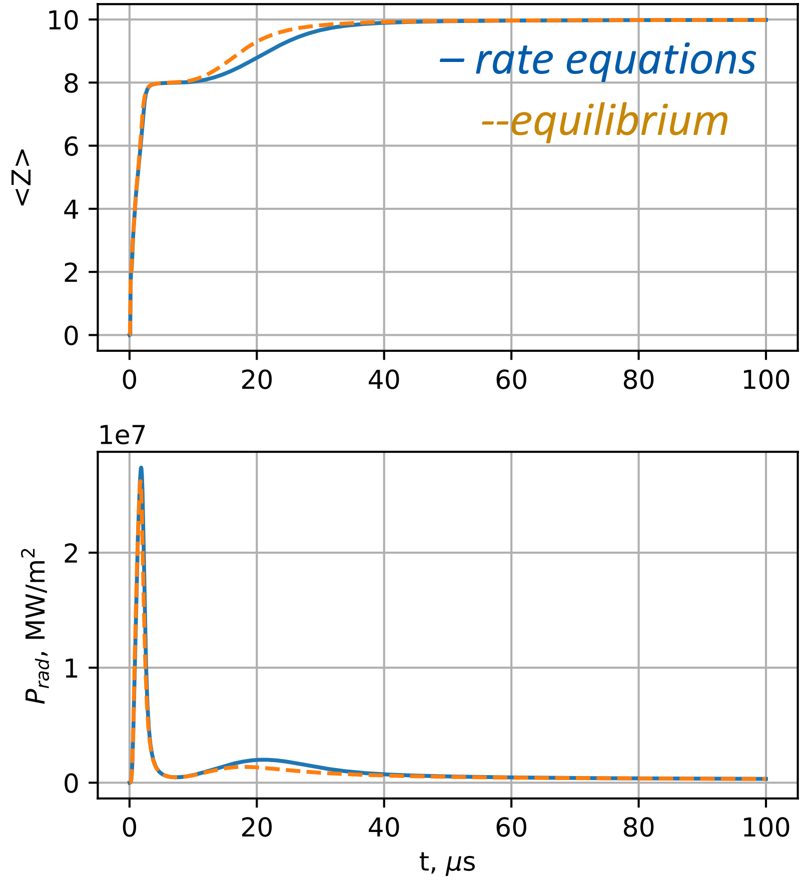
Ionization balance in a heated plasmoid

- Gradual heating of a cold and dense plasmoid ensures that ionization distribution follows closely the collisional radiative equilibrium distribution [ADAS].

- We solve a set of time-dependent ionization-recombination rate equations assuming temperature and density dependence given by the self-similar equations: $n \sim N_l t^{-\frac{3}{2}}$ and $T = \tau t$ (radiation is ignored).

- Despite a quick temperature increase (2 keV by 100 μs) the mean charge state of the time-dependent solution follows closely the equilibrium charge state. The total radiated energy (integral over 100 μs) of the time-dependent solution is only 5% higher than the equilibrium case.

Average charge state and the corresponding volumetric radiation power in a case of a Neon plasmoid ($N_l = 10^{22} m^{-2}$) in an ambient plasma with temperature 10 keV and density $n^a = 10^{20} m^{-3}$.



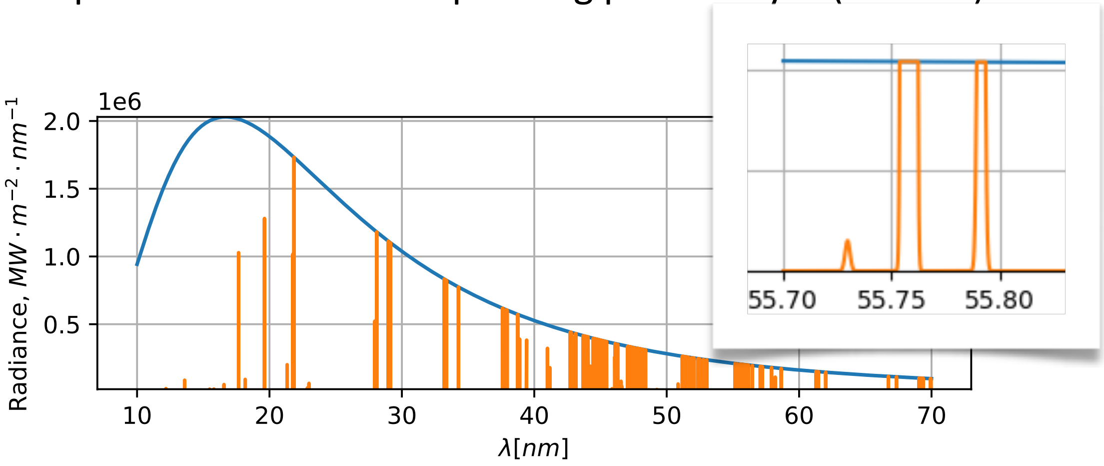
Radiation losses

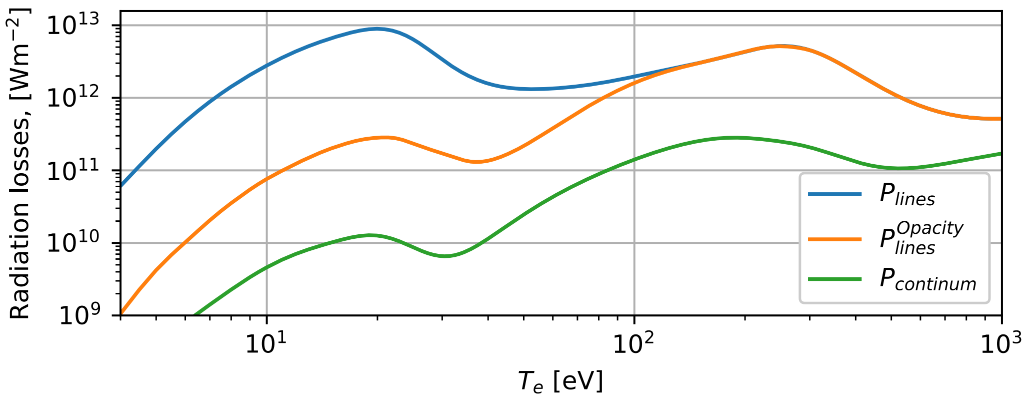
- In plasma with high-Z impurities radiation is dominated by lines.
- The mean free path of a resonant photon in the line radiation process can be significantly shorter than the width of the plasmoid (~10 cm).
- Upper estimate: spectral radiance of any radiation cannot exceed that of a black body. We cut every line at Planck's law level, assuming Doppler broadening mechanism.

$$P_{rad} \approx \int \min\left(\sum_l n_i^{k_l} n_e \varepsilon_l \frac{hc}{\lambda} P_l(\lambda) r_p, B(\lambda) \right) d\lambda$$

- The resulting radiation losses are reduced significantly for T < 100 eV.
- NB. Collisional radiative model is not applicable for high densities (lines trapping is not accounted for).

Model spectrum intensity (from a unit surface) of a 10 cm slab of Argon plasma with $n_i = 10^{22} m^{-3}$ at 15 eV (top). Radiated power loss of a corresponding plasma layer (bottom).





Governing equations

- Expansion of a heated plasmoid is governed by the following system of hydrodynamic equations

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = S\delta(x)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{Z(T)T}{m_i}\frac{\partial \ln n}{\partial x} = 0$$

$$\frac{3}{2}\frac{d}{dt}\int nZ(T)Tdx + \frac{m_i}{2}\frac{d}{dt}\int nu^2dx = P_{heating}(t) - P_{rad}(t)$$

- which admits a self-similar Ansatz $n = N_l\sqrt{\frac{a(t)}{\pi}}\ \exp\left(-a(t)x^2\right), u = b(t)x$ for $S = 0.$

- $P_{heating}$ is given by the collisional energy exchange between Maxwellian populations (ambient and plasmoid)

- The flux surface temperature evolution is approximated using $\frac{3}{2}\int_0^A n^a\dot{T}^a dx = -Q_{heating}$
  where A is the field line length. We assume full coverage of the flux surface by the expanding plasmoid. Cases of short connection length on rational magnetic surfaces are ignored.

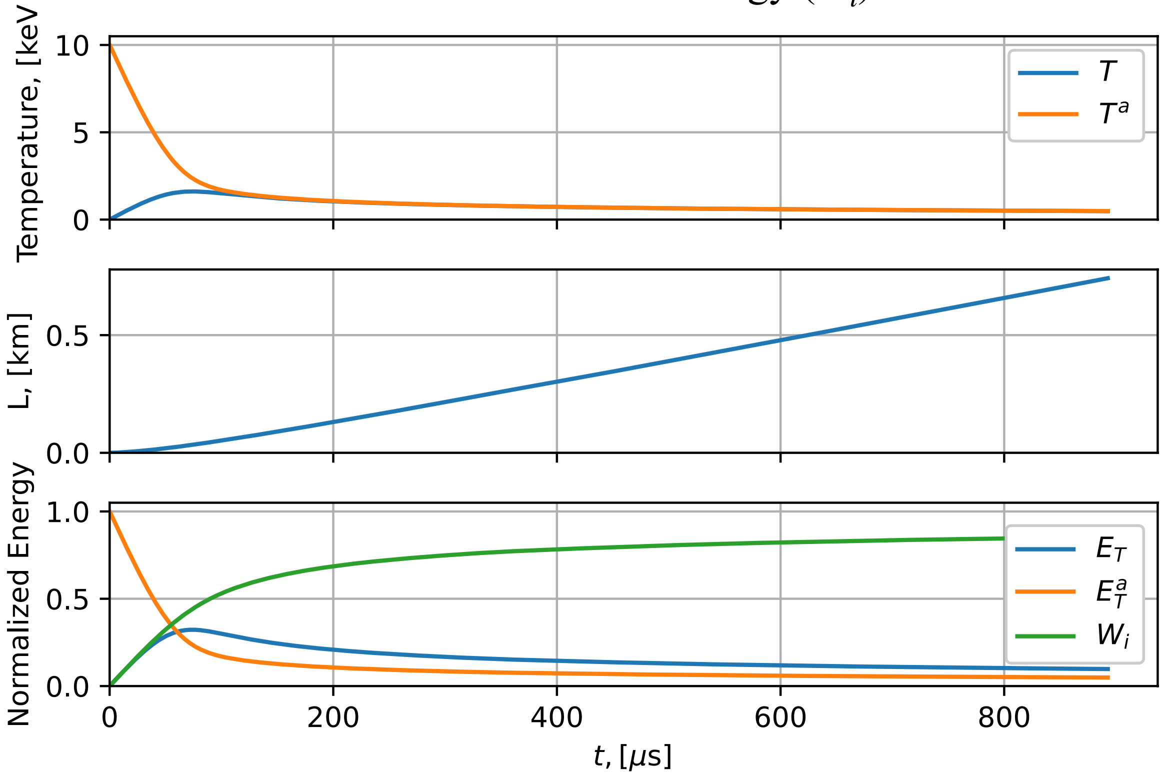
Expansion of a deuterium plasmoid

- We first consider deuterium plasmoid in $T^a = 10$ keV, $n^a = 10^{20} m^{-3}$, $N_l = 1.5 \cdot 10^{23} m^{-2}$ and $r_p = 0.3$ m which corresponds to **2x** of the pre-pellet density on a flux surface with $R = 6$ m, $r_a = 1$ m.

- This calculation is stopped when plasmoid covers the entire flux surface, by which time $W_i = 0.85$ implying that the majority of the pre-pellet electron thermal energy has been transferred to the ions. The corresponding electron temperature is $T = T^a = 480$ eV. Note that assuming a uniform injection the electron and ion temperatures after dilution would be 3333 eV.

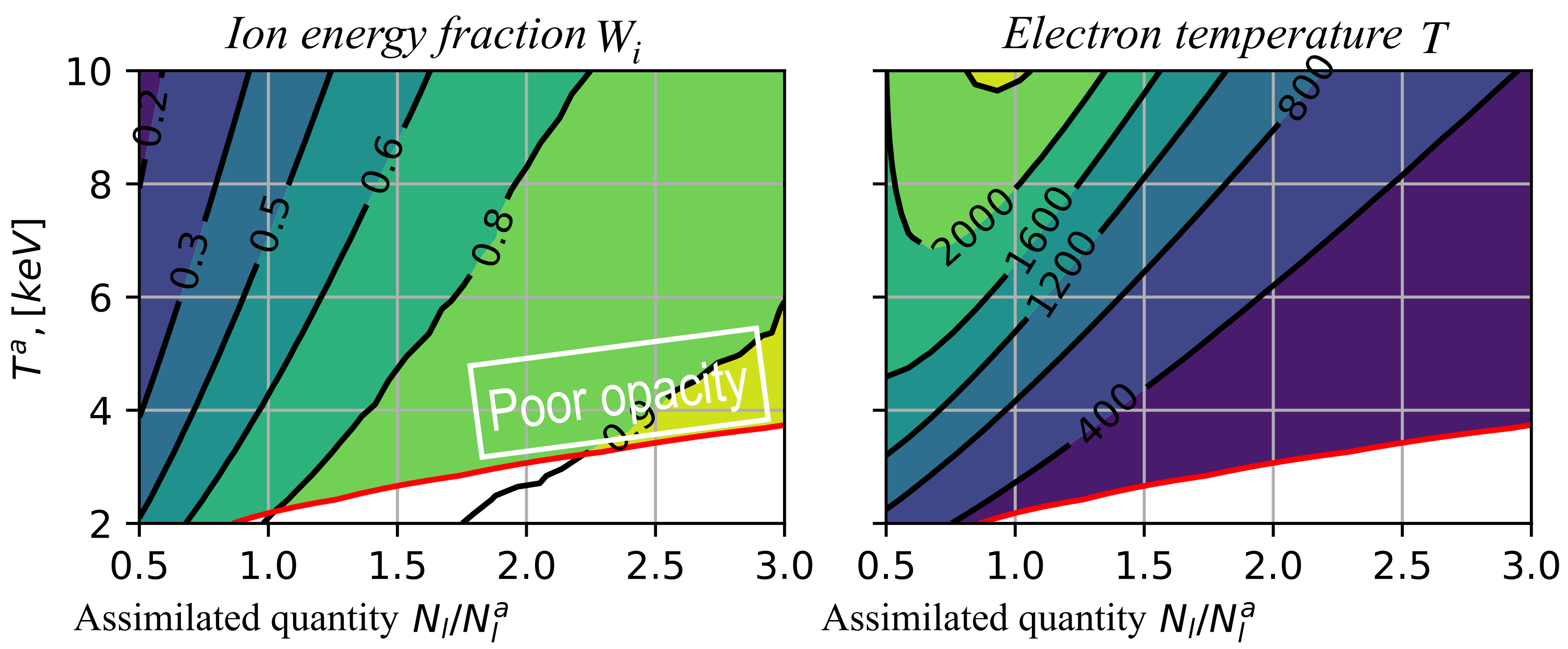
- Radiation is negligible.

Evolution of plasmoid (T) and ambient electron temperatures (T_a), plasmoid length (L), normalized plasmoid thermal energy (E_T), normalized ambient electron thermal energy (E_T^a) and the normalized ion kinetic energy (W_i).



Deuterium energy conversion fraction

Ultimate ion energy W_i ***(left)*** *and electron temperature T* ***(right)*** *as a function of pre-pellet temperature and the amount of assimilated deuterium.*



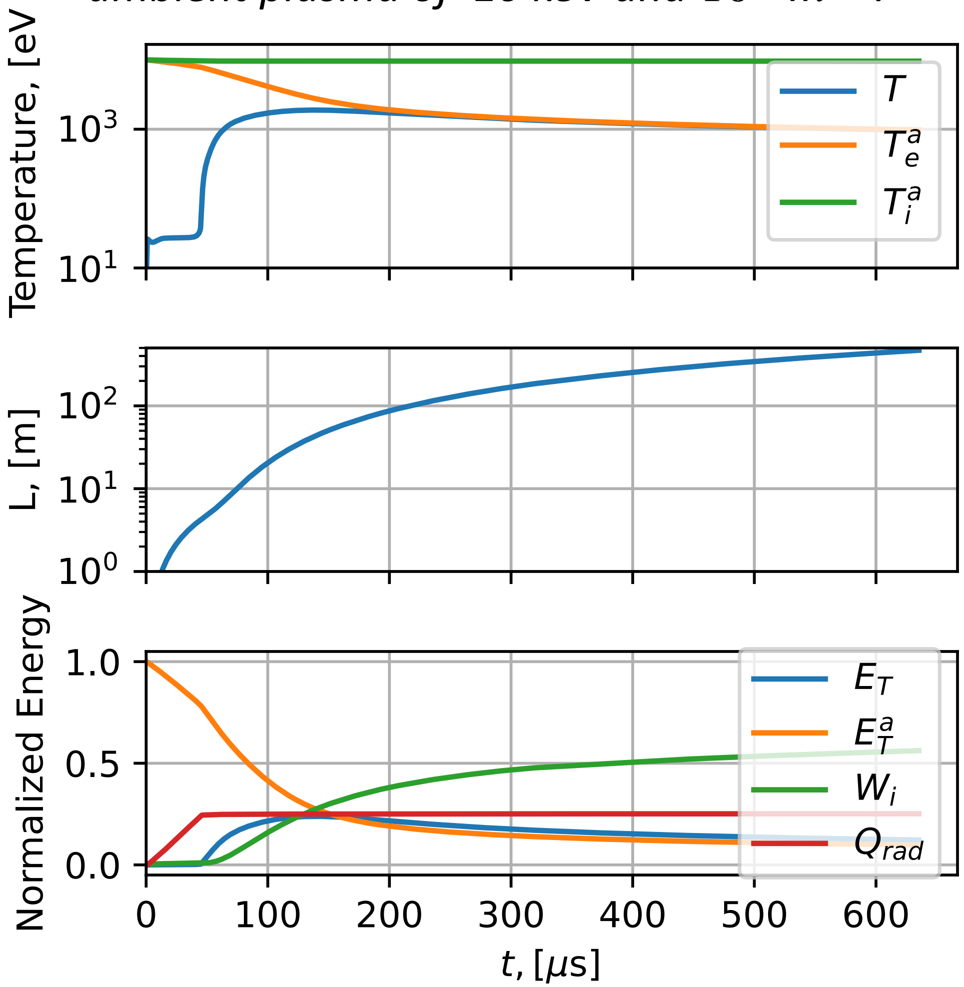
- The region where plasmoid is not transparent for the ambient electrons (attenuation $s > 1$) is marked with the red curve, close to this region our model is not valid. A hydrodynamic description of both the plasmoid and the ambient plasma is appropriate for $s \gg 1$.

Expansion of a neon plasmoid

- Temperature stays at ≈20 eV initially as the strong line emission radiates the incoming energy (despite the plasmoid is not transparent for lines).

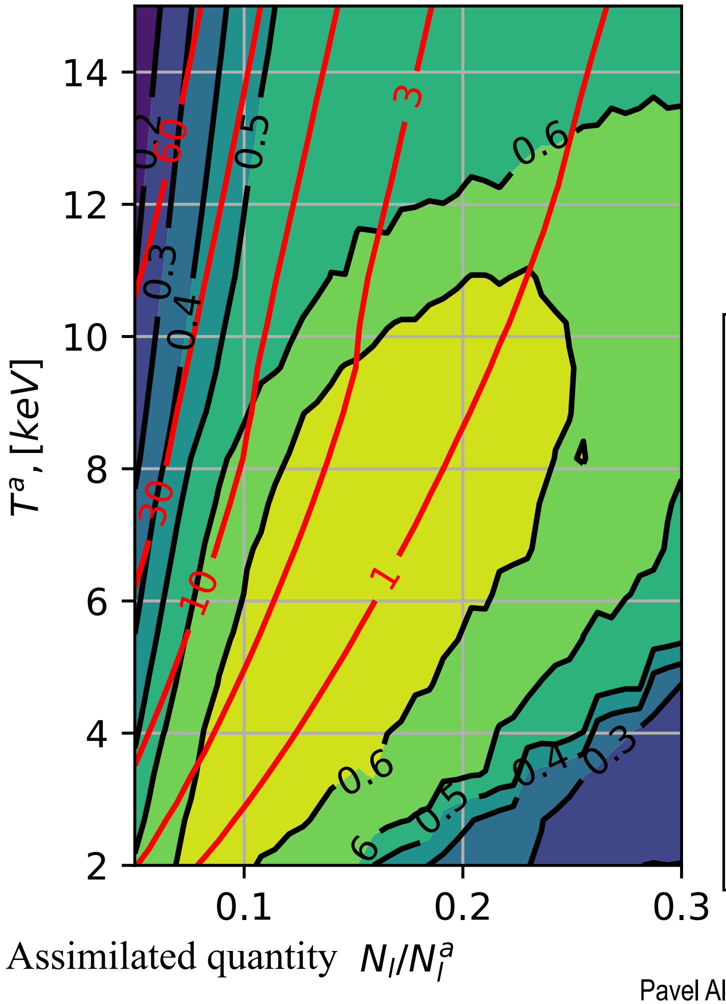
- After expanding to about 10 m, the radiation losses decrease (due to density decrease $\sim t^{-3/2}$) so heating and expansion accelerate. Ultimately the ions gain over 50% of the pre-pellet electron thermal energy.

Evolution of a neon plasmoid with $N_l = 10^{22} m^{-2}$ in an ambient plasma of 10 keV and $10^{20} m^{-3}$.



Neon energy conversion fraction

Ultimate ion energy as a function of pre-pellet temperature and the amount of assimilated neon atoms.
Red contours indicate ambient ions thermalization time in a post pellet plasma in ms.



- The ambipolar energy transfer accounts for up to 60% of the electron thermal energy.
- The remainder is radiated in the beginning.
- Ion-electron thermalization time in a post pellet plasma is short (due to low Te). Ions contribute to TQ dynamics.

Summary

- Significant transfer of pre-quench electron thermal energy to the injected ions is expected for the disruption mitigation pellets.

- The remainder of the energy is radiated by a dense plasmoid during expansion, in spite of the line emission trapping.

- The ion energy and the energy transferred to the injected ions are expected to be radiated on a longer timescale after homogenization.