Energy balance during pellet assimilation

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Motivation

• Injection of shattered pellets is a critical part of the envisaged ITER disruption mitigation system.

• Rapid deposition of a large amount of material is expected to result in a controlled cooling of the entire plasma. Unlike in the case of uniform gas injection, a considerable transfer of thermal energy from plasma electrons to the injected ions accompanies a localised material injection, due to ambipolar parallel expansion of the pellet produced plasmoid.

• The present work quantifies this energy transfer.

• Not considered: self-consistent ablation process, plasmoid drifts
Pellet cloud formation

- Consider a fast* pellet which crosses a field line

- Only a thin layer (~ mean free path) is heated and evaporated by the plasma heat flux

- This evaporated over-dense cloud initially expands with the ion sound speed in three dimensions. 3D expansion stops when the cloud is ionised and its hydrodynamic pressure becomes lower than the magnetic pressure

- Then the cloud expands along the field line

* A case of slow pellets is considered in [Arnold, A.M., Aleynikov, P., Helander, P., Self-similar expansion of a plasmoid supplied by pellet ablation, Accepted to PPCF (2021)]
Expansion of a heated plasma into vacuum

- In the simplest case of cold plasmoid ions and constantly heated electrons the expansion is governed by the hydrodynamic equations:

\[ \frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nu) = 0, \]
\[ m_i \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = -T(t) \frac{\partial \ln n}{\partial x}, \]
\[ \frac{d}{dt} \int_{-\infty}^{\infty} \left( \frac{3nT^2}{2} + \frac{m_i n u^2}{2} \right) dx = \int_{-\infty}^{\infty} Q(t) ndx \]

- with a solution [1]:

\[ n(x, t) = n_0 \sqrt{\frac{3m_i}{8\pi \tau t^3}} \exp \left( -\frac{3m_i x^2}{8\tau t^3} \right), \]
\[ u(x, t) = \frac{3x}{2t}, \]
\[ T(t) = t\tau, \quad \tau = \frac{1}{3n_0} \int_{-\infty}^{\infty} nQ \, dx. \]

Half of the energy transmitted to the plasmoid by the ambient plasma is in the kinetic energy of the plasmoid ions

Attenuation of the ambient plasma

• The cold plasmoid opacity is different for the ambient hot plasma electrons and ions.

• Because the stopping power of the hot ions on cold electrons is very high, the cold plasmoid is not transparent for the ambient ions before it is heated.

• Modelling shows that plasmoid pressure quickly becomes higher than the ambient (in under 1 µs). As plasmoid expands, its pressure starts to decrease and becomes comparable to the ambient pressure.

• The plasmoid becomes transparent to the ambient ions when it reaches 100eV (within a few µs).

\[
\text{Attenuation } s = \int_{-L/2}^{L/2} \frac{1}{\lambda} \, dx \text{ of 5 keV plasma in } 2 \cdot 10^{22} \text{m}^{-2} \text{ plasmoid.}
\]
More complete model for early stage

- In order to capture the early stage of expansion accurately we developed a fluid + kinetic Lagrangian code [A. Runov, P. Aleynikov, A. M. Arnold, B. N. Breizman, and P. Helander, 2021 Modelling of parallel dynamics of a pellet produced plasmoid, Accepted to JPP]

- In the model the plasmoid is treated with the Braginskii equations (two temperatures)

- Slowing down of the incident ambient particles within the plasmoid is treated with a kinetic equation

\[ v_{\parallel} \frac{\partial f}{\partial x} = -f \nu^s, \]

where \( \nu^s \) is the slowing-down frequency (Eq. (18.5) from [Trubnikov 1965]).

- Kinetic momentum and energy sources in Braginskii equations are:

\[ S_V = m_e \int (f_e \nu_{ee} + f_e \nu_{ei}) v_{\parallel} d^3v + m_i \int (f_i \nu_{ii}^e + f_i \nu_{ie}^e) v_{\parallel} d^3v \]

\[ S_T^e = \frac{1}{2} \int (m_e f_e \nu_{ee} + m_i f_i \nu_{ii}^e) v^2 d^3v \]

\[ S_T^i = \frac{1}{2} \int (m_i f_i \nu_{ii}^i + m_e f_e \nu_{ei}^e) v^2 d^3v \]
Initially, when the plasmoid is not transparent to the ions, the total momentum source is approximately equal to the ambient ion hydrodynamic pressure.

As the plasmoid is heated, the ion mean free path increases and the friction forces from the left and the right ambient fluxes cancel each other.

At later stages of expansion the solutions of complete and simplified modes agree very well. Detailed study is in [Runov et al. 2021]
Ionization balance in a heated plasmoid

- Gradual heating of a cold and dense plasmoid ensures that ionization distribution follows closely the collisional radiative equilibrium distribution [ADAS].

- We solve a set of time-dependent ionization-recombination rate equations assuming temperature and density dependence given by the self-similar equations: $n \sim N_l t^{-\frac{3}{2}}$ and $T = \tau t$ (radiation is ignored).

- Despite a quick temperature increase (2 keV by 100 µs) the mean charge state of the time-dependent solution follows closely the equilibrium charge state. The total radiated energy (integral over 100 µs) of the time-dependent solution is only 5% higher than the equilibrium case.
Radiation losses

- In plasma with high-Z impurities radiation is dominated by lines.

- The mean free path of a resonant photon in the line radiation process can be significantly shorter than the width of the plasmoid (~10 cm).

- Upper estimate: spectral radiance of any radiation cannot exceed that of a black body. We cut every line at Planck’s law level, assuming Doppler broadening mechanism.

\[
P_{\text{rad}} \approx \int \min \left( \sum_{l} n_{l}^{i} n_{e} \varepsilon_{l} \frac{h c}{\lambda} P_{l}(\lambda) r_{p}(\lambda) B(\lambda) \right) d\lambda
\]

- The resulting radiation losses are reduced significantly for \( T < 100 \text{ eV} \).

- NB. Collisional radiative model is not applicable for high densities (lines trapping is not accounted for).
Governing equations

- Expansion of a heated plasmoid is governed by the following system of hydrodynamic equations

\[
\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = S\delta(x)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{Z(T)T}{m_i} \frac{\partial \ln n}{\partial x} = 0
\]

\[
\frac{3}{2} \frac{d}{dt} \int nZ(T)Td\mathbf{x} + \frac{m_i}{2} \frac{d}{dt} \int nu^2 d\mathbf{x} = P_{heating}(t) - P_{rad}(t)
\]

- which admits a self-similar Ansatz \( n = N_l \sqrt{\frac{a(t)}{\pi}} \exp \left( -a(t)x^2 \right) \), \( u = b(t)x \) for \( S = 0 \).

- \( P_{heating} \) is given by the collisional energy exchange between Maxwellian populations (ambient and plasmoid)

- The flux surface temperature evolution is approximated using \( \frac{3}{2} \int_0^A n^a \tilde{T}^a d\mathbf{x} = -Q_{heating} \)

where \( A \) is the field line length. We assume full coverage of the flux surface by the expanding plasmoid. Cases of short connection length on rational magnetic surfaces are ignored.
Expansion of a deuterium plasmoid

- We first consider deuterium plasmoid in $T^a = 10$ keV, $n^a = 10^{20} \text{m}^{-3}$, $N_l = 1.5 \cdot 10^{23} \text{m}^{-2}$ and $r_p = 0.3 \text{ m}$ which corresponds to 2x of the pre-pellet density on a flux surface with $R = 6 \text{ m}$, $r_a = 1 \text{ m}$.

- This calculation is stopped when plasmoid covers the entire flux surface, by which time $W_i = 0.85$ implying that the majority of the pre-pellet electron thermal energy has been transferred to the ions. The corresponding electron temperature is $T = T^a = 480 \text{ eV}$. Note that assuming a uniform injection the electron and ion temperatures after dilution would be 3333 eV.

- Radiation is negligible.

![Graphs showing evolution of plasmoid temperature, length, normalized thermal energies, and ion kinetic energy over time.](image-url)
Deuterium energy conversion fraction

Ultimate ion energy $W_i$ (left) and electron temperature $T$ (right) as a function of pre-pellet temperature and the amount of assimilated deuterium.

- The region where plasmoid is not transparent for the ambient electrons (attenuation $s > 1$) is marked with the red curve, close to this region our model is not valid. A hydrodynamic description of both the plasmoid and the ambient plasma is appropriate for $s \gg 1$.
Expansion of a neon plasmoid

- Temperature stays at \( \approx 20 \text{ eV} \) initially as the strong line emission radiates the incoming energy (despite the plasmoid is not transparent for lines).

- After expanding to about 10 m, the radiation losses decrease (due to density decrease \( \sim t^{-3/2} \)) so heating and expansion accelerate. Ultimately the ions gain over 50\% of the pre-pellet electron thermal energy.

**Evolution of a neon plasmoid with \( N_i = 10^{22} \text{ m}^{-2} \) in an ambient plasma of 10 keV and \( 10^{20} \text{ m}^{-3} \).**
Ultimate ion energy as a function of pre-pellet temperature and the amount of assimilated neon atoms. Red contours indicate ambient ions thermalization time in a post pellet plasma in ms.

- The ambipolar energy transfer accounts for up to 60% of the electron thermal energy.
- The remainder is radiated in the beginning.
- Ion-electron thermalization time in a post pellet plasma is short (due to low Te). Ions contribute to TQ dynamics.
Summary

- Significant transfer of pre-quench electron thermal energy to the injected ions is expected for the disruption mitigation pellets.

- The remainder of the energy is radiated by a dense plasmoid during expansion, in spite of the line emission trapping.

- The ion energy and the energy transferred to the injected ions are expected to be radiated on a longer timescale after homogenization.