New Observations of Magnetic Island Flux Tunneling, Heteroclinic Bifurcation and Seeding by Non-Linear Three-Wave Coupling

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with Todd E. Evans[1],
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Theory and Simulation of Disruptions

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GENERAL ATOMICS

1. **Flux Tunneling Between Magnetic Island Chains**
   L. Bardóczi and T.E. Evans 2021 Nucl. Fusion **61** 074001 (2021)

2. **Magnetic Island Heteroclinic Bifurcation**
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3. **NTM Seeding by Nonlinear Three-Wave Interactions**
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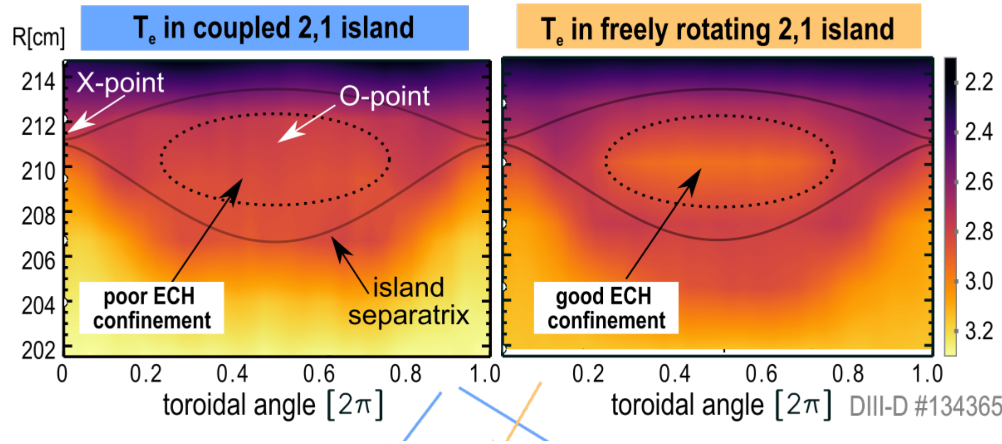
Topological Bifurcation of Magnetic Islands Can Bring/ Explain Challenges for Active Tearing Mode Stabilization

Background/Motivation:

- 2/1 magnetic islands cause disruptions.
- Stabilization with ECCD is a commonly used technique, but it requires good confinement at the O-point.



- Recent theory predicts topological bifurcation within magnetic islands due to **flux tunneling** between adjacent co-rotating islands of different helicity [1]. Never tested in experiments before.

Importance:

- Stabilization of coupled islands by ECCD may be harder (or impossible)

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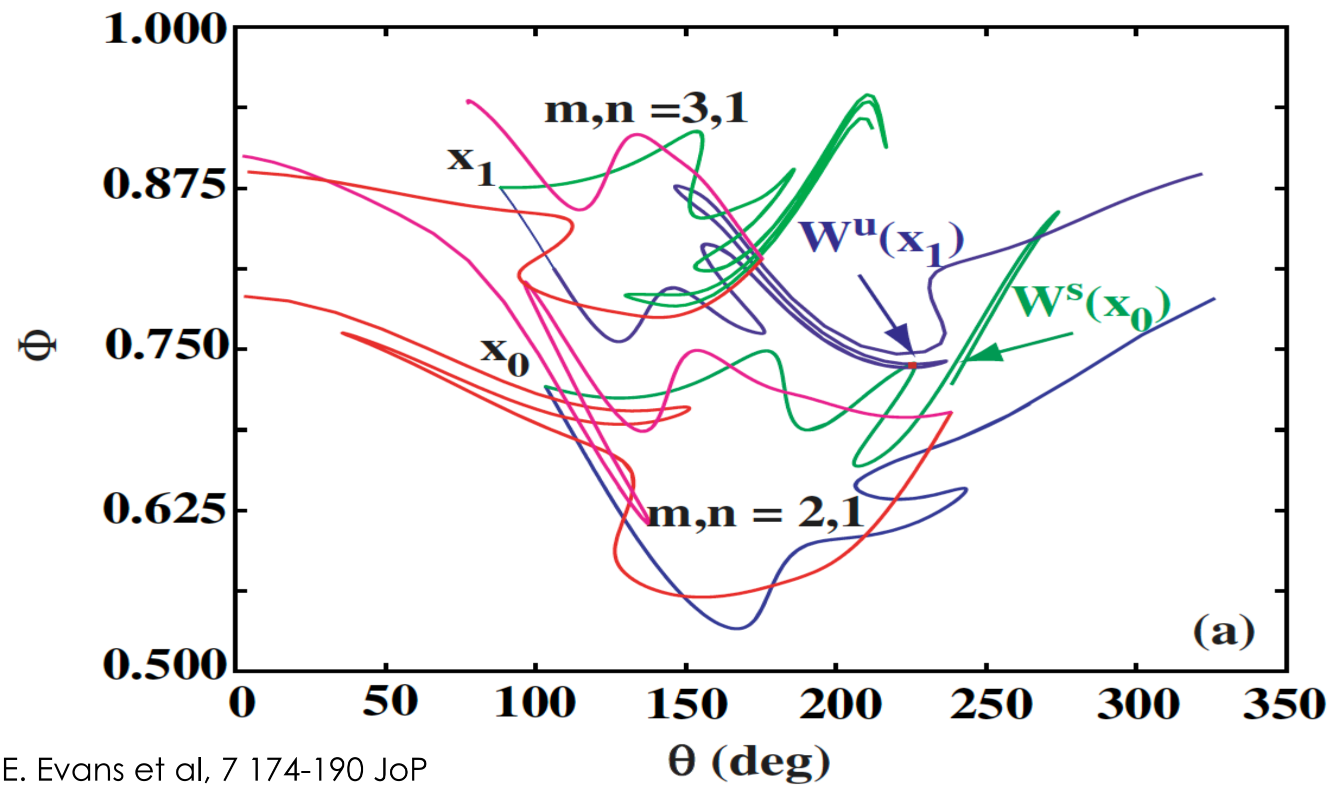
[1] T. E. Evans et al, 7 174-190 JoP

Large Coupled Island Chains Form Stochastic Regions and Exchange Magnetic Field Lines

- Flux tunneling was predicted in numerical simulations [1]
- Intersections of Wu(x1) & Ws(x0) define lobes called homoclinic tangles.
- Overlaps of homoclinic tangles cause stochastic mixing near the X points.
- At sufficiently large perturbation amplitude, homoclinic tangles associated with different islands intersect and field lines are exchanged between the island chains,



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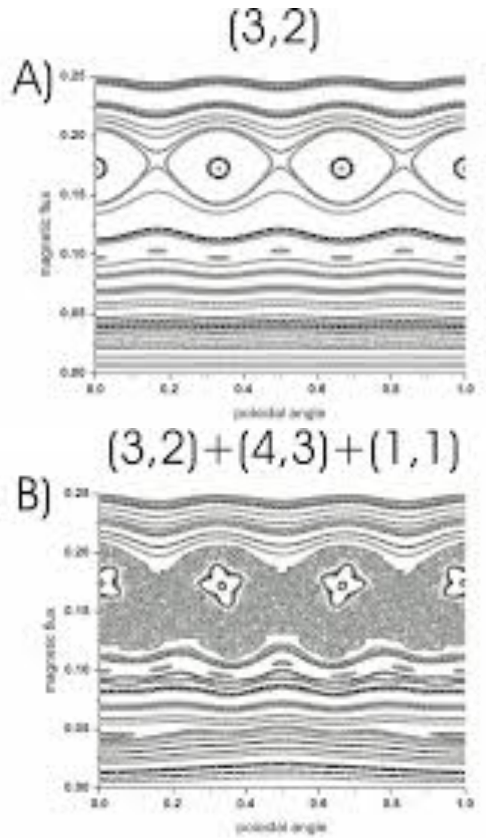
[1] T. E. Evans et al, 7 174-190 JoP

Topological Bifurcation *Can* Be Monitored in Experiments by Measurements of T_e in the Presence of Heat Sources

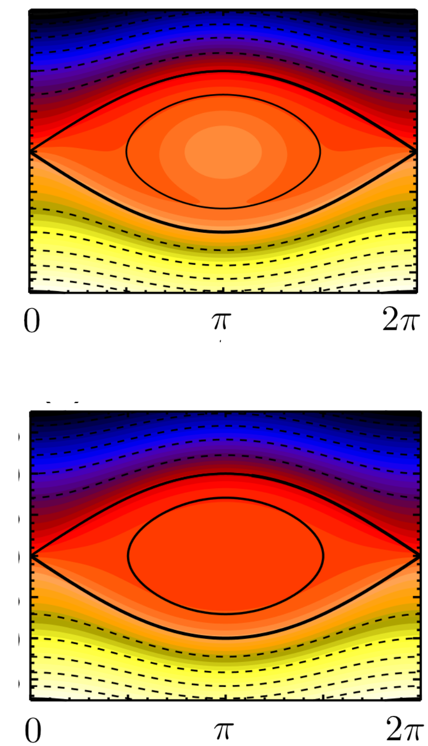
- Field lines can't be measured but T_e is determined by the magn. geometry + transport:



- **Nested** surfaces & Q>0 → **peaked** T_e

$$+\quad \chi_\perp \nabla_\perp^2 T_e + \chi_{||} \nabla_{||}^2 T_e = -Q(r) \quad \longrightarrow$$

- **Stochastic** flux & Q>0 → **flat** T_e



ECH offers ability to differentiate between nested & stochastic magnetic configurations using T_e data.

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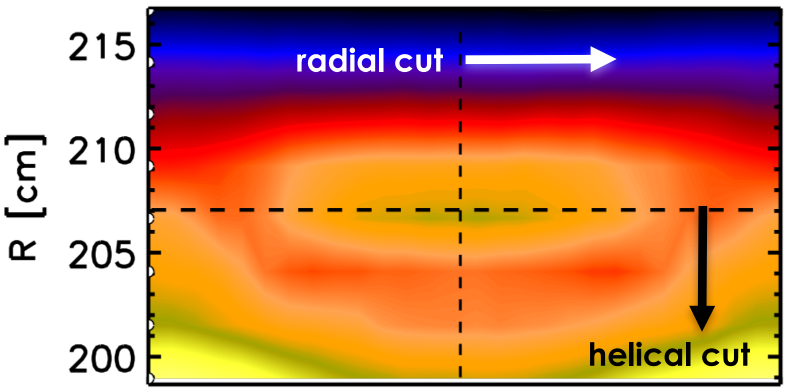
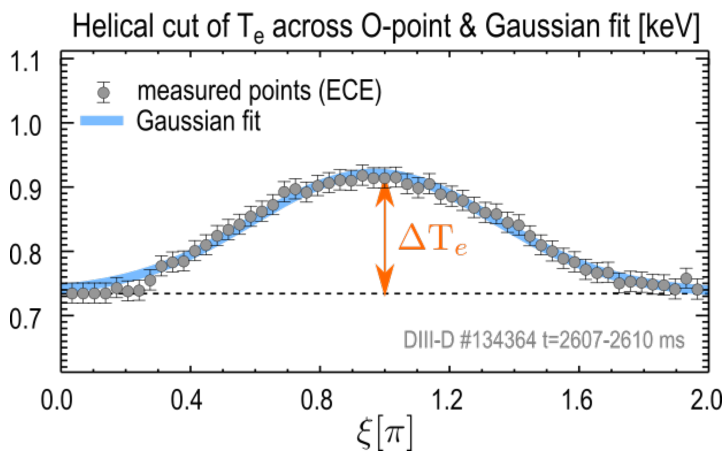
Analysis technique: use helical profile for accurate and detailed characterization of the $\triangle T_e$ structure at the O-point

ECE electron temperature [keV]





- ECE is probed with 0.5 MHz → at 10kHz rotation Te(ξ) has 50 samples per island cycle. At 1ms resolution one can average 10X
- Use helical profile in experiment to determine $\triangle T_e$. Needs rotating islands.

$\xi = m\theta - n\phi$

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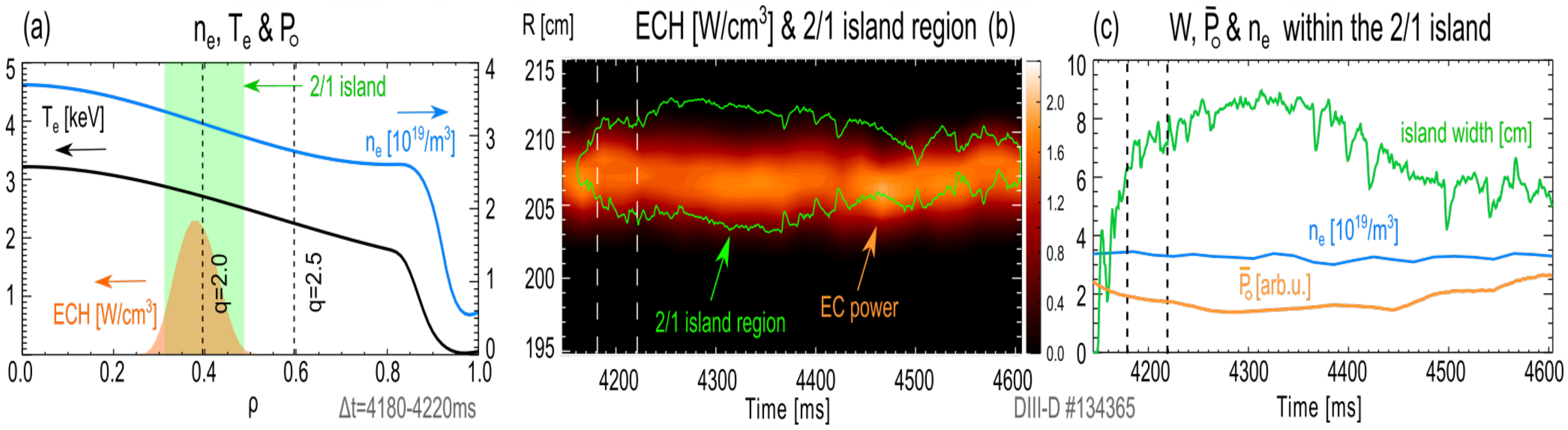
$\triangle T_e$ derived from phase-locked ECE data is used to characterize the thermal confinement within magnetic islands.

EC Heated 9 cm 2/1 Islands & 5/2 Islands with Coupled and Decoupled Phases



- Constant 3MW ECH & 60kA ECCD
- Constant ECH density and electron density within the island
- Linear diffusion predicts $\Delta T_e = P_o W^2/(n_e \chi_\perp)$

O-point T_e perturbation dynamics should go as ~$W^2(t)$

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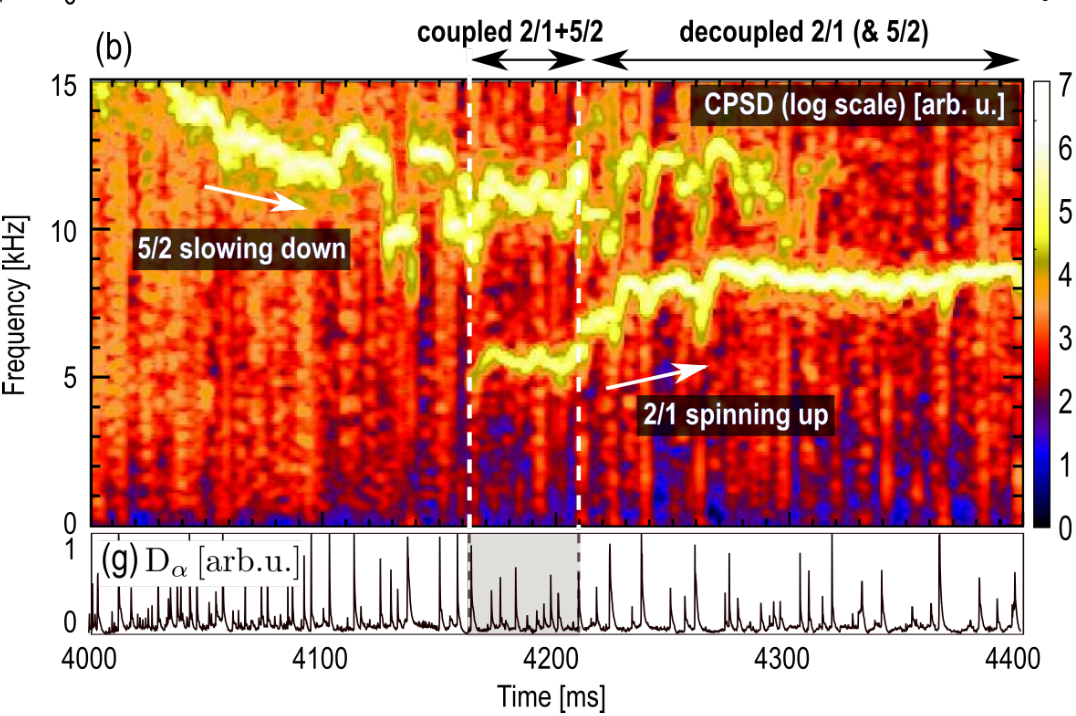
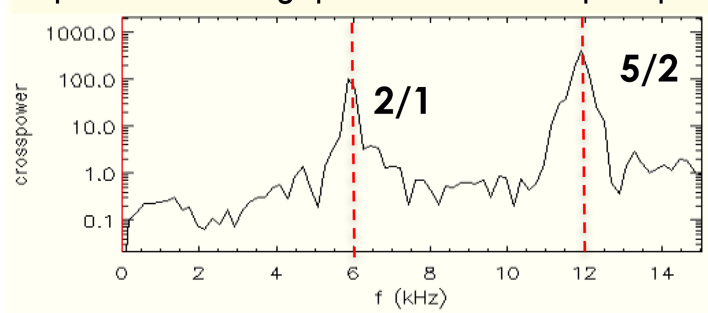
Topological Bifurcation in 2,1 Island: $\triangle T_e/T_e$ ~ 0% When Coupled to 5,2 island, $\triangle T_e/T_e$ ~ 0% Grows to 8% After Decoupling



- 5,2 slowing down
- 2/1 grows locked in the 5,2 frame
- Coupling persists for ~50 ms

Spectrum of mag. probes in the coupled phase



$$f_{2/1}/n_1=f_{5/2}/n_2$$

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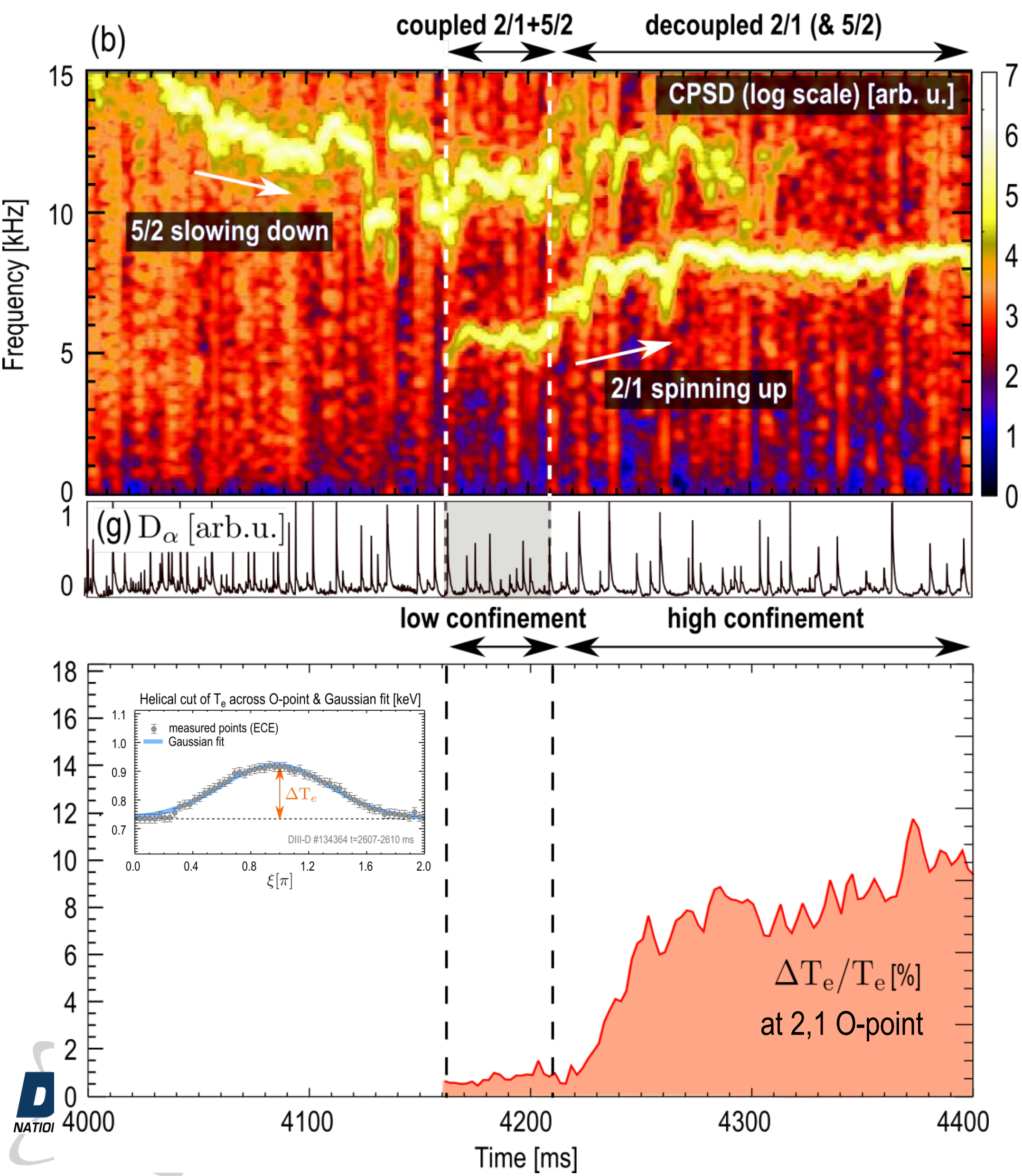

- 5,2 slowing down
- 2/1 grows locked in the 5,2 frame
- Coupling persists for ~50 ms
- ΔT_e doesn't grow in coupled islands
- ΔT_e grows after decoupling
- Recall: ECH density & n_e are ~constant in 2,1 island.

Coupling between the 2,1+5,2 islands degrades EC wave energy confinement in the 2,1 island

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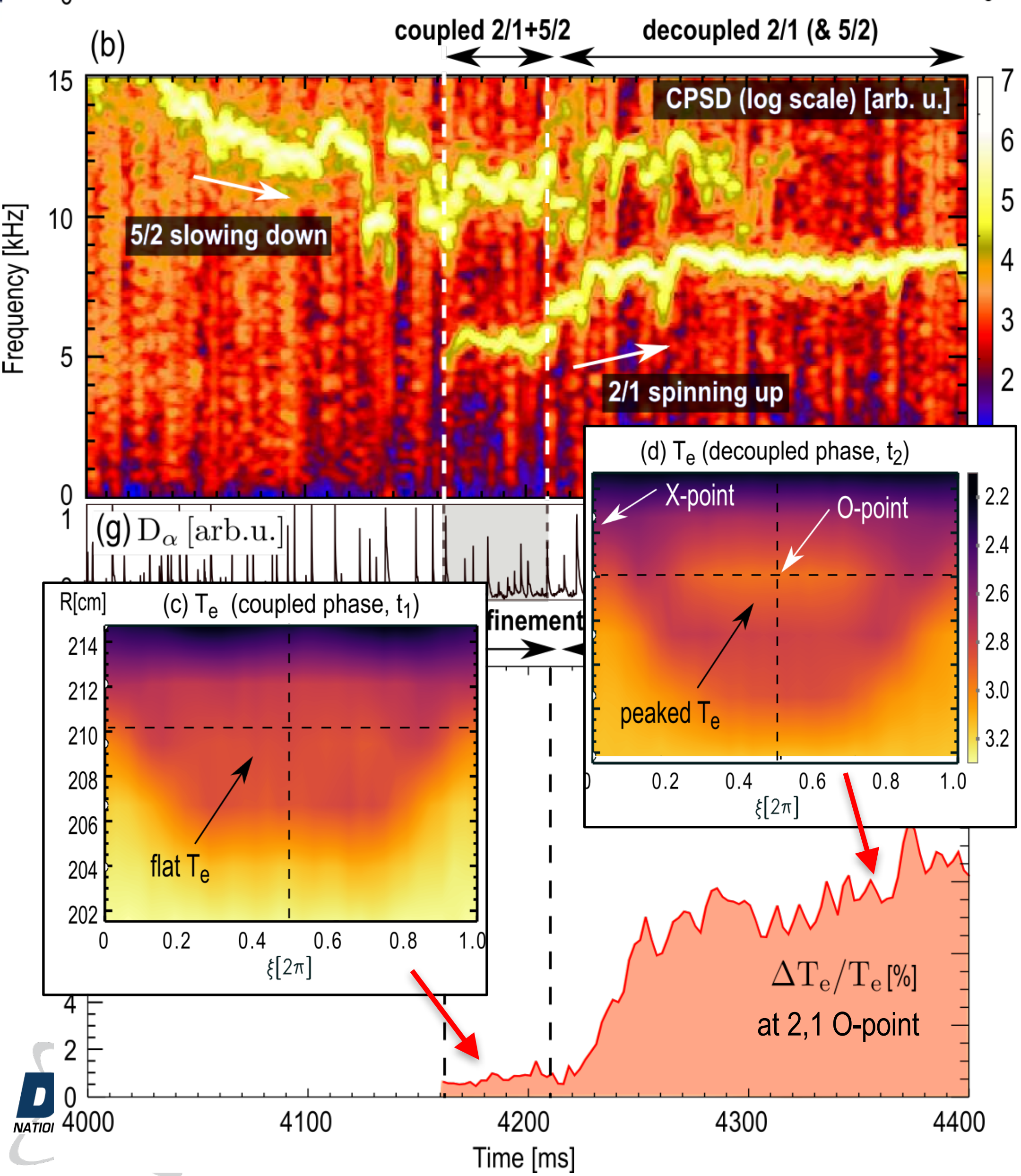

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ORBIT is Used to Map Out the Structure of Vacuum Islands

- Vacuum island structures are calculated with field line mapping via ORBIT [1].
- Non-axisymmetric field perturbation:

$$\tilde{\mathbf{B}}(\mathbf{r},\xi,\mathbf{t}) = \nabla \times (\hat{z}\Psi(r,\xi,t))$$

- Use radially localized helical current:

$$\tilde{\mathbf{j}}(r,\xi) = \tilde{\mathrm{j}}_{\circ}\cos(\xi)\delta(r_s - r)\mathbf{e}_z$$

- Ampere's law gives [2,3]:

$$\Psi_\delta(r,\xi) = \frac{\mu_\circ \tilde{\mathrm{j}}_\circ}{m^2} f(r)\cos(\xi)$$

$$f(r) = \frac{r^2}{r_s} \quad \text{at} \quad r < r_s, \quad f(r) = \frac{r_s^3}{r^2} \quad \text{at} \quad r > r_s$$

Islands are represented in ORBIT by helical current filaments whose parameters are fully constrained by magnetic measurements.

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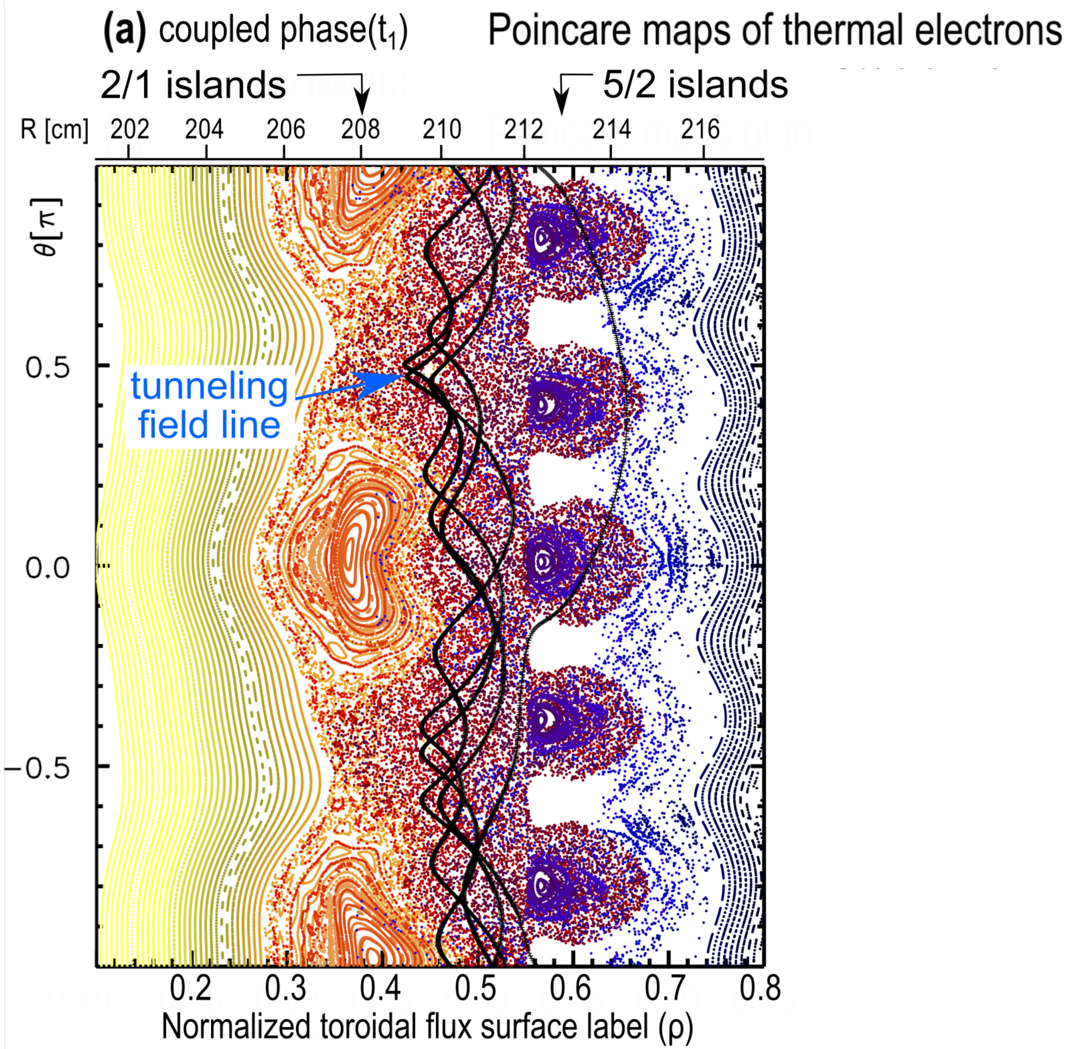
[1] R. B. White et al. Phys. Fluids, 27(2455), 1984.
[2] J. Wesson, Tokamaks, Oxford Uni. Press (2011), [3] L. Bardoczi, PPCF 61 055012 (2019)

ORBIT Shows Topological Bifurcation Occurs at the Time of Coupling/Decoupling, in Agreement with Local T_e Data

- **Coupled phase:** stochastic 2/1 island & field lines tunnel into the 5/2 island.



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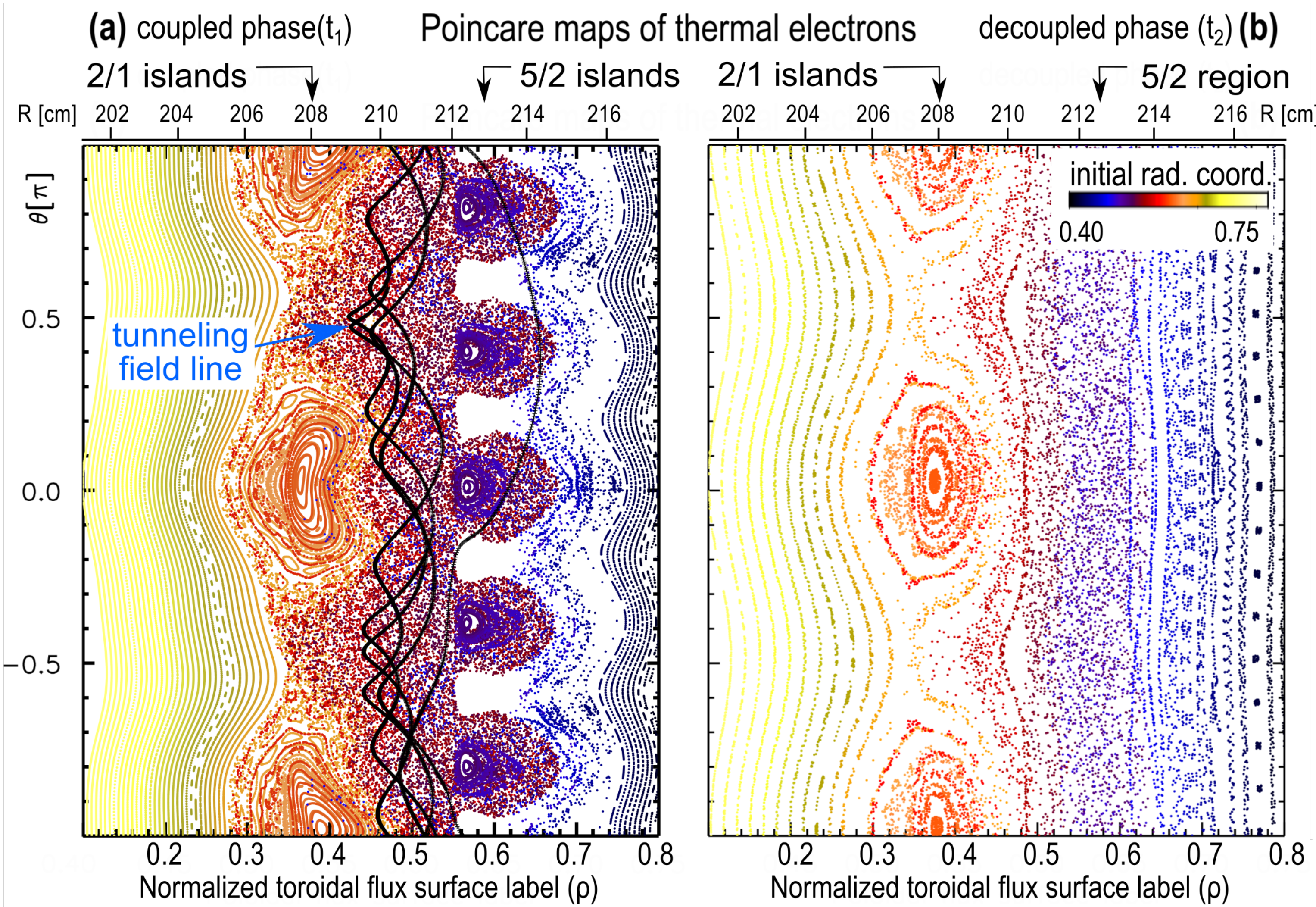
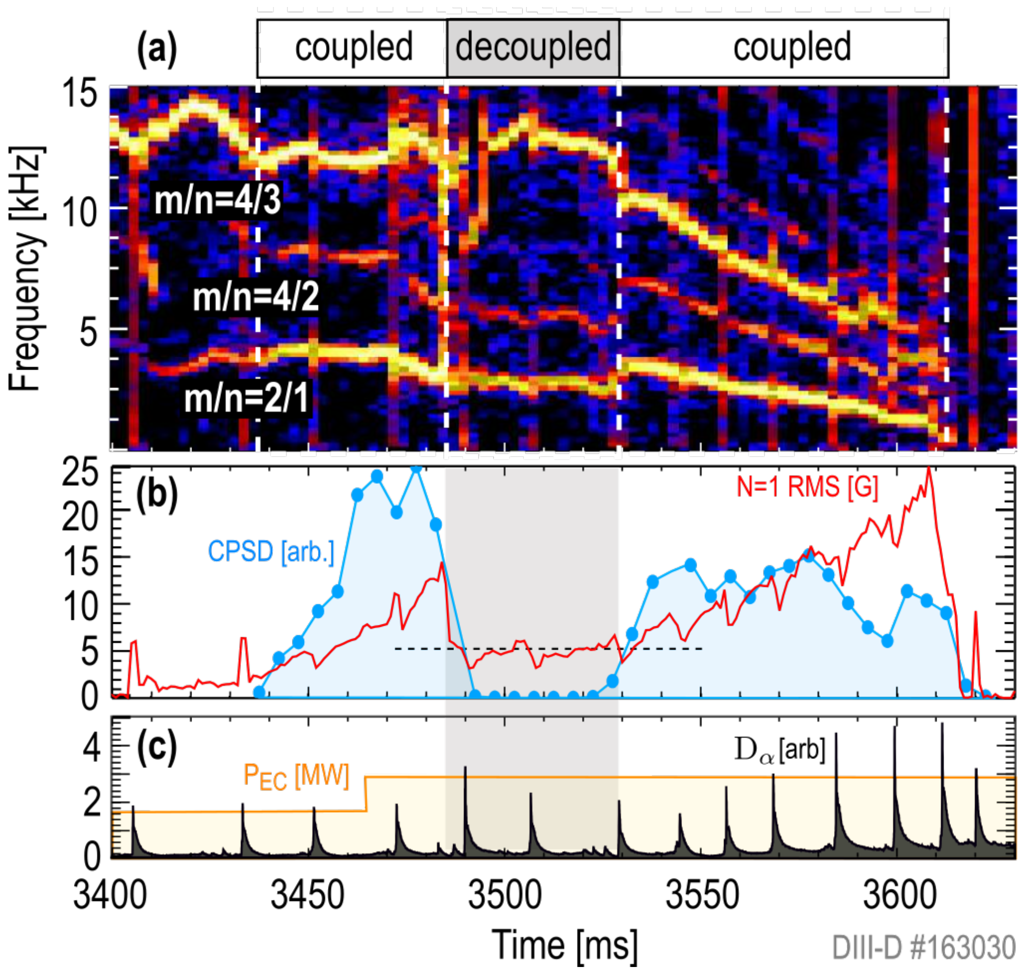
ORBIT Shows Topological Bifurcation Occurs at the Time of Coupling/Decoupling, in Agreement with Local T_e Data

- **Coupled phase:** stochastic 2/1 island & field lines tunnel into the 5/2 island.
- **Decoupled phase:** nested flux surfaces in the 2/1 island.



Island Response to ECCD Correlates with Coupling Events



- Constant 2.1MW ECH
- ITER baseline scenario plasma
- 4/3 slowing down
- 2/1 grows to 10 G

  locked in the 4/2 frame
- ELM brakes coupling, 2/1 shrinks
- Islands re-couple at another ELM
- 2/1 grows, locks & terminates

  the H-mode.

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Stochastic magnetic geometry in the 2,1 island due to coupling to islands in nearby rational surfaces can completely inhibit ECCD stabilization.

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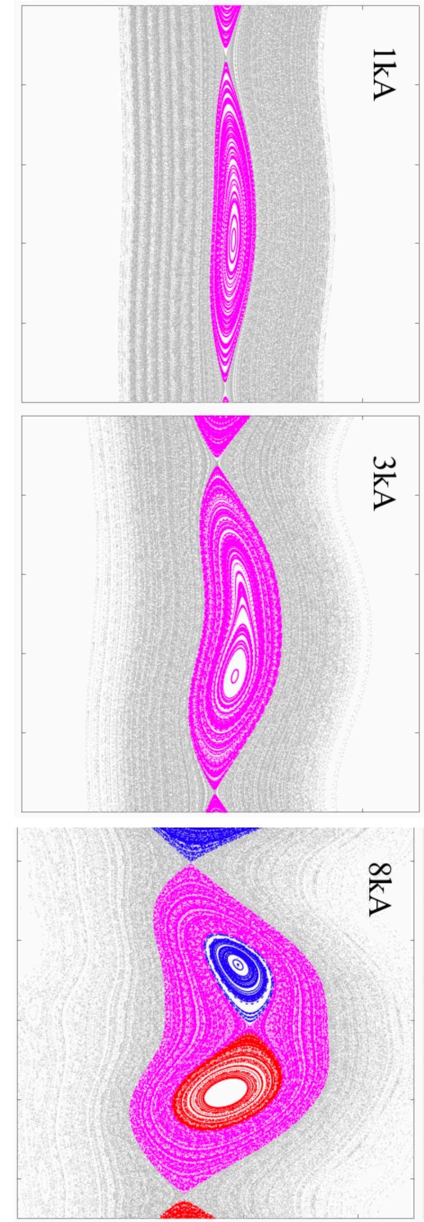
Heteroclinic Bifurcation of Magnetic Islands Can Bring Challenges for Active Tearing Mode Stabilization

Background/Motivation:

- Recent theory [1,2] predicts a **new class of bifurcations** forming heteroclinic islands caused by coupled TMs with same m/n, for example, a 2,1 island can bifurcate into a 4,2.
    - This theory has never been tested before.

- This can bring/explain existing challenges for active stabilization:
    - Rotating islands: ECCD splits between heteroclinic O-points.
    - Locked islands: can drive ECCD only in one O-point at a time.



[1] T. E. Evans et al, ArXiv 1805.10394v2 (2018)

[2] W. Wu, T. E. Evans et al, NF **59** 066010 (2019)

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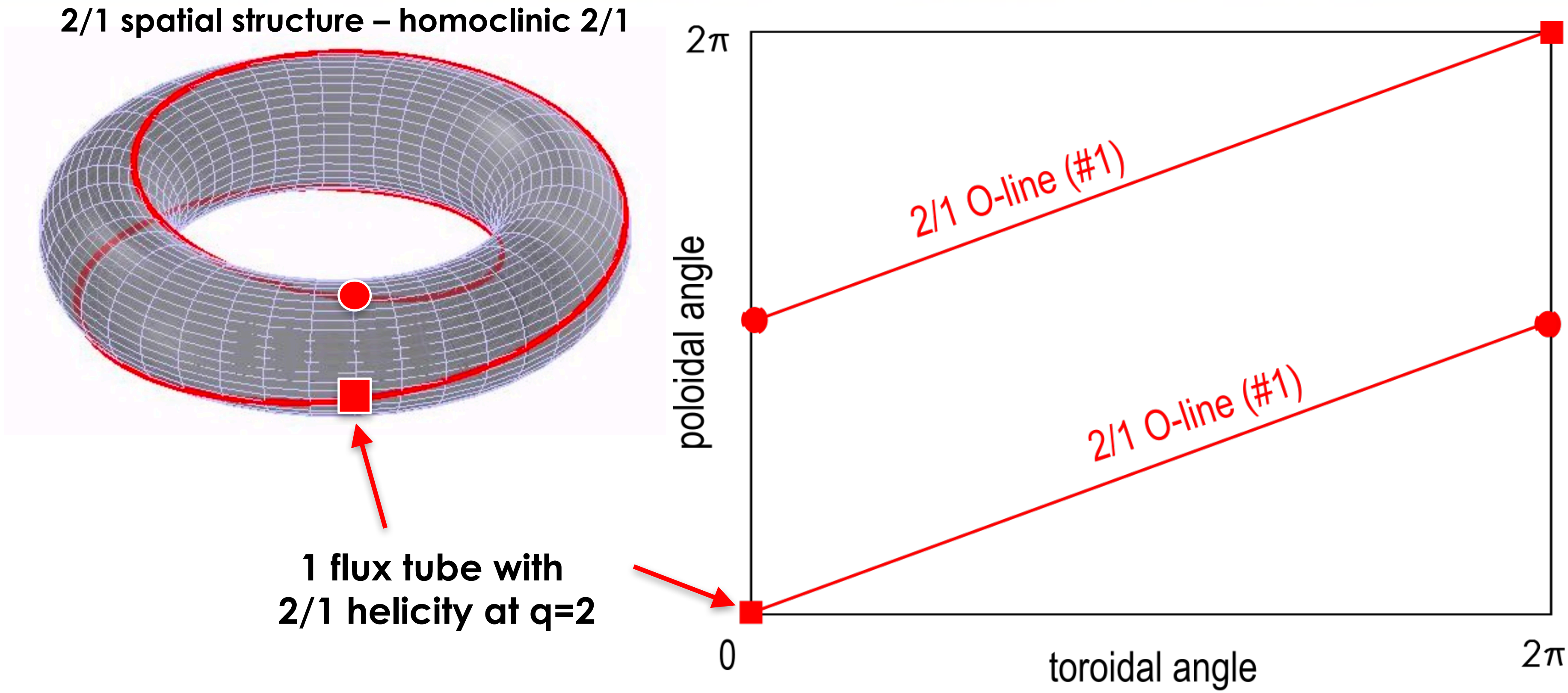
4/2 Structure in Magnetic Probes Comes From 2 Set of Heteroclinic 2/1 islands; Homoclinic 4/2 Islands Don't Exist



- Going around toroidally 2x will close the line
- This will generate 2 (1) islands in the poloidal (toroidal) plane
- Subsequent toroidal cycles will run along the same line, without mapping more islands

Homoclinic island with 2/1 helicity has 2/1 spatial structure.

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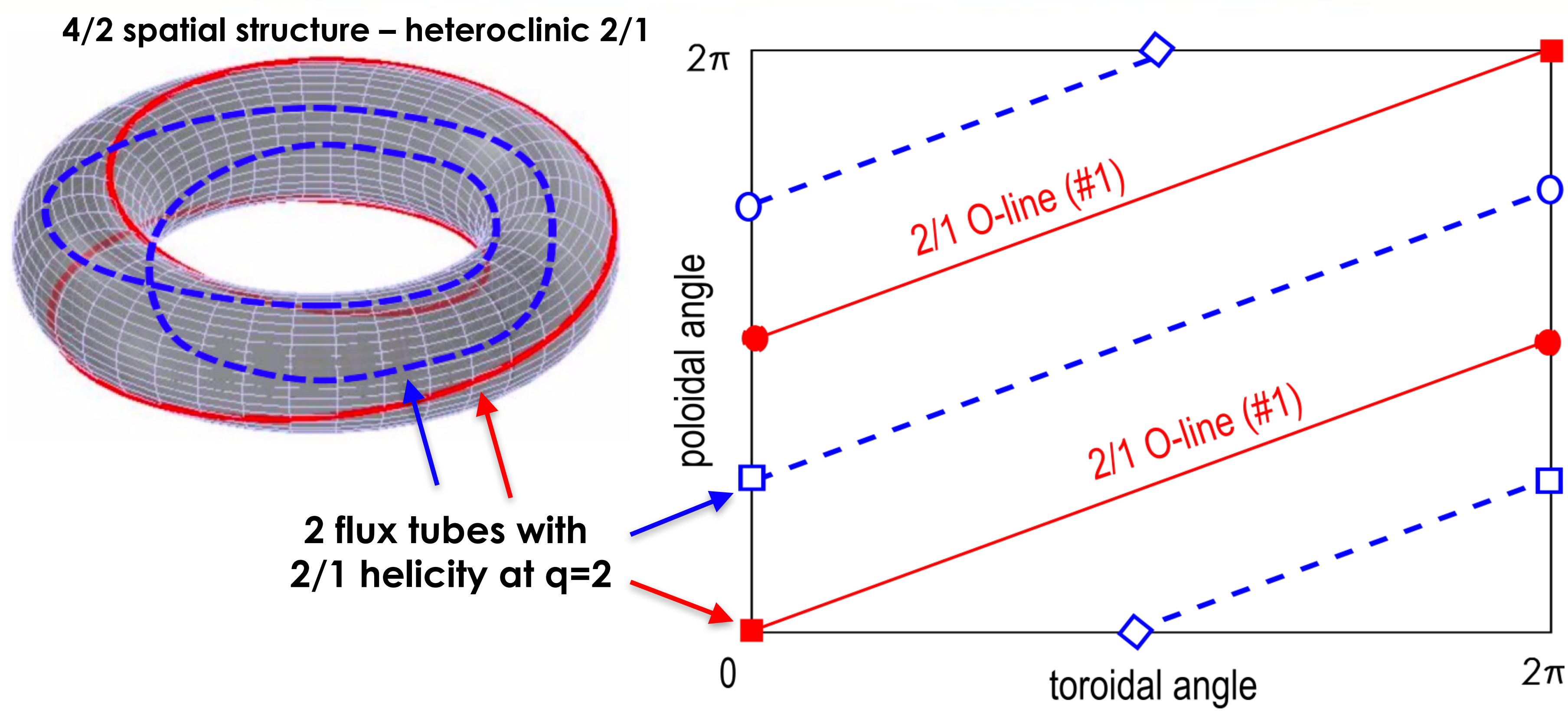
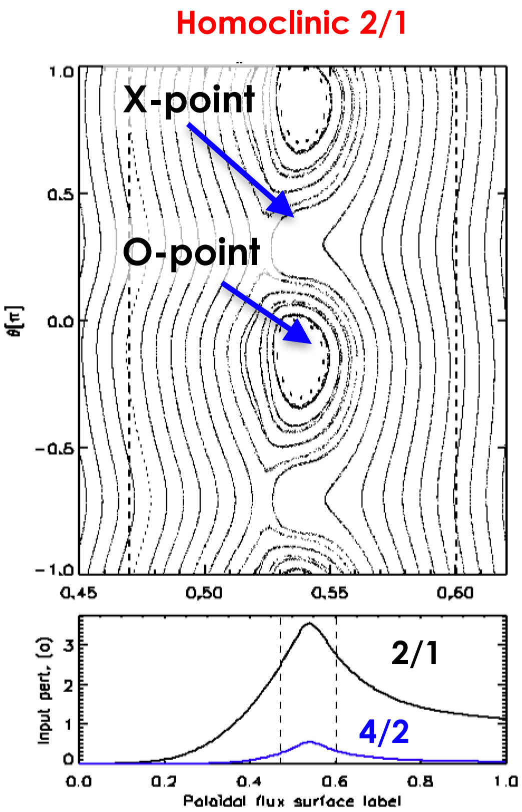


- 4 (2) islands can be mapped by two disjoint O-lines, both with 2/1 helicity
- 4/2 spatial structure is from two 2/1 flux tubes / heteroclinic islands of 2/1 helicity
- There are no 4/2 flux tubes "homoclinic 4/2 islands" (see isotopy classes of embedded closed curves in a torus)

4,2 spatial structure can not originate from a single flux tube of 4,2 helicity but only from 2 sets of heteroclinic 2,1 islands.

Heteroclinic Bifurcation Occurs When Multiple TMs of Different Helicity Compete to Form Islands at the Same q=m/n



- Solo 2/1 TMs form homoclinic islands with nested flux surface topology.

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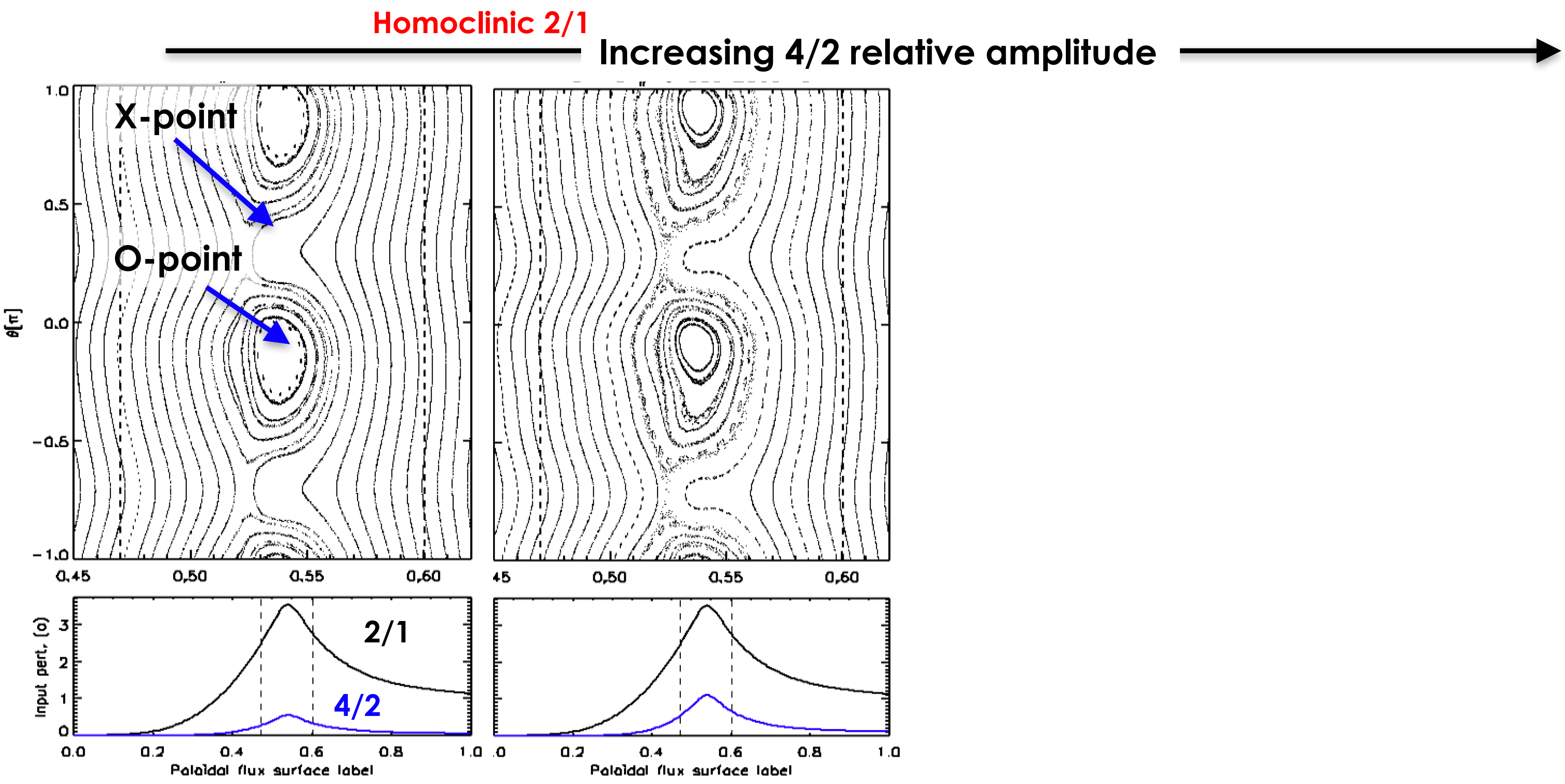

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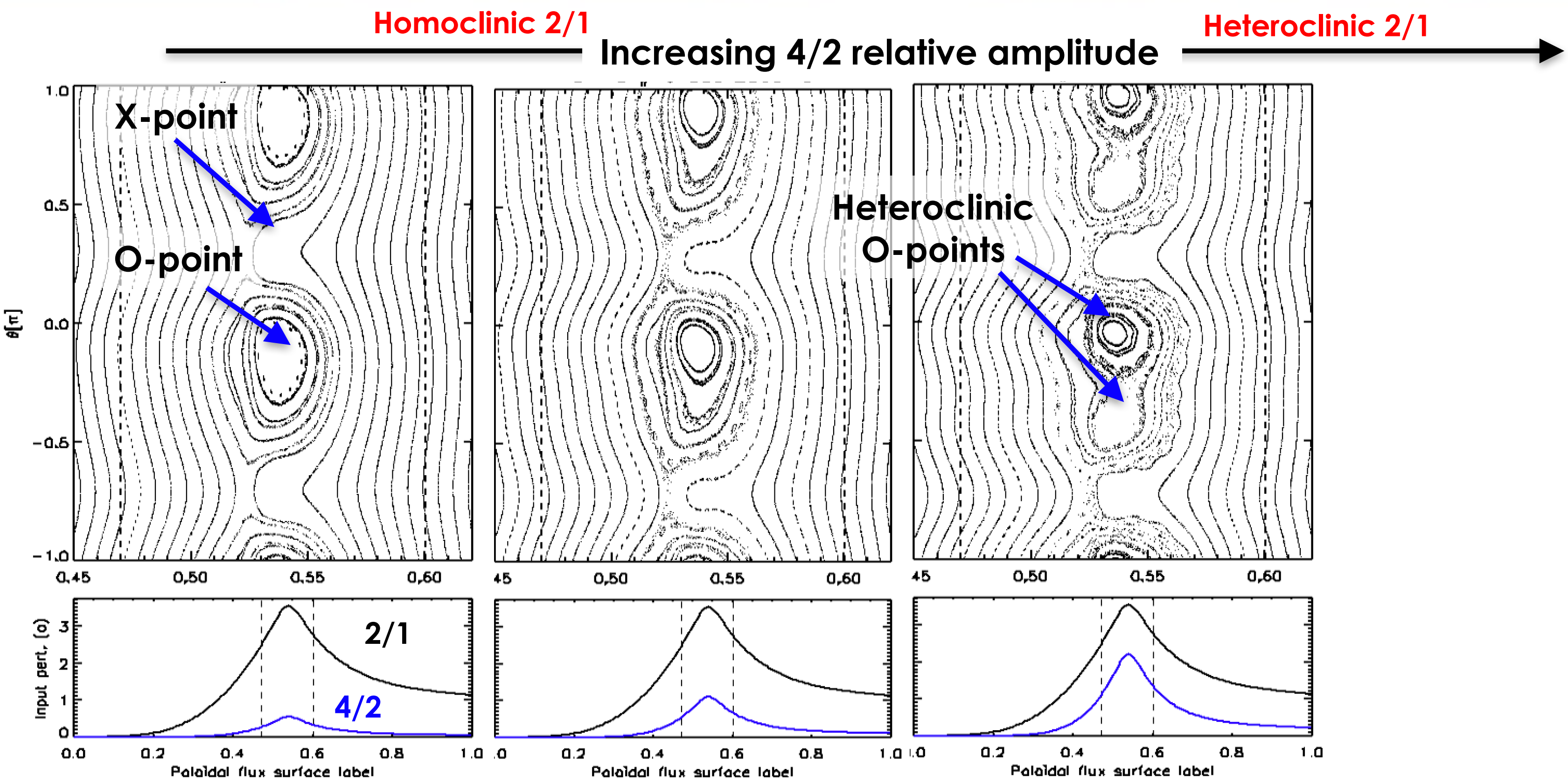

- Solo 2/1 TMs form homoclinic islands with nested flux surface topology.
- Heteroclinic bifurcation occurs due to coupled TMs of different helicity at same q.
- Second, disjoint, O-line forms within the largest island.

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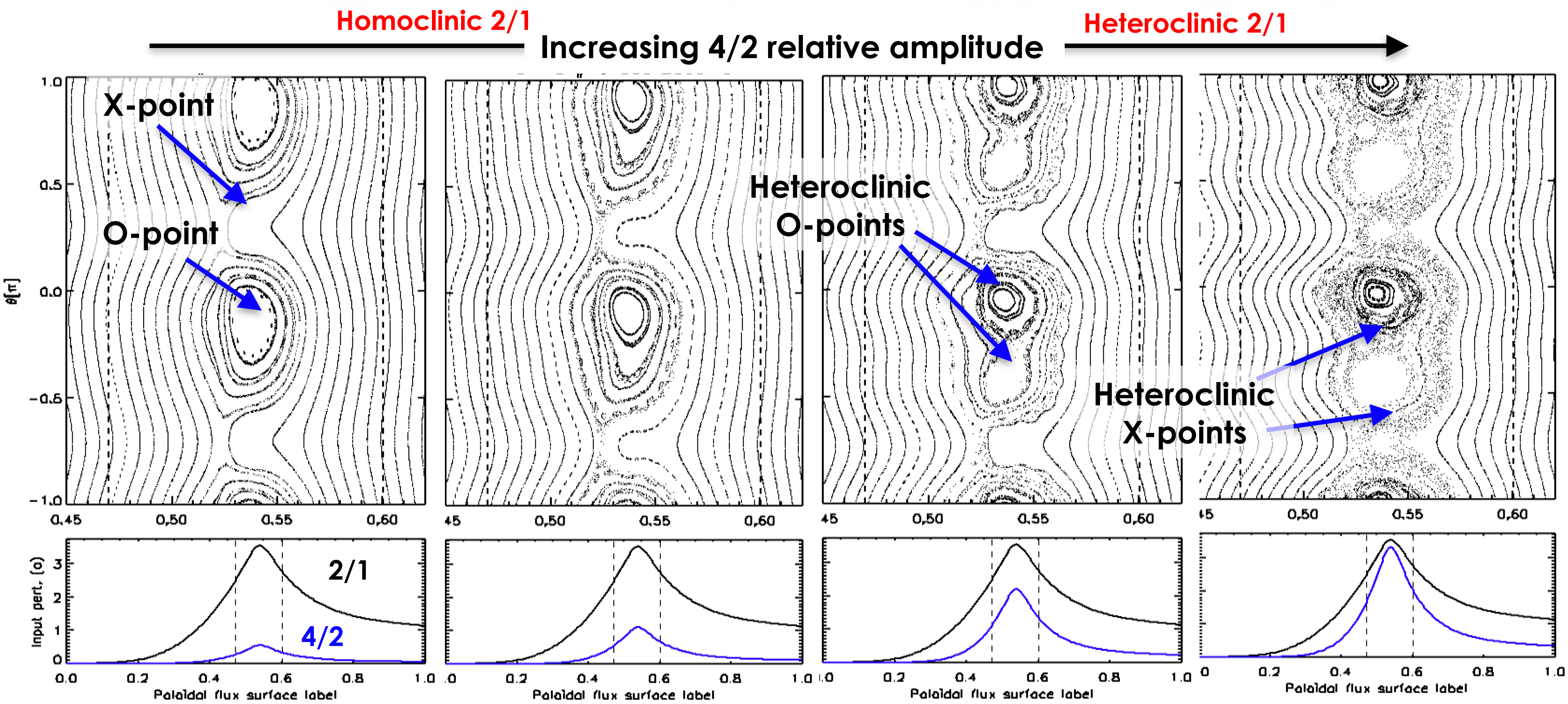

- Solo 2/1 TMs form homoclinic islands with nested flux surface topology.
- Heteroclinic bifurcation occurs due to coupled TMs of different helicity at same q.
- Second, disjoint, O-line forms within the largest island.
- In the analyzed discharge the threshold is at $A_{4/2}/A_{2/1}$~80% relative amplitude at r_s.

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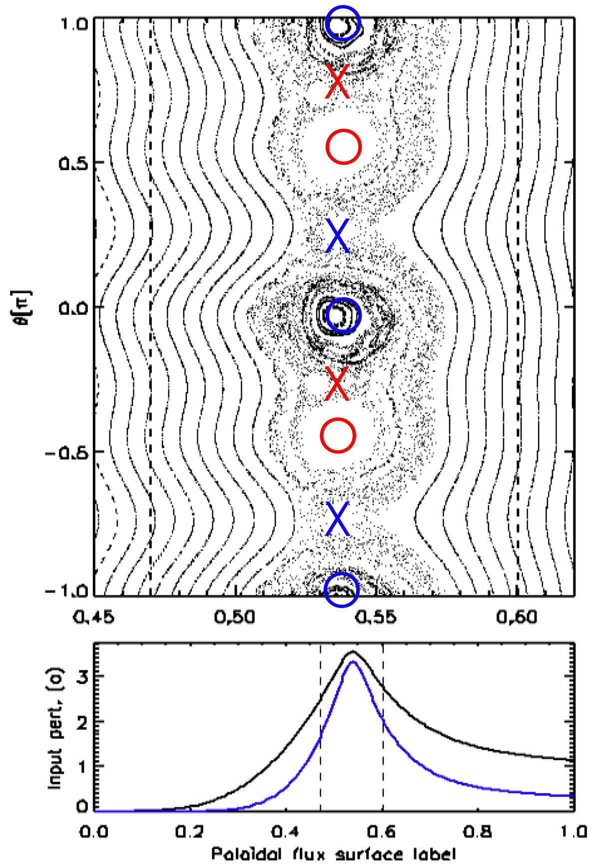
Higher Order Heteroclinic Bifurcations Can Occur Due to Additional TMs with Higher (m,n) With the Same m/n



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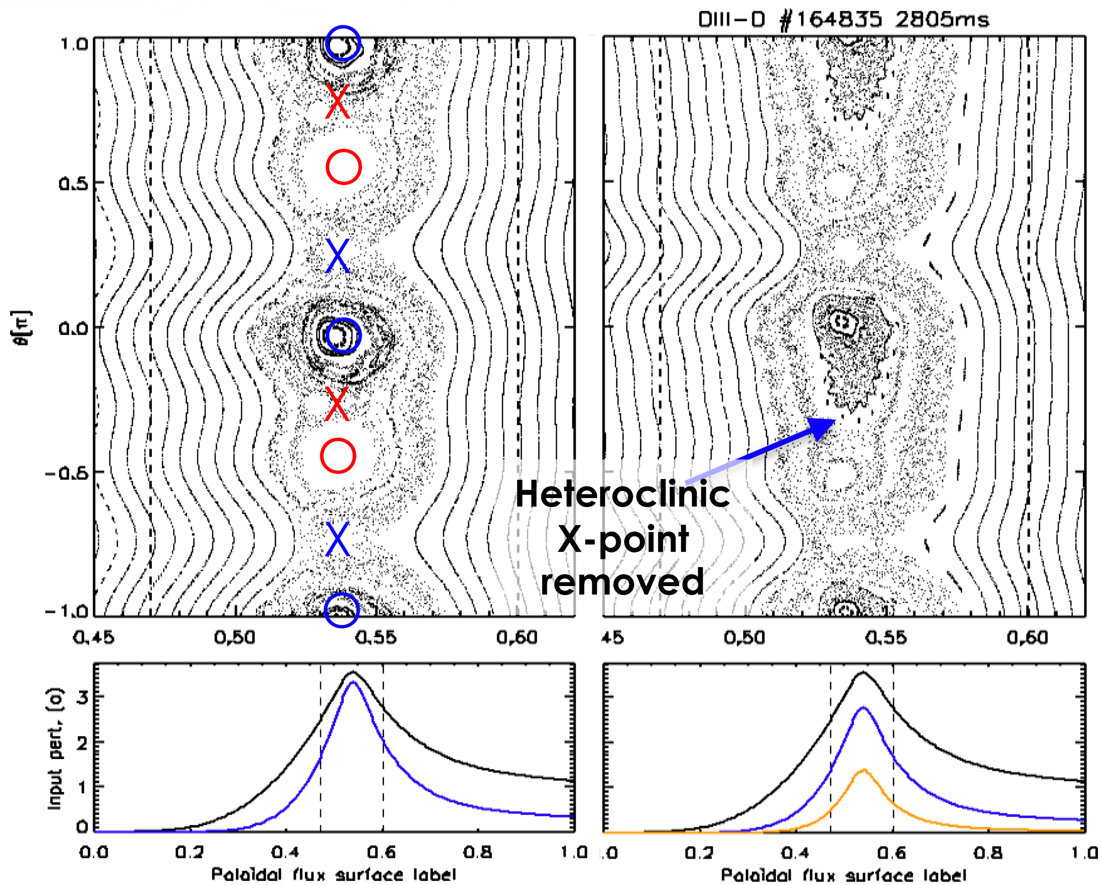

- Higher (m,n) tearing modes can further modify the internal structure:
  - 4% (at the wall) 6/3 removes the internal X-point

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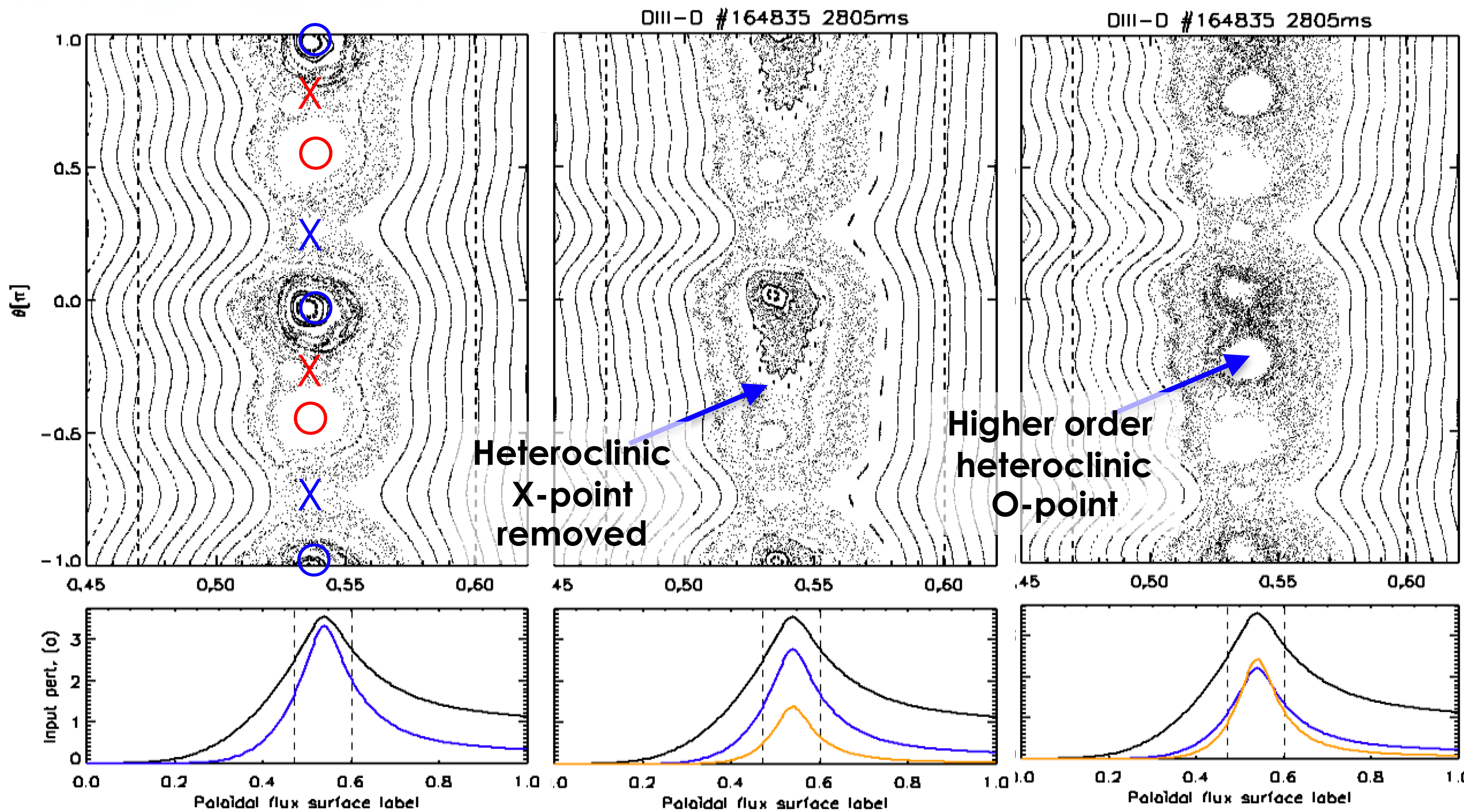

- Higher (m,n) tearing modes can further modify the internal structure:
  - 4% (at the wall) 6/3 removes the internal X-point
  - 7% (att the wall) 6/3 turns the internal X-point into a 3rd heteroclinic O-point

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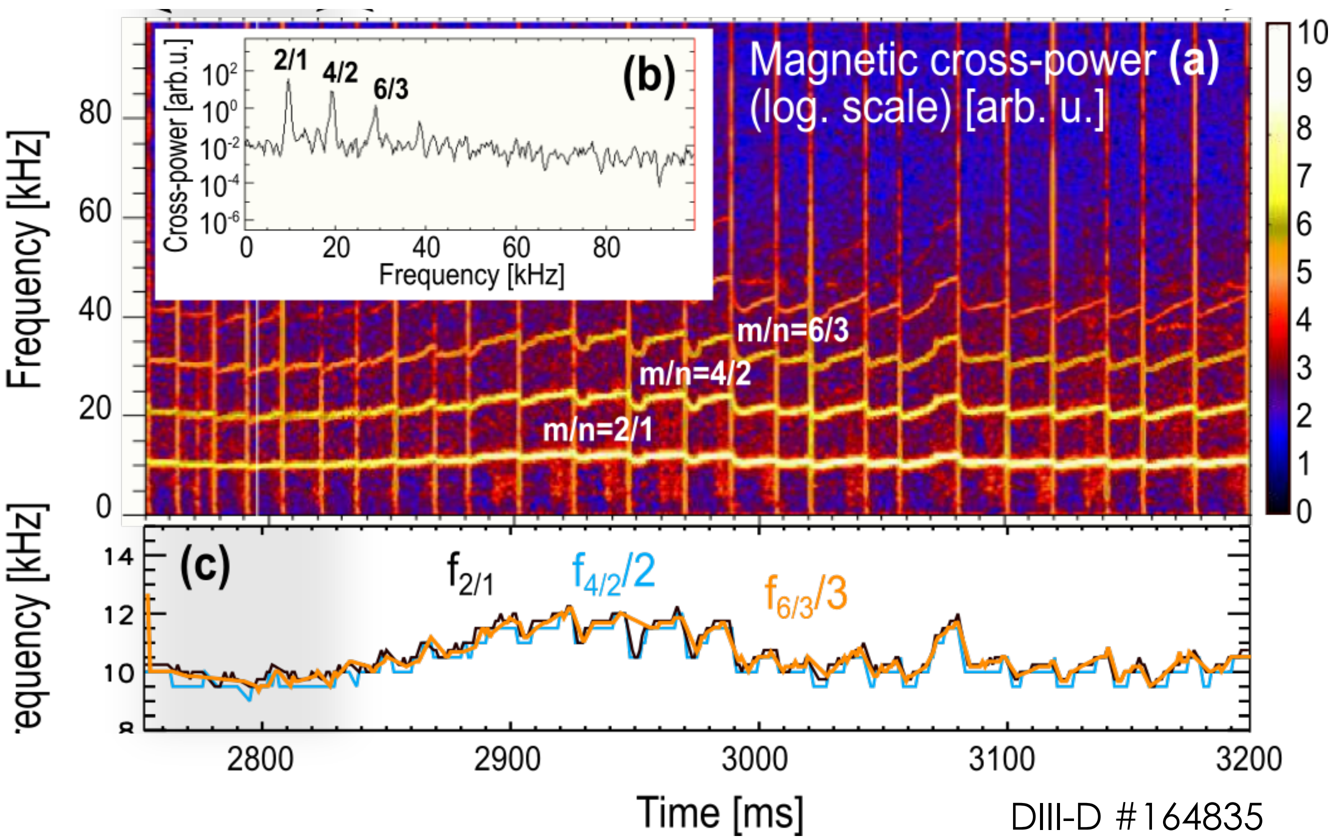
Candidate Discharge: Large Coupled 2/1, 4/2 & 6/3 TMs



DIII-D #164835

- Large & long lived & coupled 2/1, 4/2 & 6/3 TMs [identified through their toroidal structure from multiple toroidally placed probe signals]
- q=2 is subject to ECH (not shown)
- Good ECE data: no cutoff or 3rd harmonic contamination (not shown)

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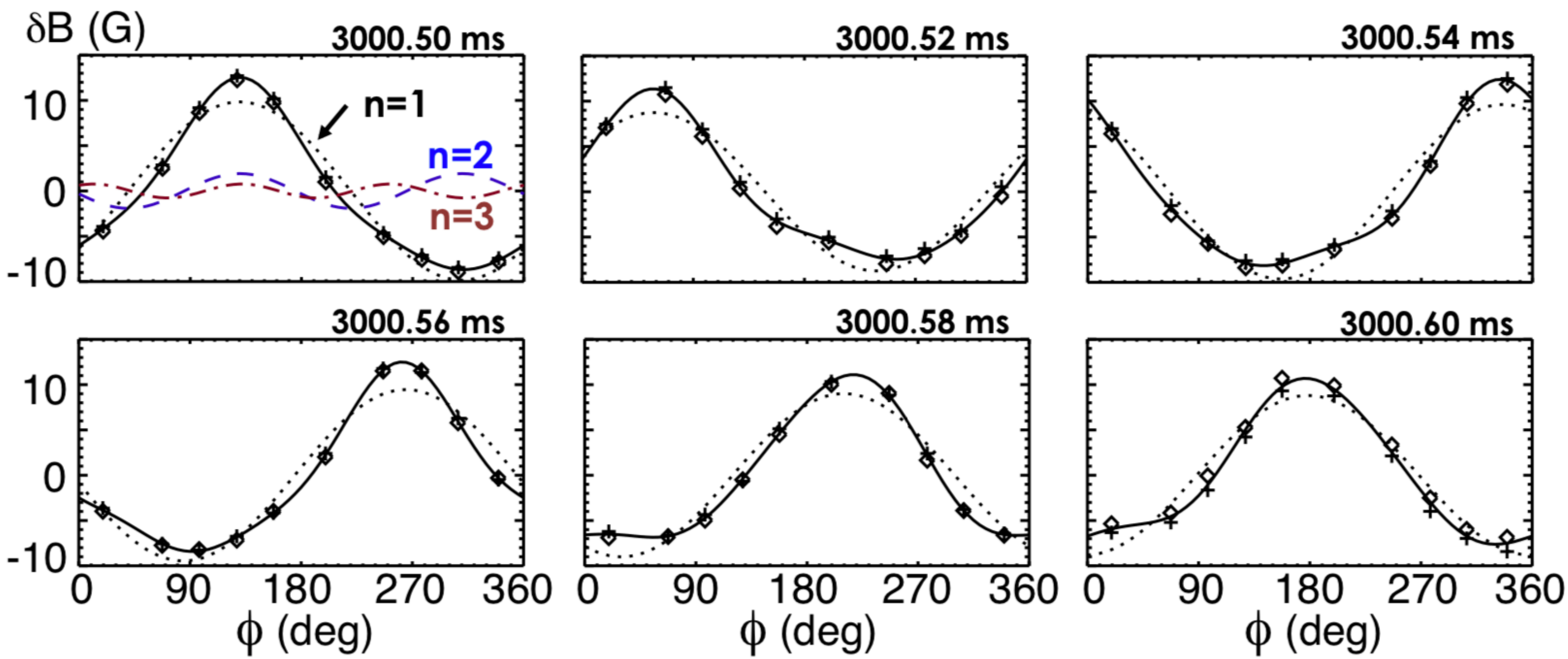
Detailed fits confirm non-sinusoidal spatial structure

- **Harmonics n=1,2,3 rotate together ⇒ constant spatial structure**
- **Alignment of the maxima for the three harmonics creates**
  - Narrow maximum
  - Broader minimum



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Slide from: E. Strait / 3DSP Meeting/ May 4, 2020

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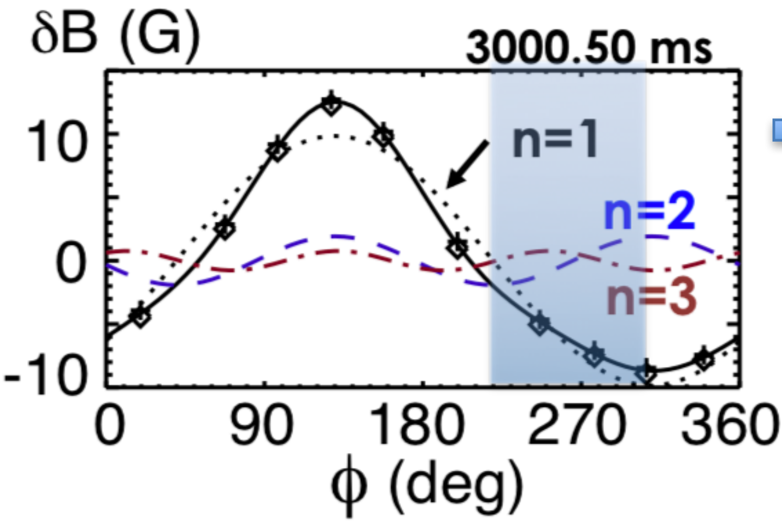

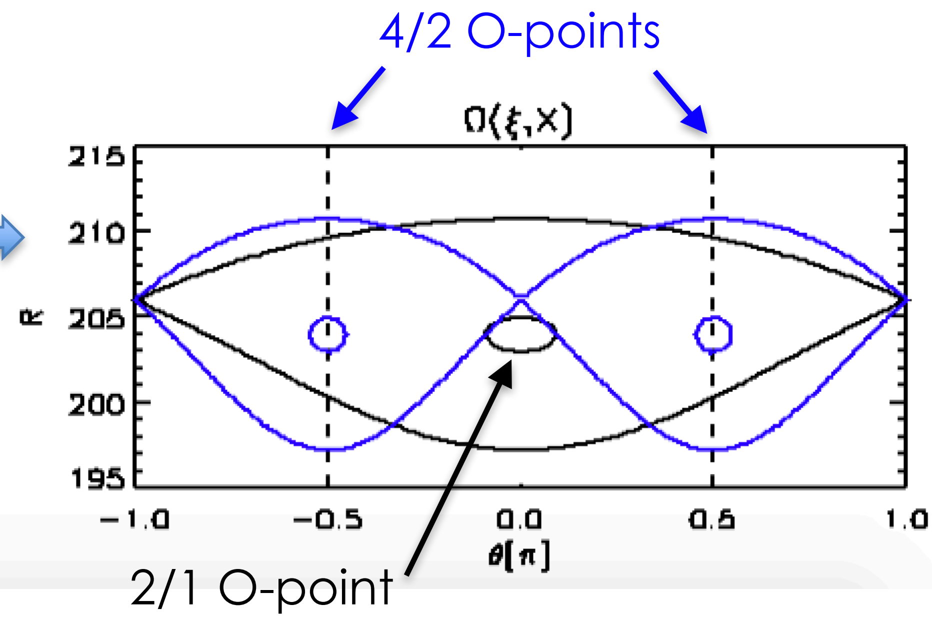

4/2 O-points are shifted with respect to the 2/1 O-point by +/- 90°

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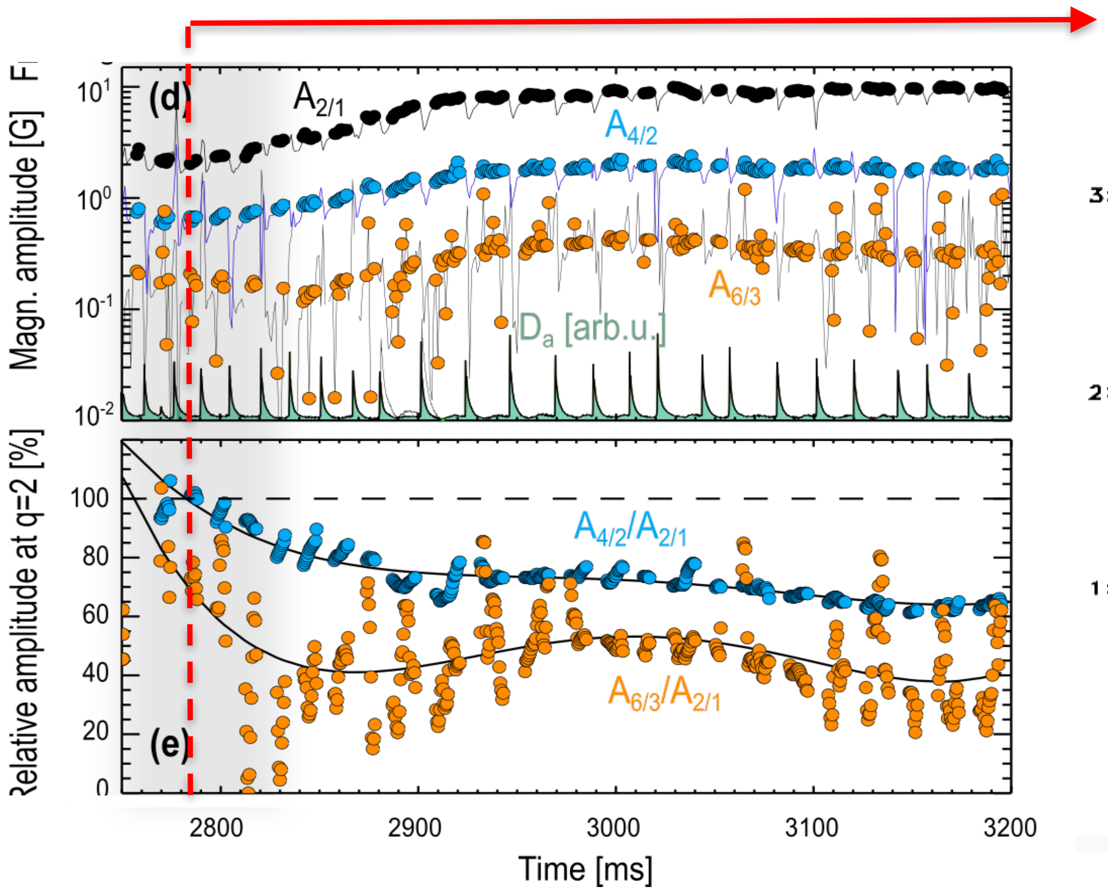
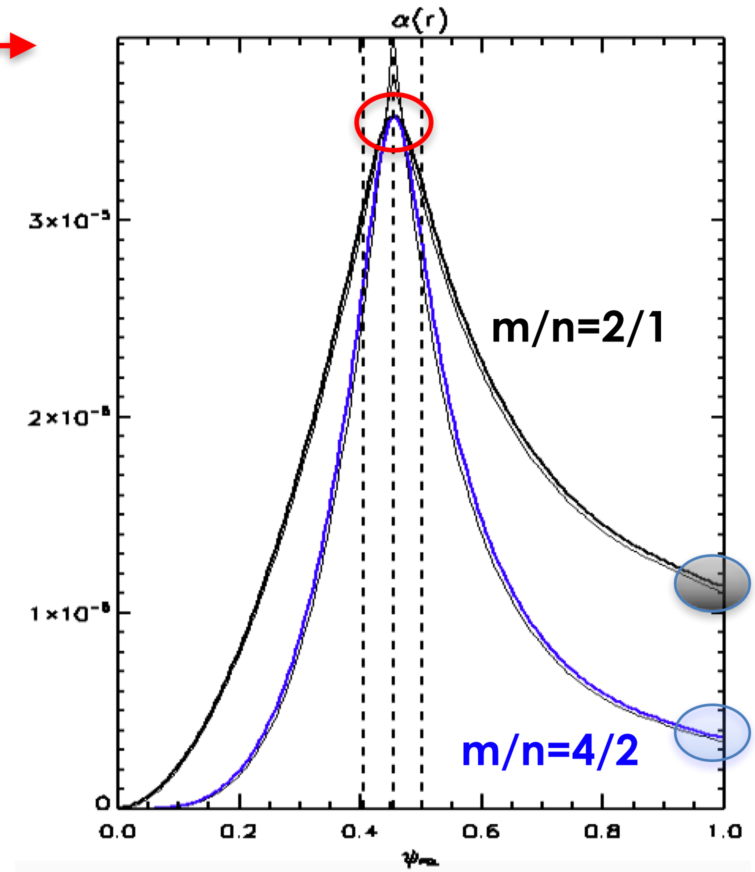
Faster 2/1 Growth Yields Scan of $A_{4/2}/A_{2/1}$ Relative Amplitude in a Range Where the Heteroclinic Bifurcation Should Occur





Early in the shot 4/2 is as large as the 2/1 at q=2 and this ratio decreases to ~60% in the saturated state. A transition from heteroclinic to homoclinic structure should be occurring in this shot.

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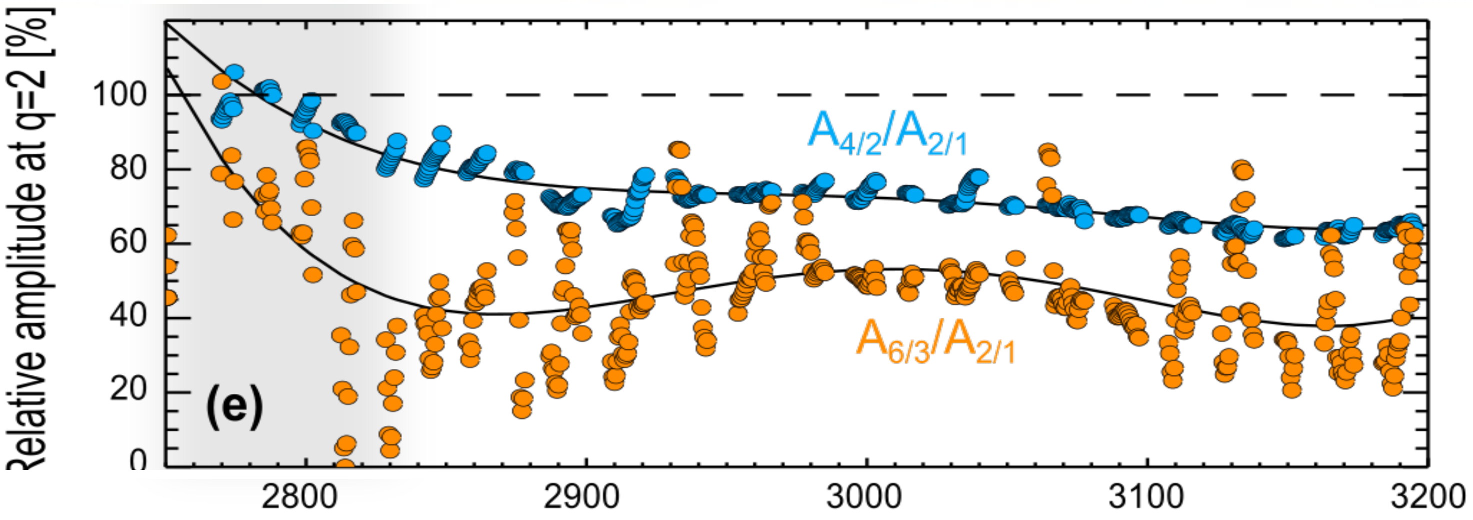
Candidate Discharge Meets All Conditions for the Heteroclinic Bifurcation to Occur



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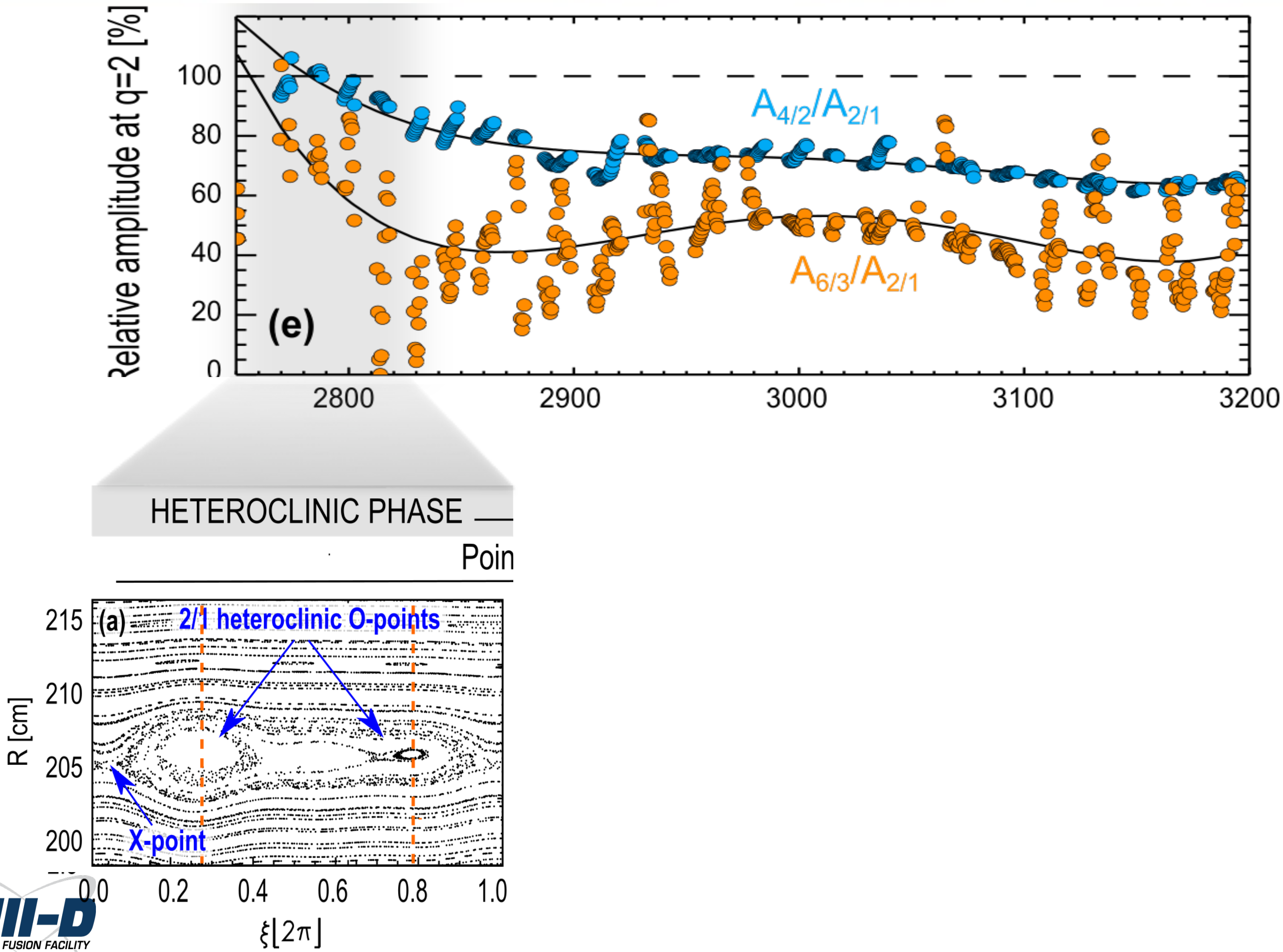

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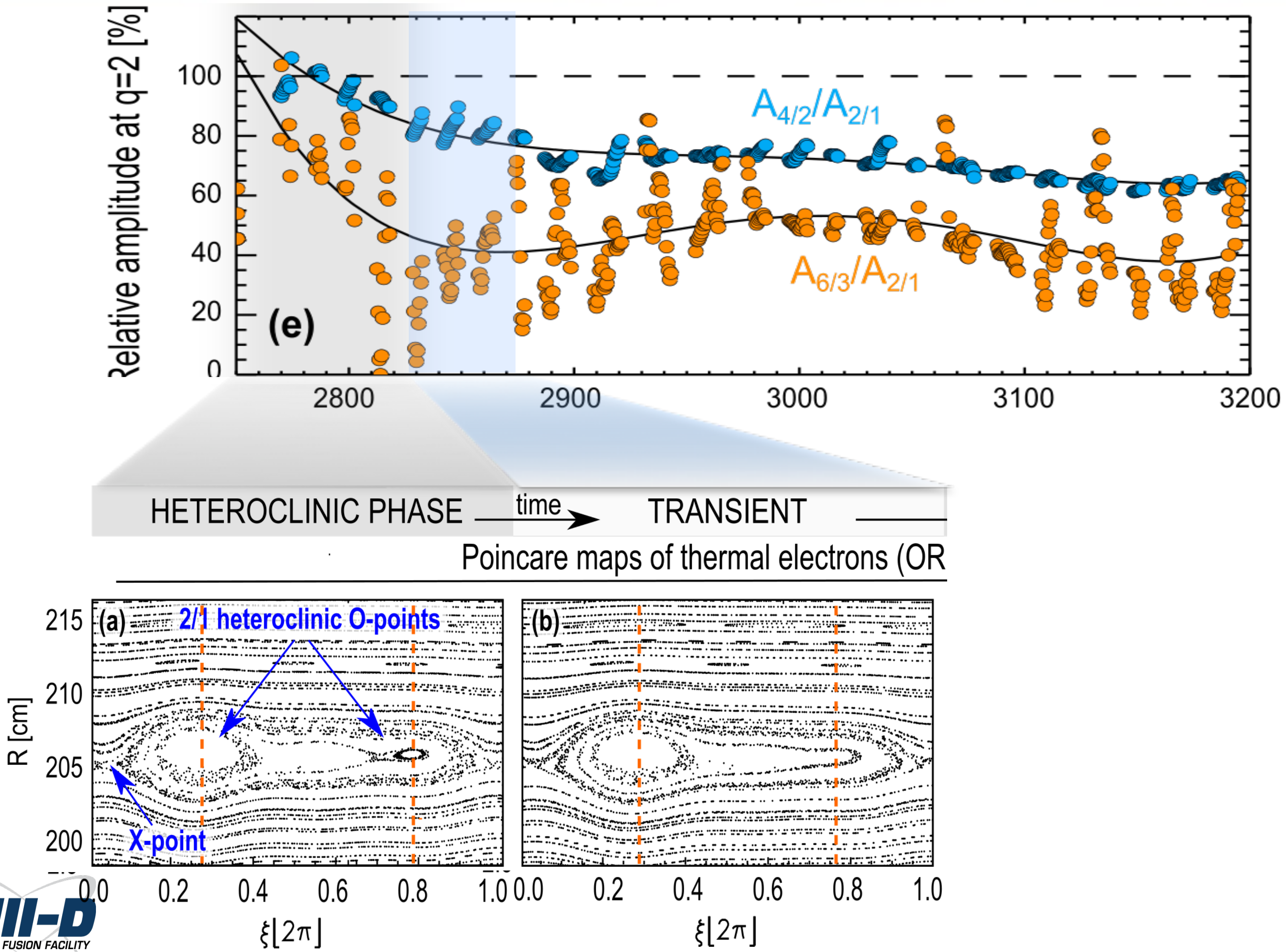

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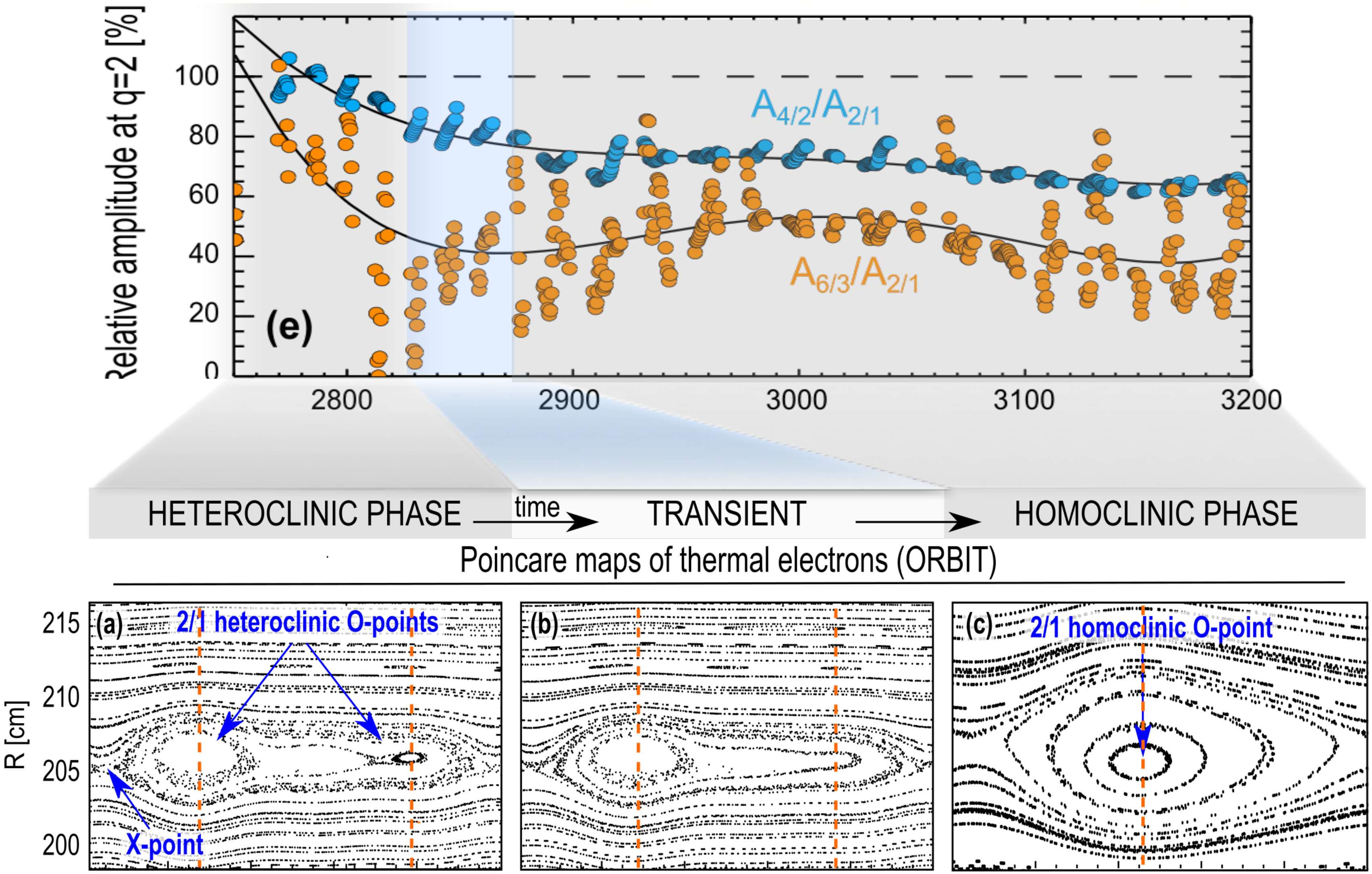
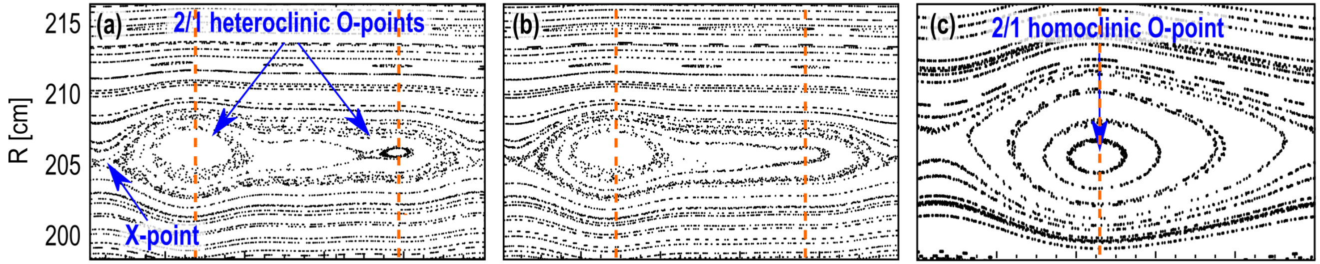


Simulations show bifurcation should be occurring in this DIII-D shot.
Local measurements are needed for confirmation.

T_e Distribution Within the EC Heated Island is Consistent With Bifurcation From Heteroclinic to Homoclinic Structure

HETEROCLINIC PHASE —time→ TRANSIENT → HOMOCLINIC PHASE

Poincare maps of thermal electrons (ORBIT)

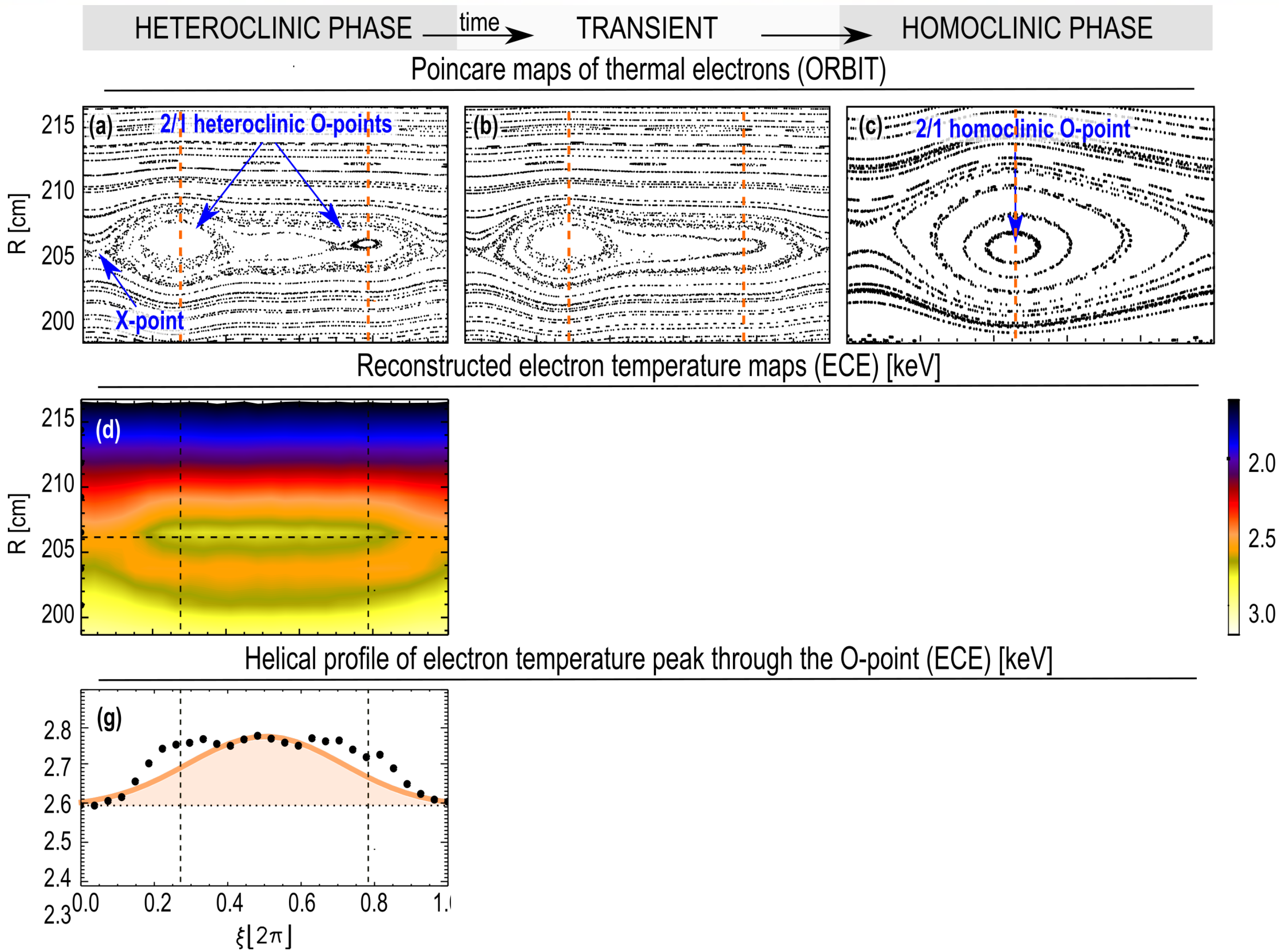


Reconstructed electron temperature maps (ECE) [keV]

Helical profile of electron temperature peak through the O-point (ECE) [keV]

- Measured Pi/2 phase shift between 4/2 and 2/1 O-points is a strong constraint and is well matched by T_e early in the evolution.
- Te data supports bifurcation from heteroclinic to monoclinic phase

T_e Distribution Within the EC Heated Island is Consistent With Bifurcation From Heteroclinic to Homoclinic Structure



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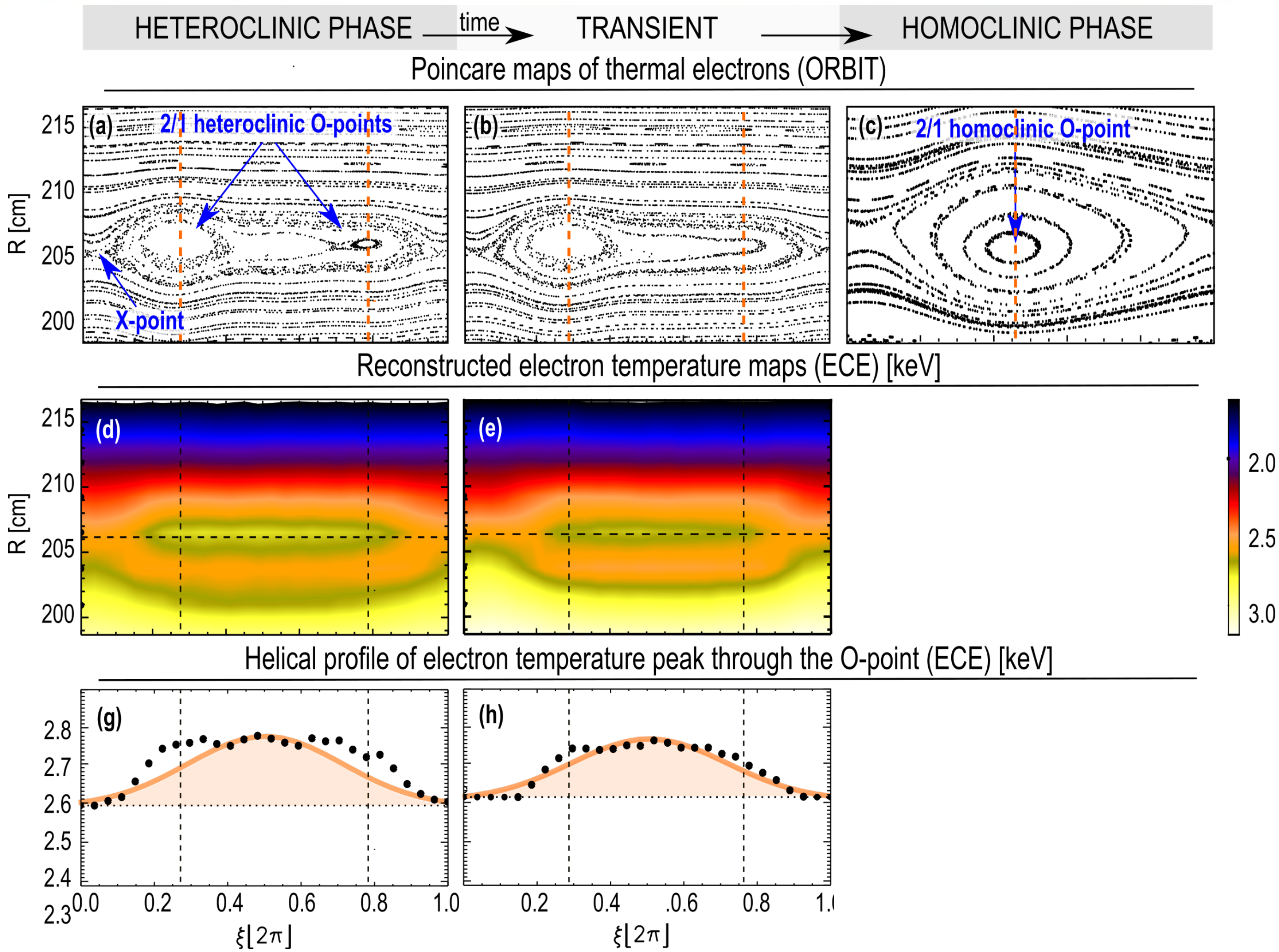

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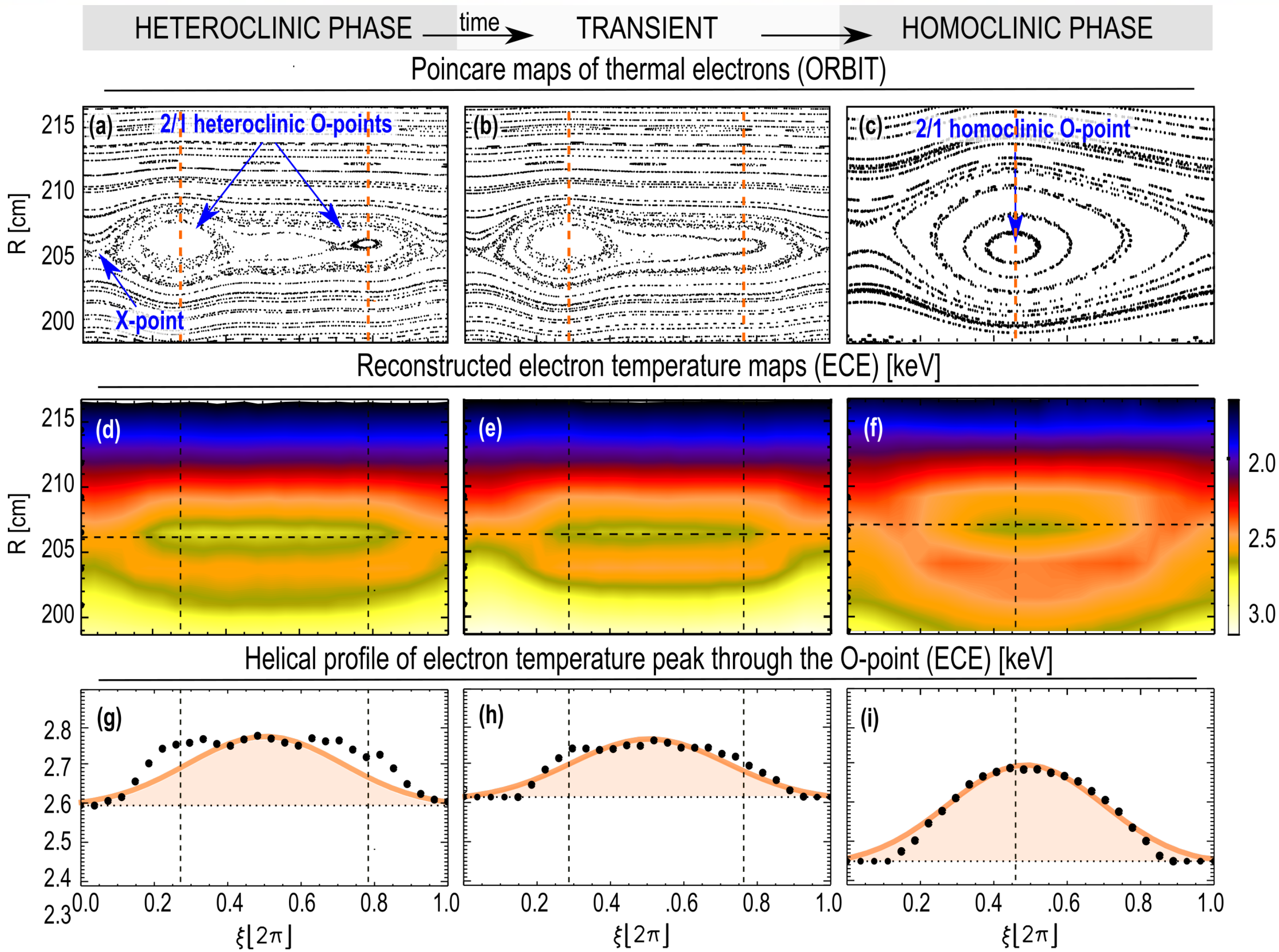

- Measured Pi/2 phase shift between 4/2 and 2/1 O-points is a strong constraint and is well matched by T_e in the early evolution.
- T_e data supports bifurcation from heteroclinic to monoclinic phase

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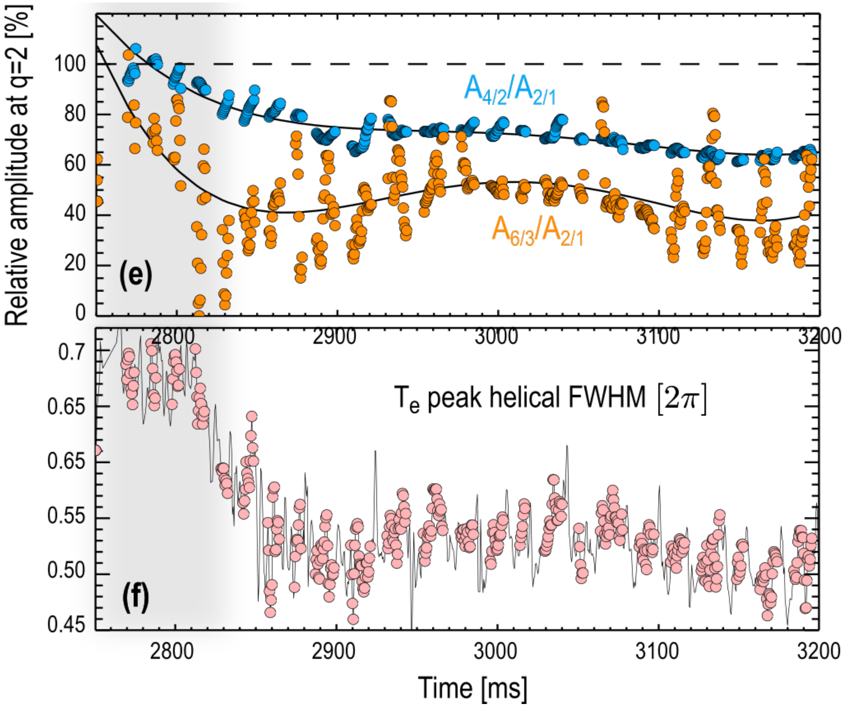
Time Trance of $\triangle T_e$ Width Shows 2 Preferred Solutions, Transition Correlates With 4/2 Relative Amplitude



- There are two solutions for the ΔT_e width
- ΔT_e is consistent with double O-points when heteroclinic structure is expected based on the TM amplitudes.

- Below a threshold relative amplitude of ~80% ΔT_e is consistent with the island having a single O-point.

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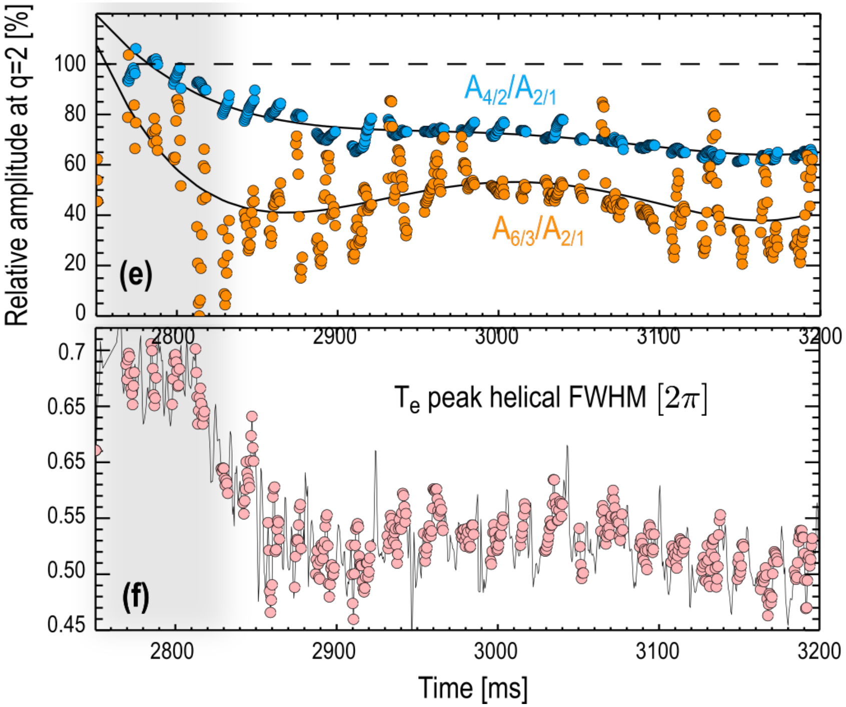

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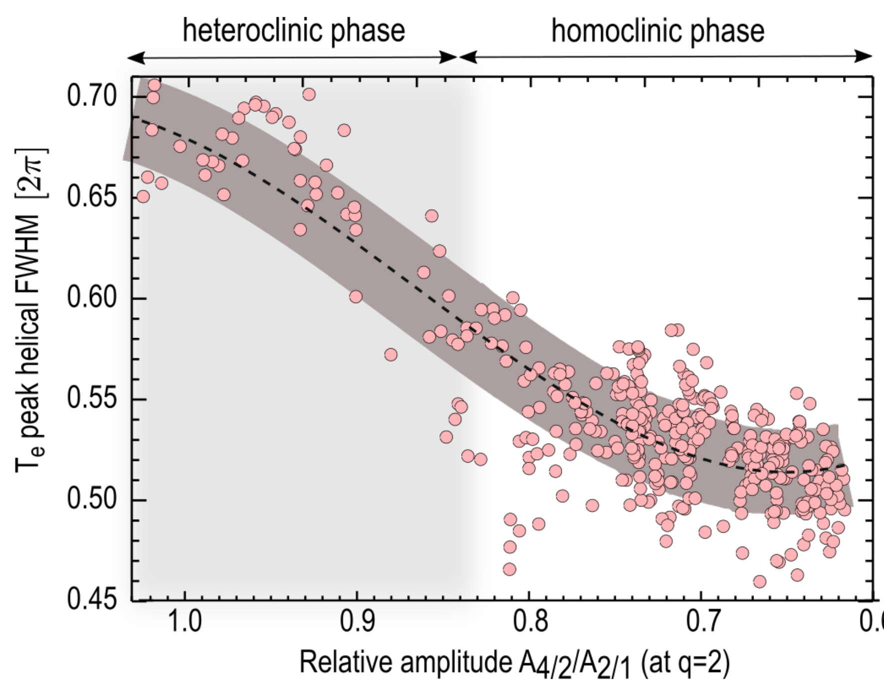

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1. **Flux Tunneling Between Magnetic Island Chains**
   L. Bardóczi and T.E. Evans 2021 *Nucl. Fusion* **61** 074001 (2021)

2. **Magnetic Island Heteroclinic Bifurcation**
   L. Bardóczi and T. E. Evans Phys. Rev. Lett. **126**, 085003 (2021)

3. **NTM Seeding by Nonlinear Three-Wave Interactions**
   L. Bardóczi, N. C. Logan and E. J. Strait, Phys Rev. Lett. awaiting publication (2021)

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Nonlinear Three-Wave Coupling of Magnetic Islands Predicts New Mechanism for Disruptive 2,1 NTM Seeding

Background/Motivation:

- NTM prevention by removal of 2,1 seeding mechanisms is important for stable operation of future reactors (e.g. sawtooth and ELM control) [1].
- The theory of nonlinear 3-wave coupling applies to MI triplets [2-7].

  → Potential new type of disruptive NTM seeding in tokamaks. e.g. 3,2 – 1,1 → 2,1.

Importance:

- Complicates NTM prevention by removal of 2,1 seeding mechanisms. Calls for high differential rotation and avoidance of all tearing activity as much as possible.

[1] O. Sauter et al, PRL **88**, 105001 (2002); **[2]** C. C. Hegna, PoP **3** 4646 (1996);
[3] R. Fitzpatrick, PoP **22** 042514 (2015)**;** **[4]** S. Asadi, et al. PRL , **69** 2 (1992);
[5] B. Tobias, et al. PoP, **23** 056107 (2016); **[6]** E. J. Strait, et al. PRL, **62** 11 1282, (1989) ;
[7] M.F.F. Nave et al, NF **43** 179 (2003);

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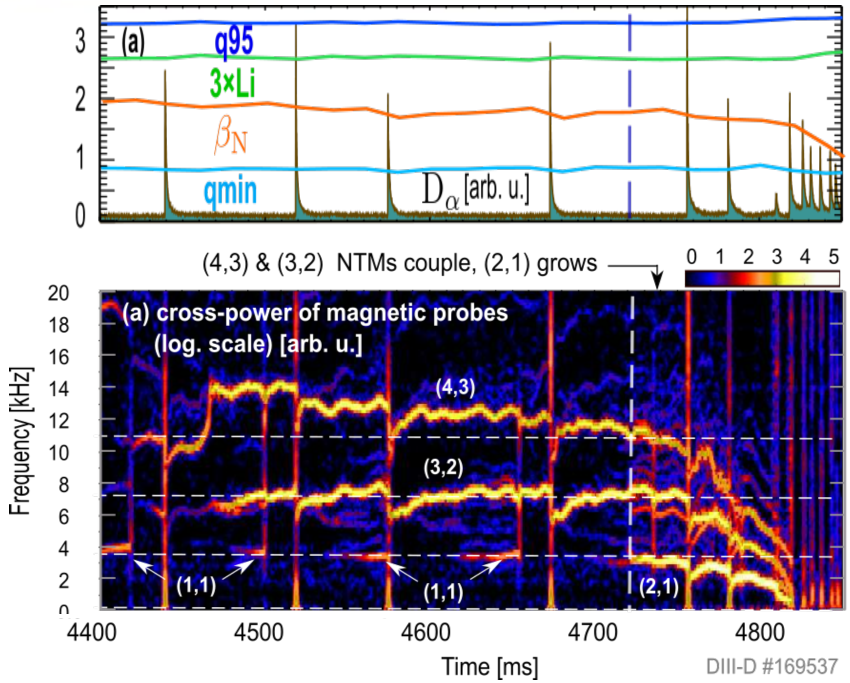
2,1 NTMs are Seeded by Nonlinear Three-Wave Interactions

- Discharges with ELM and sawtooth crashes
- β in flattop & and j(r) fully relaxed.
- The plasma is robustly stable to classical tearing modes (RDCON [1])



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[1] A. H. Glasser. Phys. Plasmas, 23(072505), 2016.

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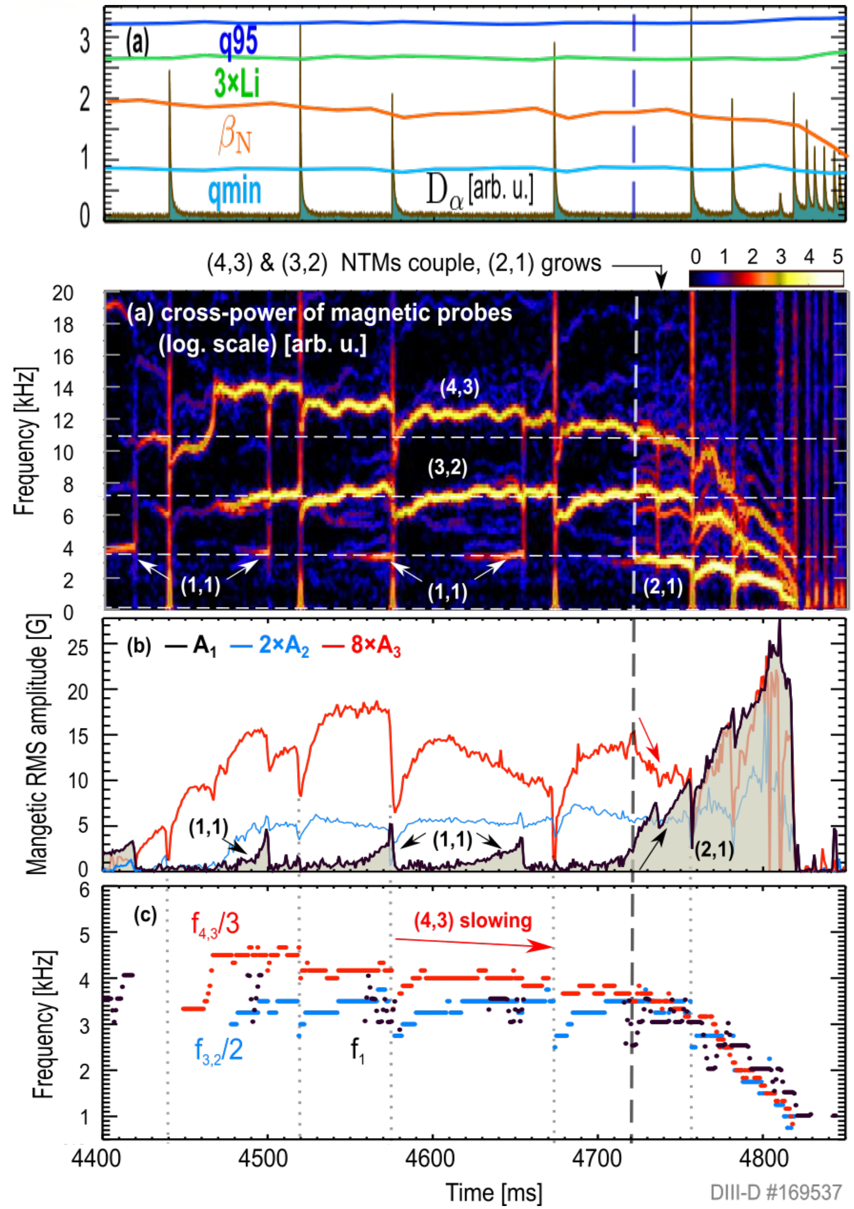
- The plasma is robustly stable to classical tearing modes (RDCON [1])

- 4,3 slows & couples to 3,2 when 2,1 is seeded
- The following 3-wave relations are satisfied:

$$(m,n=4,3)-(m,n=3,2)\longrightarrow(m,n=1,1) \qquad (1)$$
$$(m,n=3,2)-(m,n=1,1)\longrightarrow(m,n=2,1). \qquad (2)$$

- 4,3 amplitude drops when 2,1 grows,
  → consistent with the 4,3 driving the 2,1



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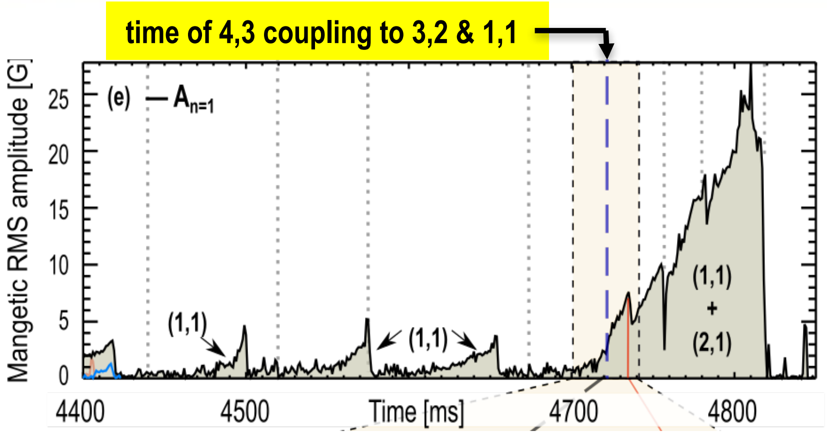
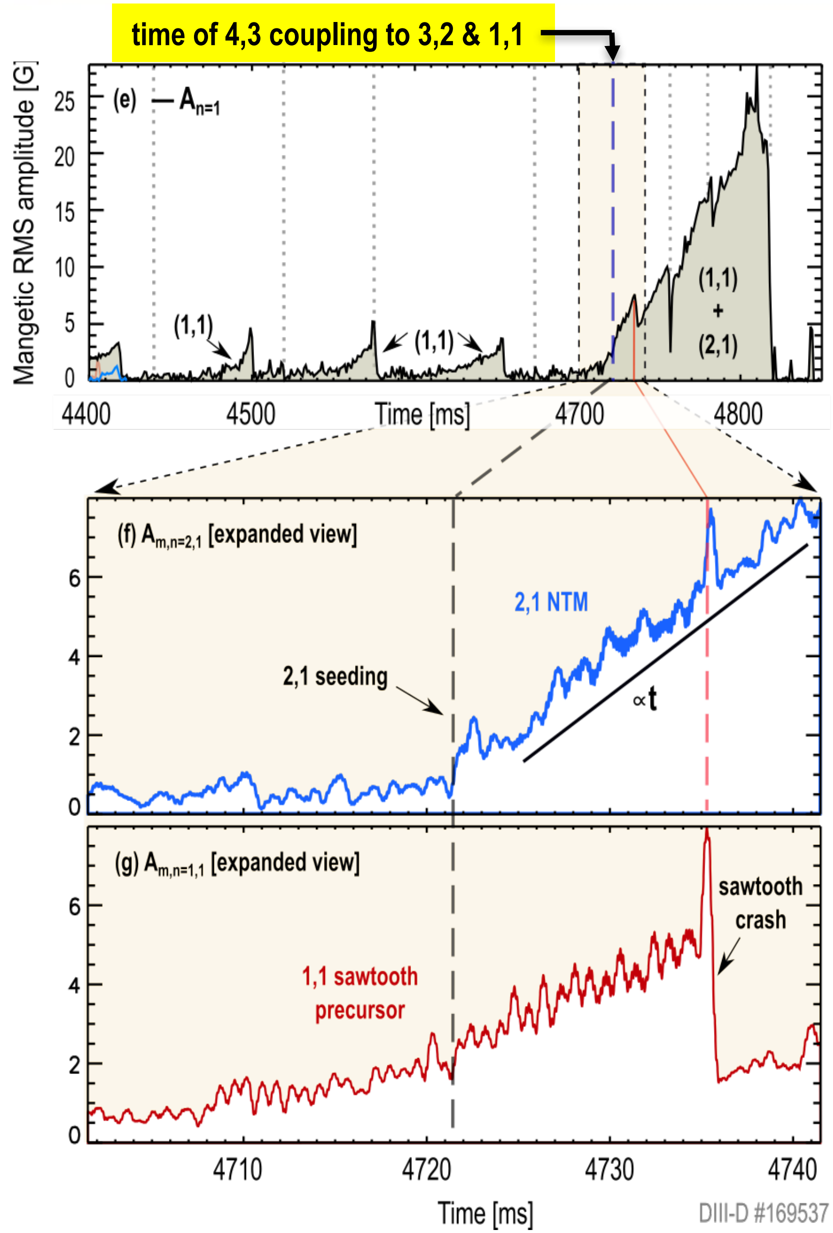


2,1 NTMs are Seeded by Nonlinear Three-Wave Interactions

- Combination of HFS and LFS magnetic probes enable to isolate m=1 & m=2 in the n=1 signal
- Seeding: 2,1 rises by 1G when modes couple
- 3-wave relations are satisfied, as 1,1 mode exists in the 3,2 frame at seeding (2G)
- 1,1 crash is not the cause of 2,1 seeding:
  (a) 1,1 crash occurs 16ms after the seeding
  (b) 2,1 is 6G at the time of the 1,1 crash and is not affected by it
- Linear 2,1 growth is consistent with NTM (classical TM grows as ~t^2)



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Magnetic Energy Balance of Cylindrical Model Shows Drop in 4,3 Amplitude Accounts for the 2,1 Seed Island

- Non-axisymmetric field perturbation:

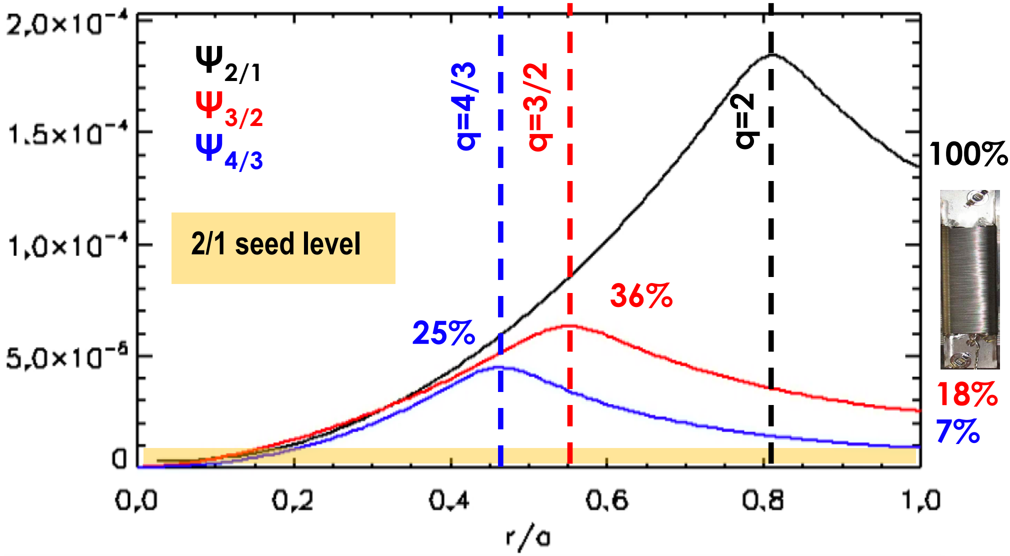
$$\tilde{\mathbf{B}}(\mathbf{r},\xi,\mathbf{t}) = \nabla \times (\hat{z}\Psi(r,\xi,t))$$

- Use radially localized helical current:

$$\tilde{\mathbf{j}}(r,\xi) = \tilde{\mathrm{j}}_{\circ}\cos(\xi)\delta(r_s - r)\mathbf{e}_z$$

- Ampere's law gives [1,2]:

$$\Psi_\delta(r,\xi) = \frac{\mu_\circ \tilde{\mathrm{j}}_\circ}{m^2} f(r)\cos(\xi)$$



- Model parameters are fully constrained by magnetic measurements.

$$f(r) = \frac{r^2}{r_s} \quad \text{at} \quad r < r_s, \quad f(r) = \frac{r_s^3}{r^2} \quad \text{at} \quad r > r_s$$

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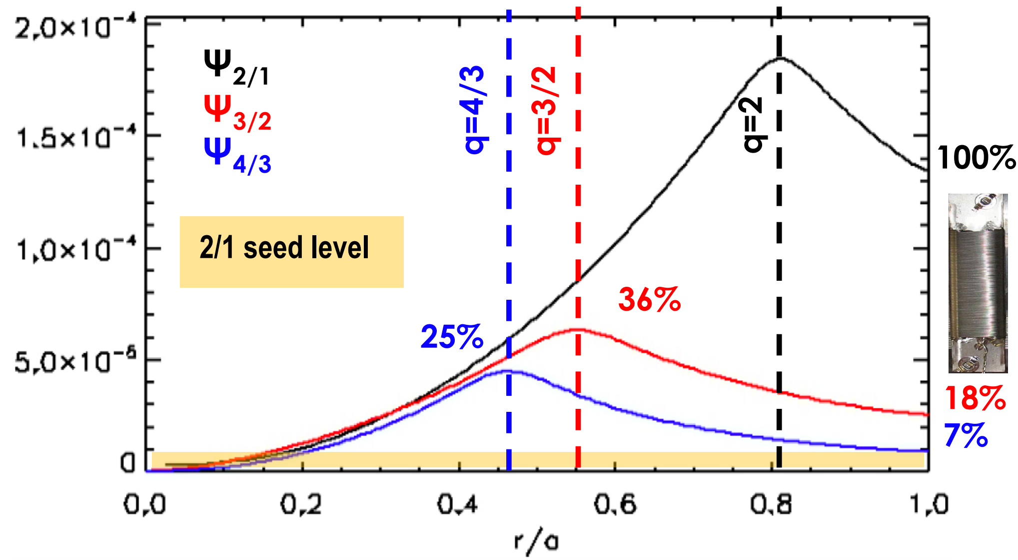

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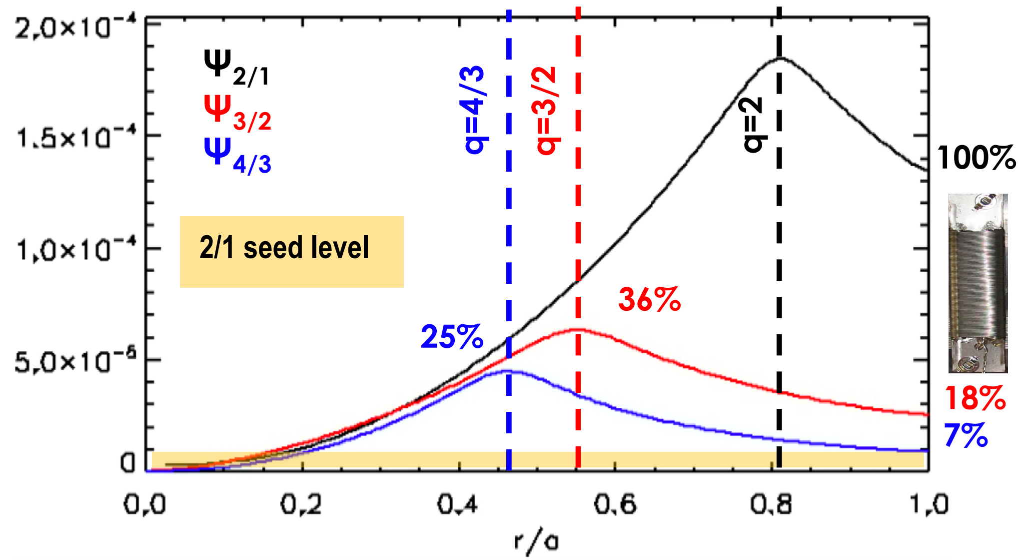

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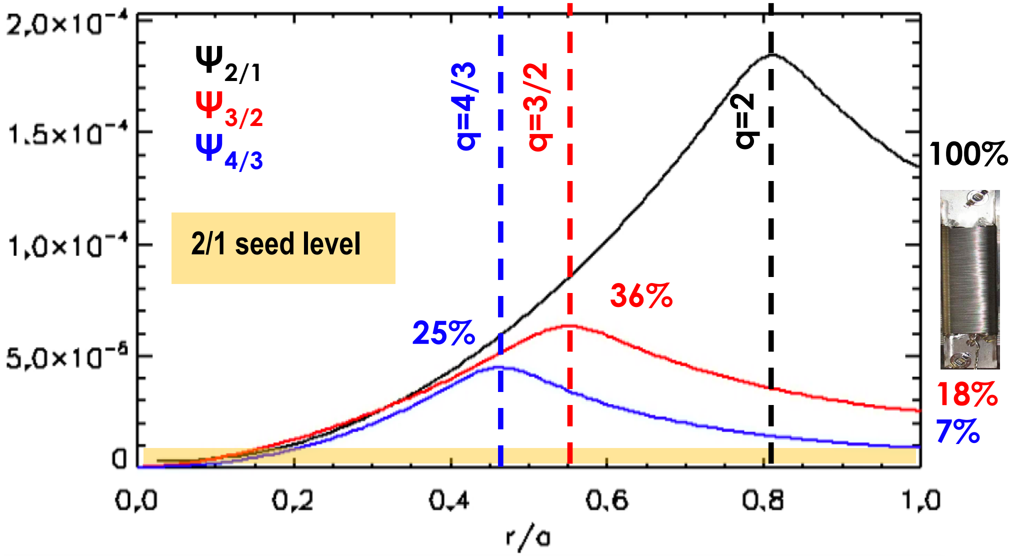

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3. **The observed drop of 4,3 magnetic energy at the time of seeding (~∫B^{2}dV) accounts for 96% of the 2,1 seed island magnetic energy.**

Fixed-Phase Relationships Are Identified by Bi-Coherence

- The bi-coherence is a statistical measure for quantifying the extent of phase coupling between frequency pairs in a single signal. Often used to identify non-linear interactions:

$$b^2 = \left\langle \frac{|\langle F_{i,n}(f_1)F_{i,n}(f_2)F^*_{i,n}(f_1+f_2)\rangle_n|^2}{\langle|F_{i,n}(f_1)F_{i,n}(f_2)|^2\rangle_n\langle|F^*_{i,n}(f_1+f_2)|^2\rangle_n} \right\rangle_i$$

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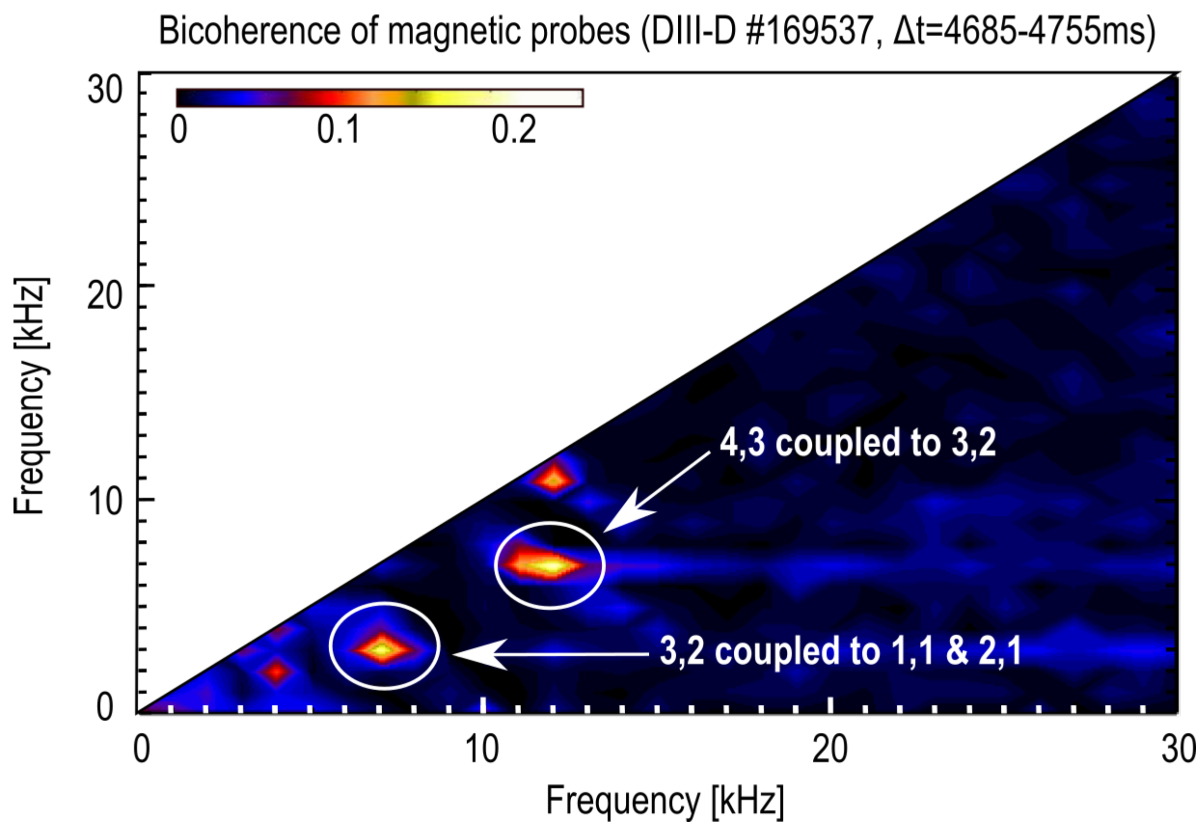
- Calculated in Δt=70ms window centered at the 2/1 seeding, averaged over 14 LFS midplane mag. probes.

Bi-coherence confirms phase-locked state between the 4,3 and 3,2, as well as the 3,2, 1,1 & 2,1.



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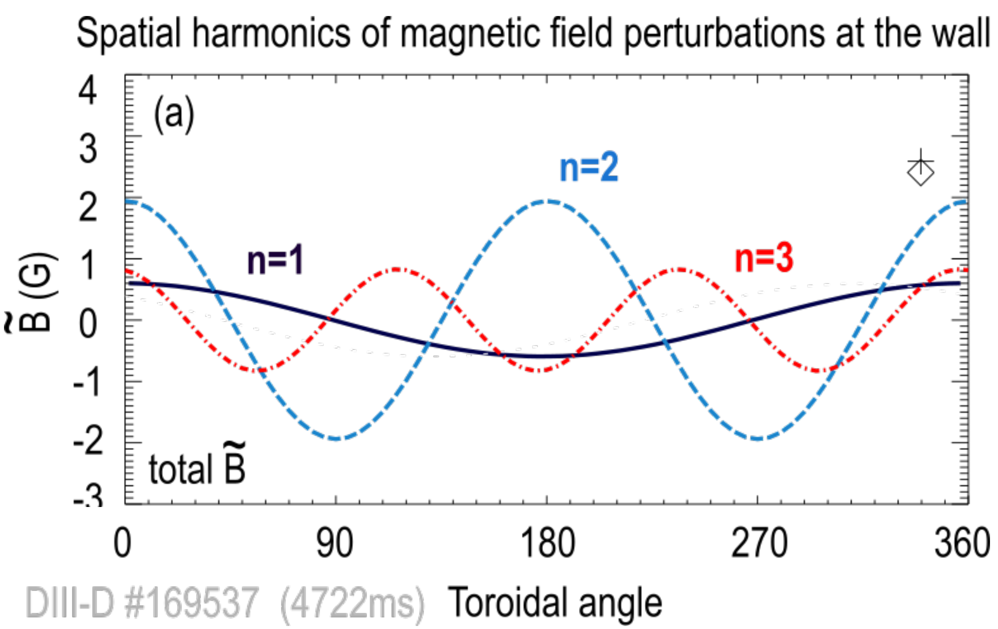
Phase Relationships Between Coupled Islands Agree With Theory

- The phase relationship between the 4, 3, 3, 2 & 2,1 is determined in the phase-locked state using the toroidal array of B_θ sensors @ LFS mid-plane.



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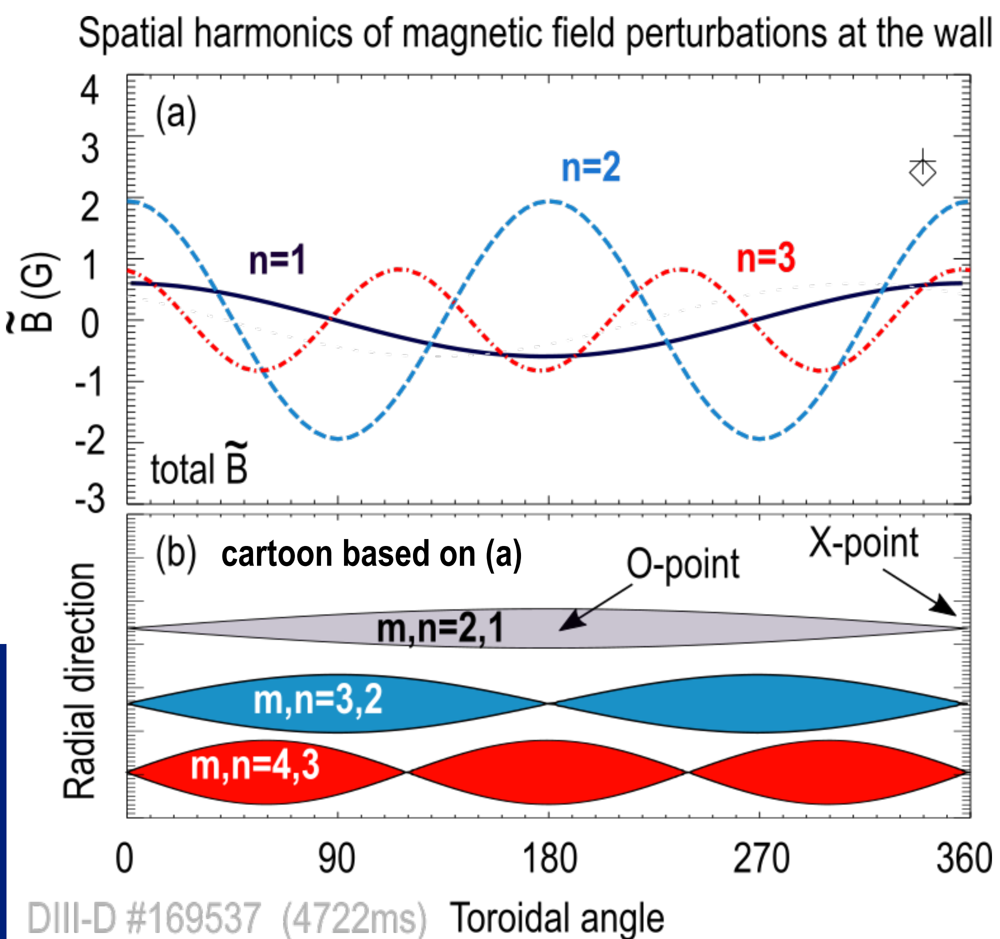
- The m,n island X-points (O-points) correspond to maxima (minima) of the corresponding n^{th} harmonic.

The phase-locked state is characterized by the alignment of one of the X-points in the outboard mid-plane, in agreement w theory [1]



[1] R. Fitzpatrick, PoP **22** 042514 (2015);

Summary

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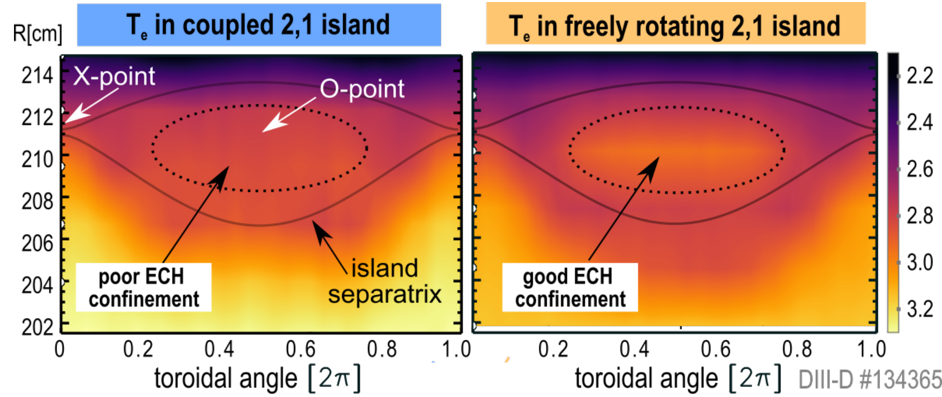

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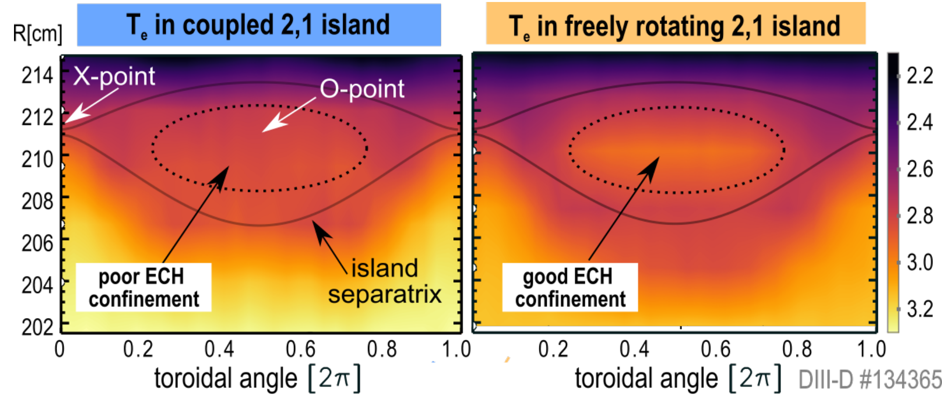

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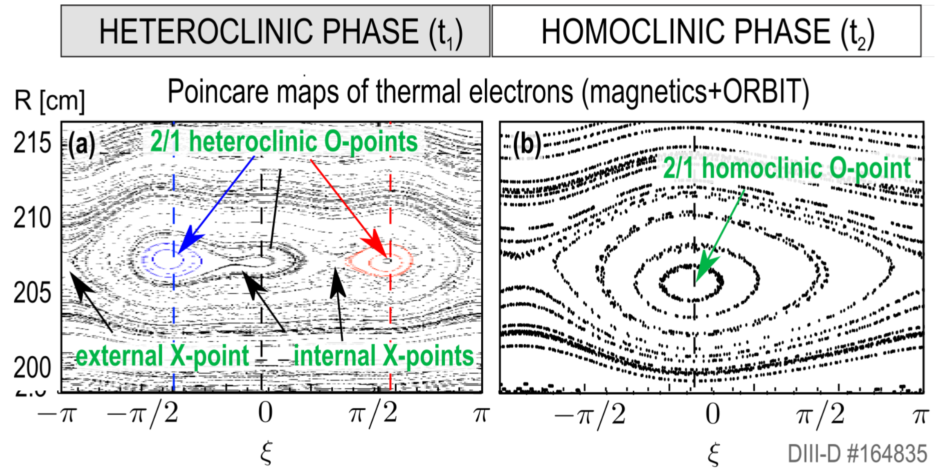

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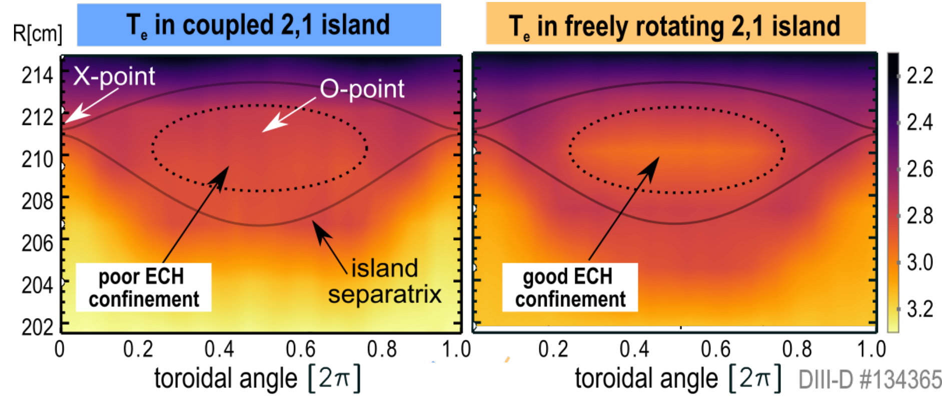

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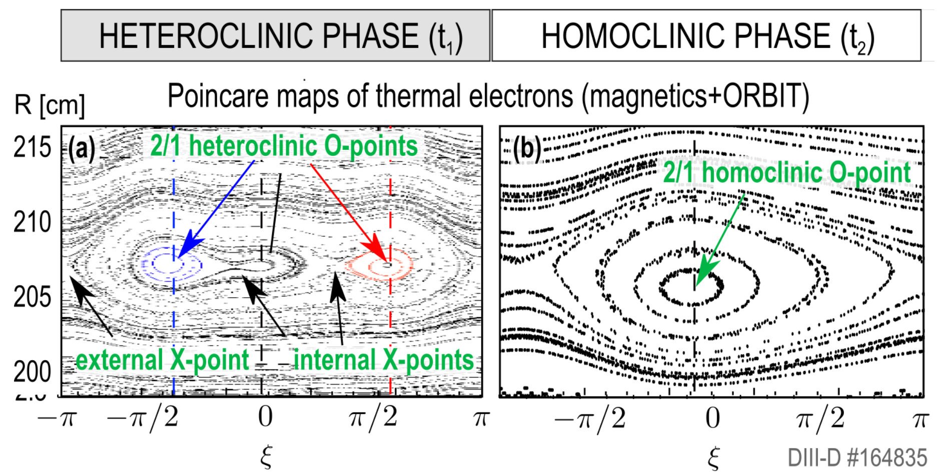

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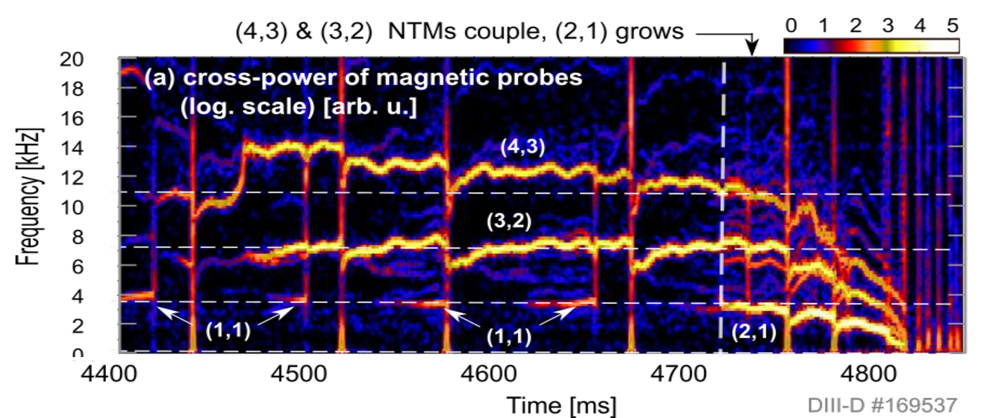

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Future work

1. **Flux tunneling**
   - Quantitatively evaluate the degree of ECCD loss due to stochastization and it's impact within the MRE.

2. **Heteroclinic bifurcation**
   - What causes heteroclinic bifurcations?
   - How does the heteroclinic bifurcation impact the ECCD efficiency?
   - Do coupling to other island chains affect the heteroclinic bifurcation?

3. **Seeding by non-linear 3-wave interactions**
   - Can 3-wave seeding be removed by
     - high(er) differential rotation at q=2?
     - reducing the 1,1 mode amplitude with central ECH?
     - removing the 3,2 mode with ECCD at q=1.5?

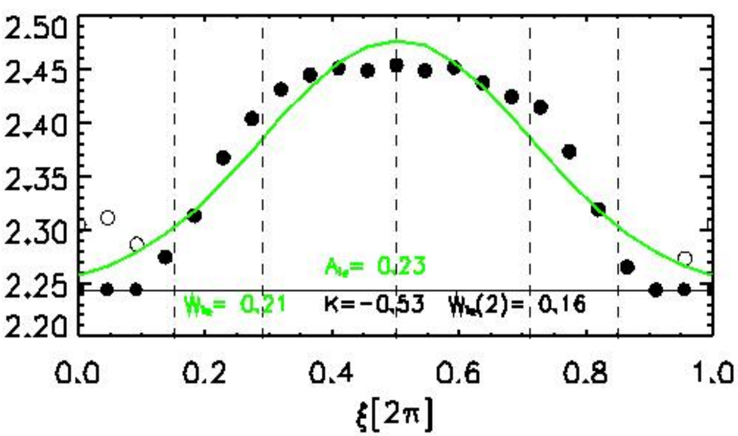
Thank You For Your Attention!

Extras

Heteroclinic Bifurcation: Result is Reproducible & Observed Only When 4/2 is Present

- Profile with split ΔT_e is seen in 50ms data in 25 thousand ECE points
- Time resolution of analysis does not affect the result.
- 2 more shots with the right conditions, both show signatures of ΔT_e splitting
- Discharges with $A_{4/2}/A_{2/1}$< 10% don't show peak splitting.

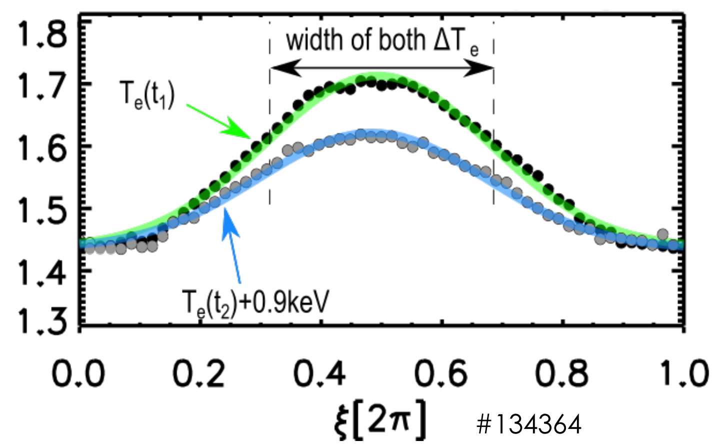
with 4/2 mode



#164831

t_{CENTER}=1715.00ms (10ms)

without 4/2 mode



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Heteroclinic Bifurcation: Possible Alternative Explanations?

1. High $\chi_\perp$ in the O-point region could cause flattening.
   - Why only in the middle, why is helically elongated, why not in the large island?

2. Large parallel diffusivity in the O-point region.
   - If so, the peak would get even more flat as the island grows because the connection length decreases with the island width.
   - It doesn't explain the elongated shape of the flat top of the ΔT_e peak.
     In an island with nested flux surfaces high $\chi_{//}$ would lead to "circular" flat top.

3. Another island with m/n not equal to 2/1 could cause stochastization and flux tunneling if it is rotation coupled.
   - There are no such islands in this plasma

4. Modulation of rotation frequency could result in fake n=2 component. Analysis of spatial structure from toroidal array of magnetic probes confirms n=2 is real.

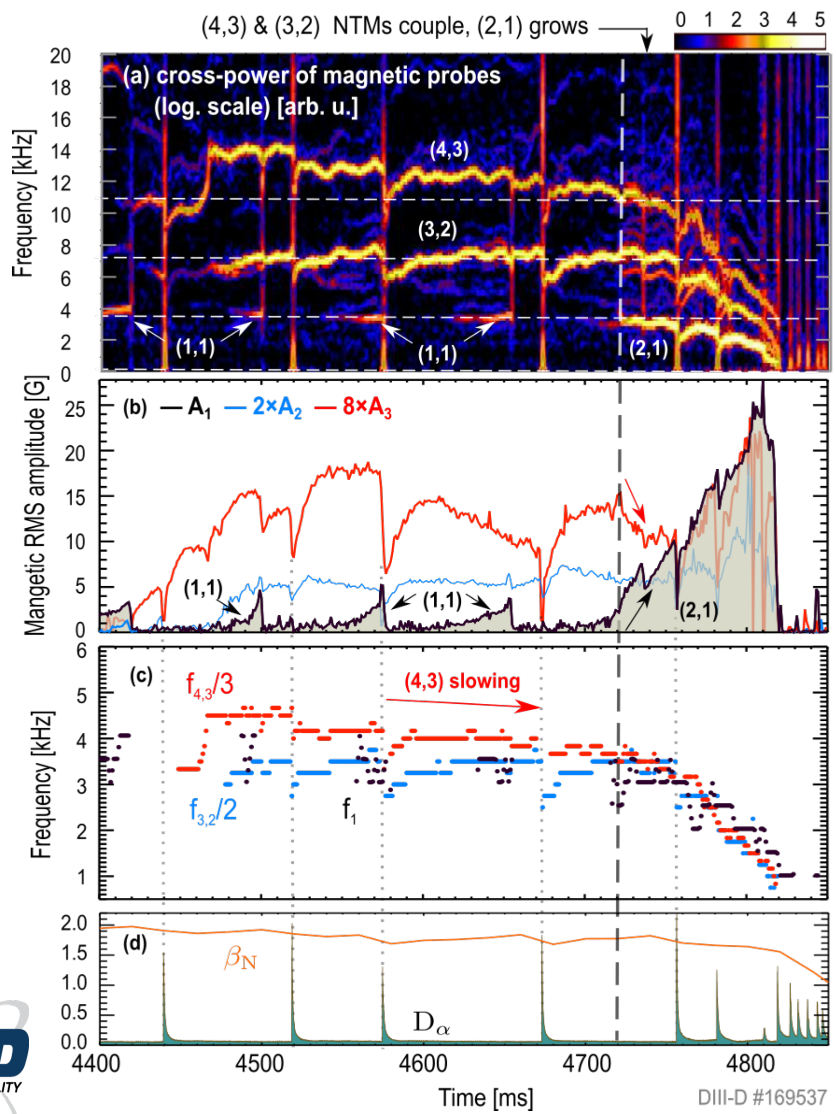
General reasons "for" that can't be explained by either of the above:

- the ΔT_e width correlates with expected O-point locations from the measured phase
- the narrowing of ΔT_e correlates with the n=2 amplitude
- 1-3 his should happen in other shots without m/n=4/2 islands

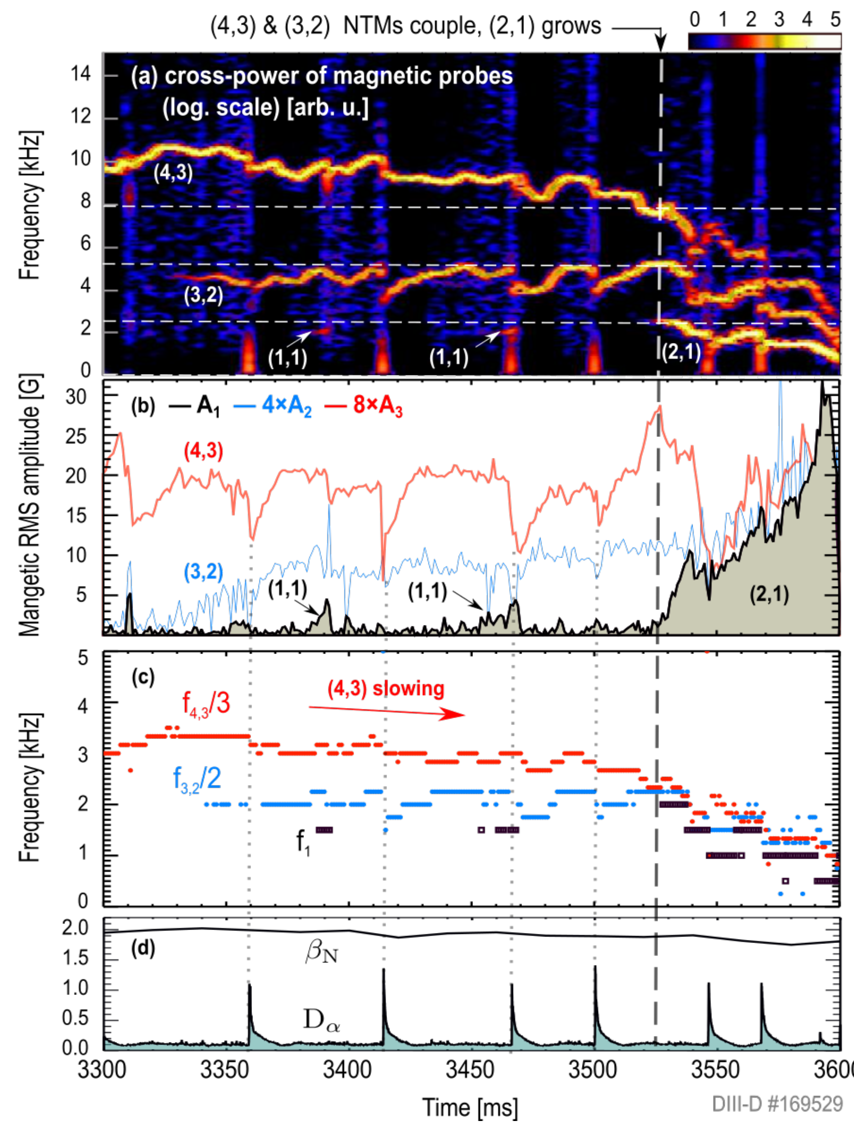
There is no better explanation of this data then the heteroclinic bifurcation.

2/1 Seeding by Nonlinear Three-Wave Interaction is Observed in Multiple Discharges

Example #1 [repeat]



Example #2



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