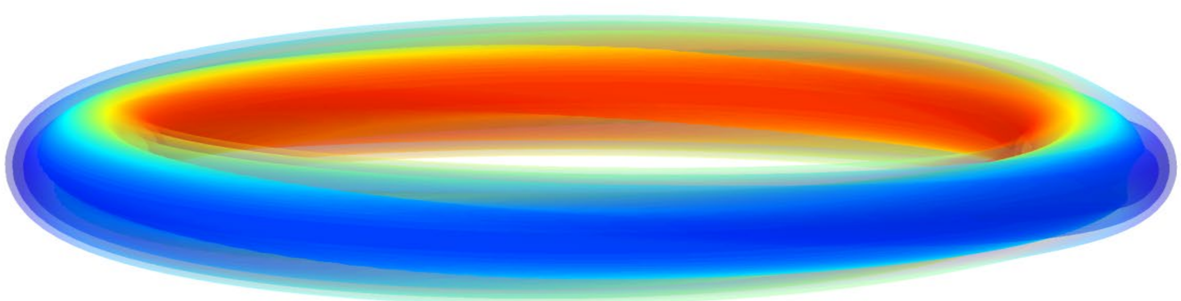
Simulation of Plateau Formation During Current Quench and MHD Instabilities with Runaway Electrons

Chen Zhao



outline

- **Introduction of M3D-C1**
- **MHD instabilities with runaway electron**
- **Runaway electron plateau during the current quench**
- **Summary**

▪ 3D Extended MHD Equations in M3D-C[1]

Density equation

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \cdot D_n \nabla n + S_n$$

Field equation

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_\perp \cdot \frac{1}{R^2}\nabla\Phi = -\nabla_\perp \cdot \frac{1}{R^2}\mathbf{E}$$

Momentum equation

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla\mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_i + \mathbf{S}_m$$

Generalized Ohm's law

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \bullet \mathbf{\Pi}_e\right) - \frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \bullet \nabla \mathbf{V}_e\right) + \mathbf{S}_{CD}$$

Pressure equations

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla \bullet \left(p_e \mathbf{V}\right)\right] = -p_e \nabla \bullet \mathbf{V} + \frac{\mathbf{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n + \mathbf{R}_c\right] + \nabla\left(\frac{\mathbf{J}}{ne}\right) : \mathbf{\Pi}_e - \nabla \bullet \mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t} + \nabla \bullet \left(p_i \mathbf{V}\right)\right] = -p_i \nabla \bullet \mathbf{V} - \mathbf{\Pi}_i : \nabla\mathbf{V} - \nabla \bullet \mathbf{q}_i - Q_\Delta + S_{iE}$$

$$\mathbf{R}_c = \eta ne\mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^\dagger\right] - 2(\mu_c - \mu)(\nabla \bullet \mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV} \qquad \mathbf{q}_{e,i} = -\kappa_{e,i}\nabla T_{e,i} - \kappa_\parallel \nabla_\parallel T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B}/B^2)\nabla \bullet \left[\lambda_h \nabla\left(\mathbf{J} \bullet \mathbf{B}/B^2\right)\right], \qquad Q_\Delta = 3m_e(p_i - p_e)/\left(M_i \tau_e\right)$$

Blue terms are 2-fluid(thermal electron) terms. Also, now have impurity, pellet, high energy particle models (both CPU & GPU version) for disruption mitigation. **NOT reduced MHD.**

▪ Fluid Runaway Electron Model

$$\frac{\partial n_{RE}}{\partial t} + \nabla \bullet (n_{RE} c \frac{\mathbf{B}}{B}) = \nabla \bullet \left(\mathbf{B} \frac{D_{RE}}{B^2} \mathbf{B} \bullet \nabla n_{RE} \right) + S_{RE}$$ RE density equation

$$\mathbf{J}_{RE} = -e n_{RE} c \frac{\mathbf{B}}{B}$$ RE current assumption

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne} (\mathbf{R}_c - \mathbf{R}_{RE} - \nabla \bullet \Pi_e) + \mathbf{S}_{CD}$$ Single fluid Ohm's law with RE

Red terms are additional runaway electron terms.

D_{RE} is the diffusion term of runaways, $\mathbf{R}_{RE} = \eta n e \mathbf{J}_{RE}$.

- In our model, the runaways move at a large speed c (~100-200 Alfven speed) parallel to the magnetic field.
- We have parallel diffusion term to RE in 3D nonlinear simulation to stabilize numerical instabilities.
- Runaway electrons are coupled to bulk plasmas through the runaway current in generalized Ohm's law.
- The runaway source includes both the Drecier term and the Rosenbluth – Putvinski avalanche term.

outline

- **Introduction of M3D-C1**
- **MHD instabilities with runaway electron**
- **Runaway electron plateau during the current quench**
- **Summary**

▪ Reduced MHD equations with runaway current

$$\omega\psi - k_{||}\phi = i\eta\left(\nabla_\perp^2\psi + j\right)$$

$$\omega\nabla_\perp^2\phi - k_{||}\nabla_\perp^2\psi = \frac{mj_0'}{r}\psi$$

$$\left(k_{||} + \omega v_A/c\right) j = \frac{mj_0'}{r}\left(\psi + v_A\phi/c\right)$$

Where $\nabla_\perp^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{m^2}{r^2}$

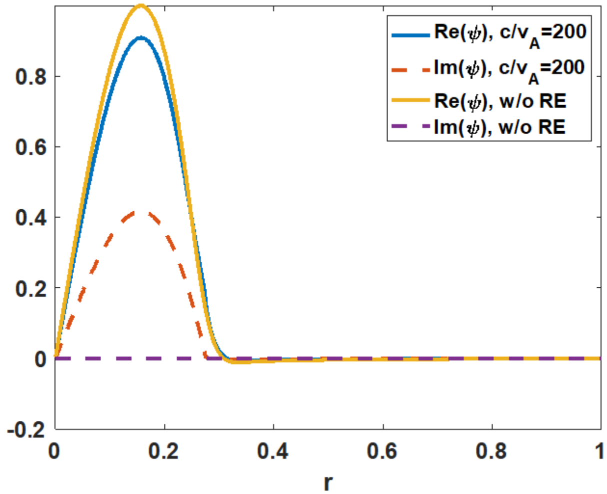
$k_{||} = \frac{nq(r)-m}{r}, \quad j_0' = \frac{d}{dr}j_0,$

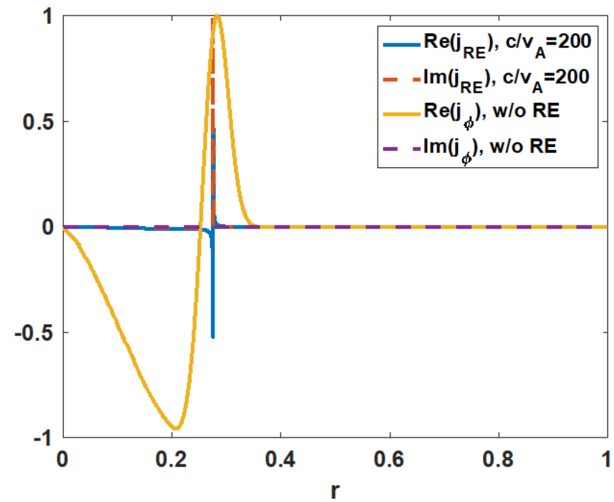
and j is runaway current.

- We transform the equations (P. Helander, 2007) to the matrix and use Matlab eigenvalue solver to get the eigenvalue ω (real frequency and growth rate) and eigenvectors Ψ, ϕ, j (mode structure) of the mode.

$$\omega\begin{bmatrix} I & 0 & 0 \\ 0 & \nabla_\perp^2 & 0 \\ 0 & 0 & v_A/c \end{bmatrix}\begin{bmatrix}\psi \\ \phi \\ j\end{bmatrix} = \begin{bmatrix} i\eta\nabla_\perp^2 & k_{||} & i\eta \\ k_{||}\nabla_\perp^2 + mj_0'/r & 0 & 0 \\ mj_0'/r & mj_0'v_A/rc & -k_{||}\end{bmatrix}\begin{bmatrix}\psi \\ \phi \\ j\end{bmatrix}$$

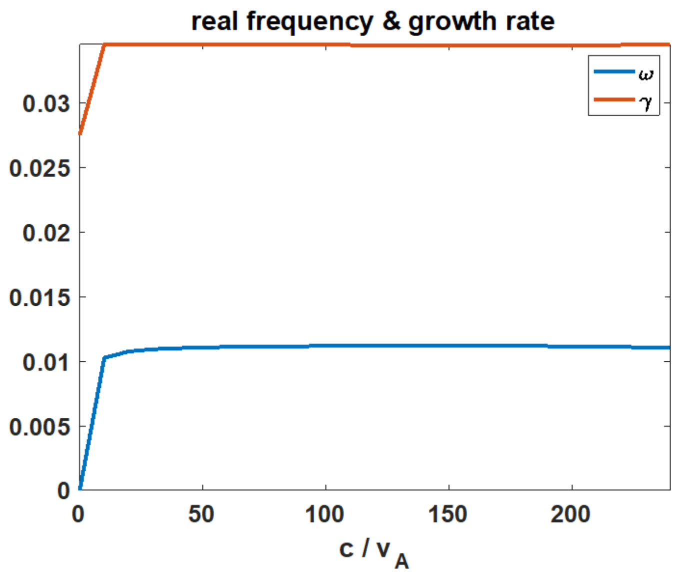
- # Eigenfunction solution of 1/1 resistive kink mode with RE

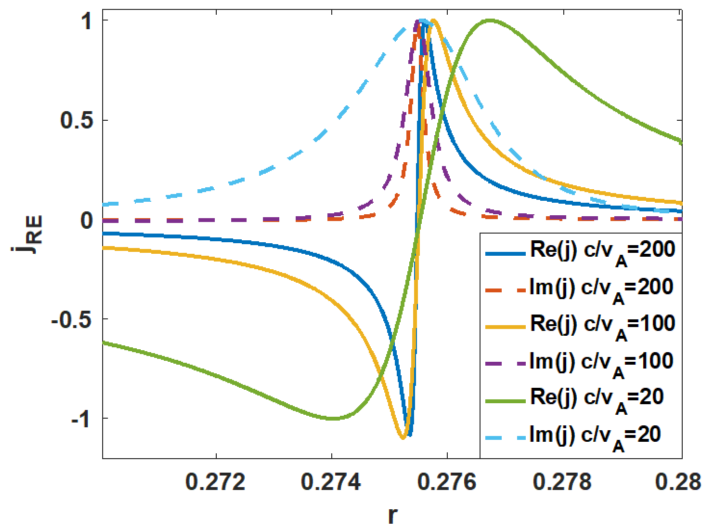




- The figures are the radial structures of magnetic flux and runaway/parallel current perturbations of 1/1 kink mode with and without runaways.
- The RE current steeply peaked around q = 1 surface. And the scale length is much smaller than the toroidal current with out runaways.

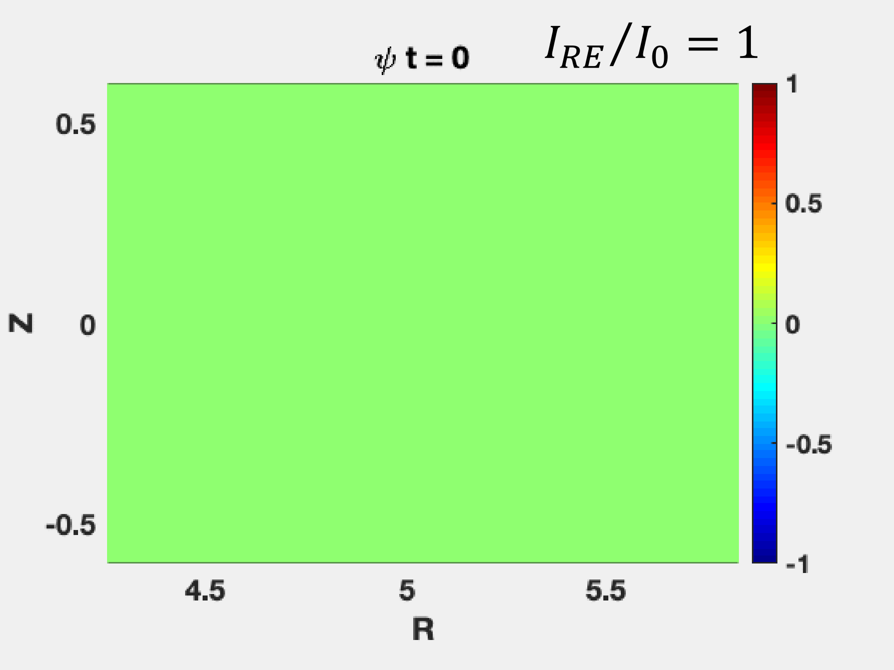
▪ Eigenvalue and Eigenvector of 1/1 resistive kink mode with different runaway velocity

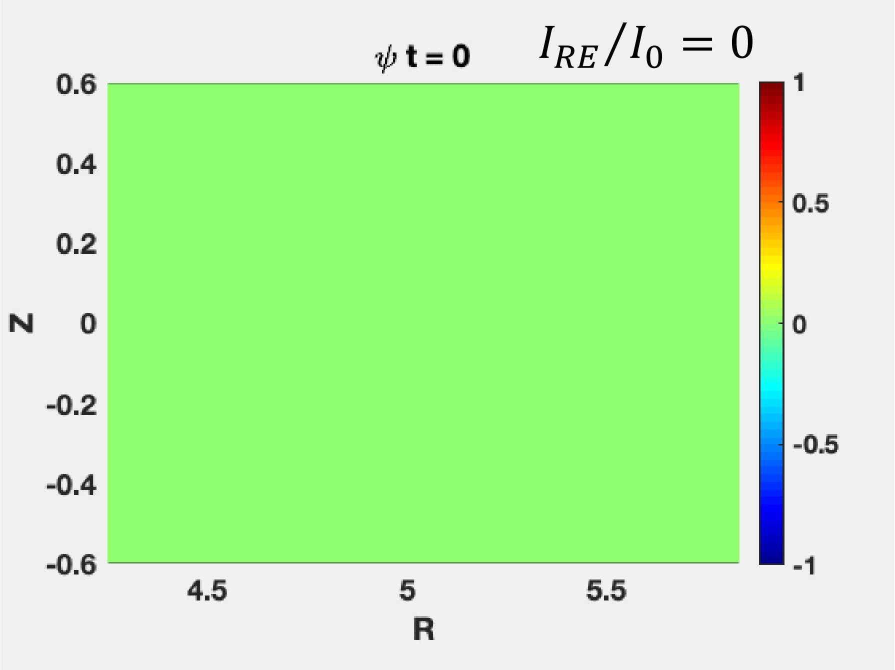




- The 1/1 mode has a non-zero and nearly constant real frequency when the RE speed lager than 10 times the Alfven velocity.
- The growth rate is also constant when RE speed lager than 10 times the Alfven velocity.
- The growth rate is about 3 times larger than real frequency.
- The runaway current scale length becomes smaller with increasing runaway speed.

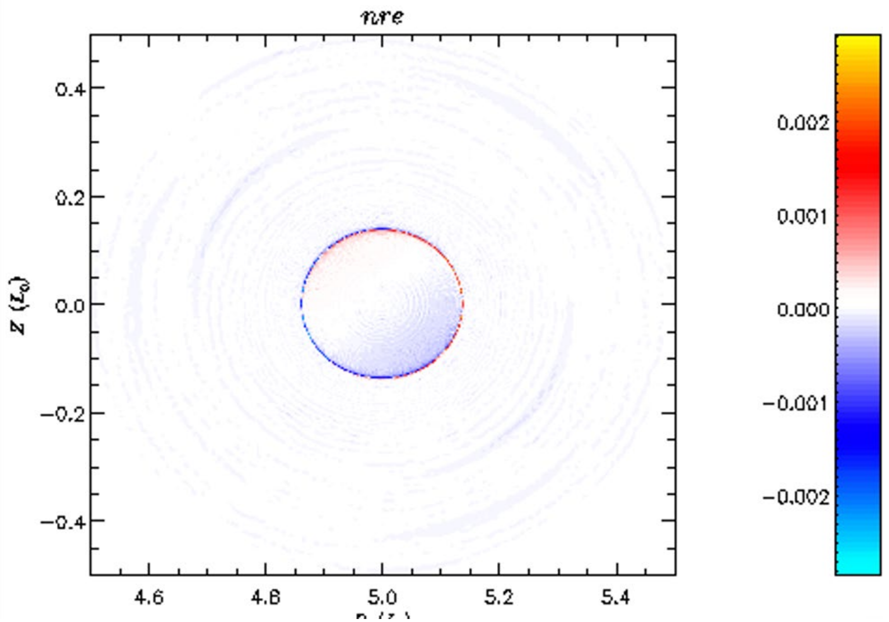
▪ Magnetic island of 1/1 resistive kink mode

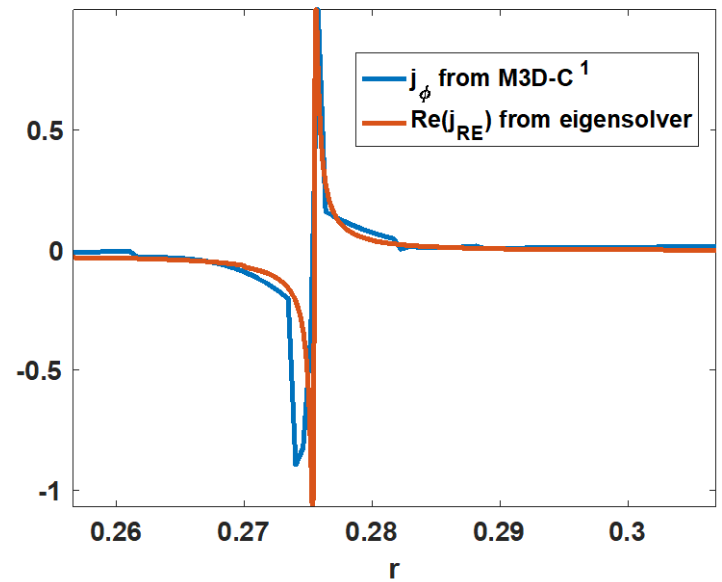




- The mode structure of 1/1 resistive kink mode with RE is similar with 1/1 kink mode with out RE.
- The runaways drive the 1/1 resistive kink mode propagate with a non-zero frequency.

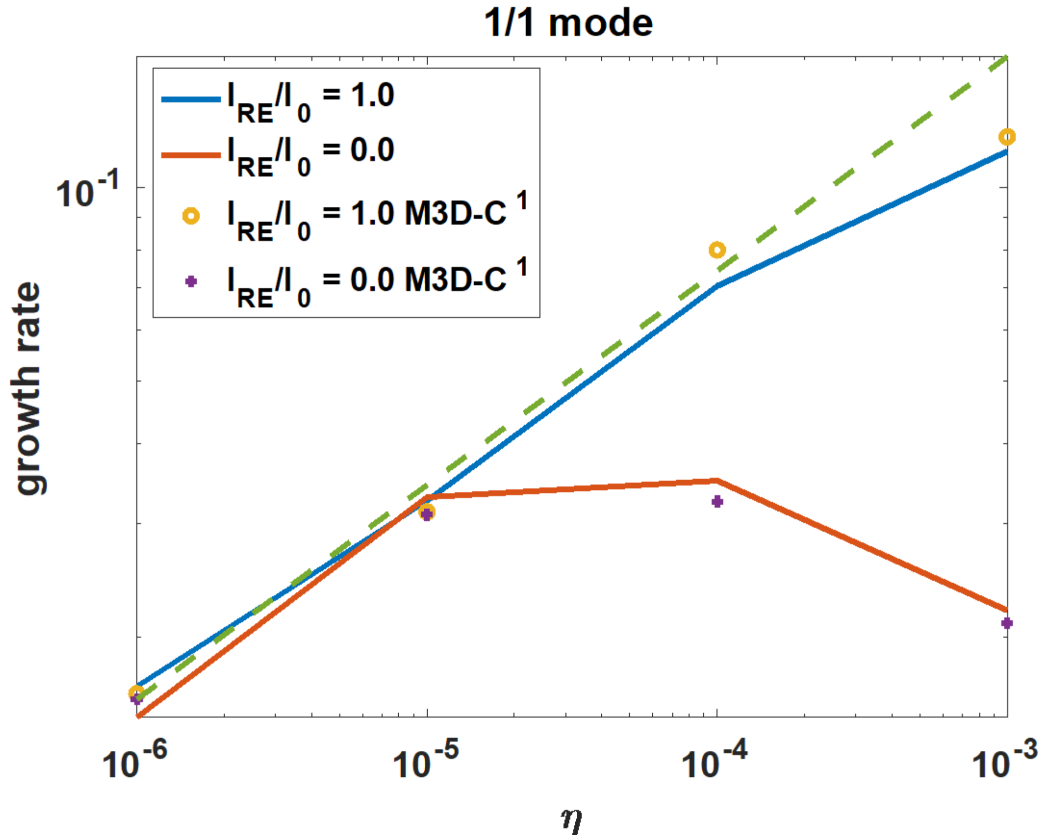
- **RE density and current perturbation of 1/1 resistive kink mode**





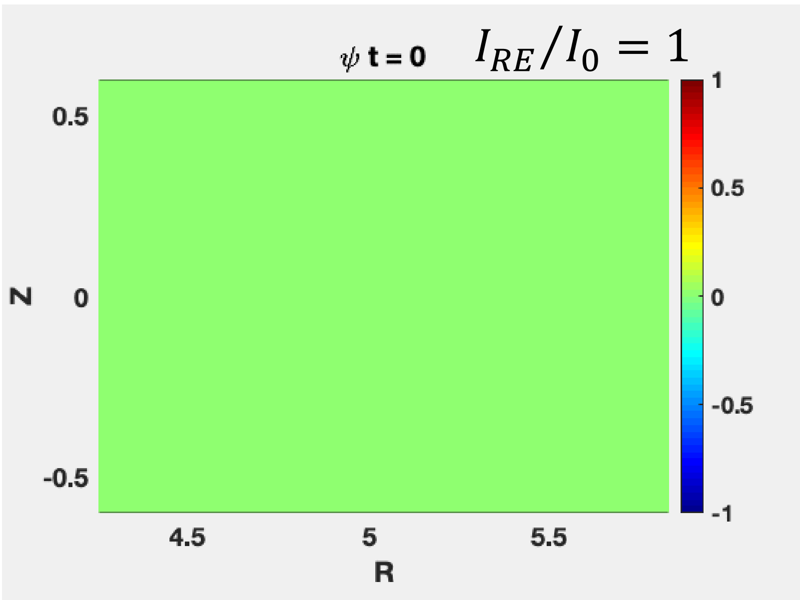
- The perturbed RE density is peaked around the q=1 surface and drives a toroidal current peaked around the rational surface. $\delta J_\phi \sim \delta J_{RE} = -e\delta n_{RE} c$
- The structure of RE current from M3D-C[1] is consistent with eigenvalue solver.

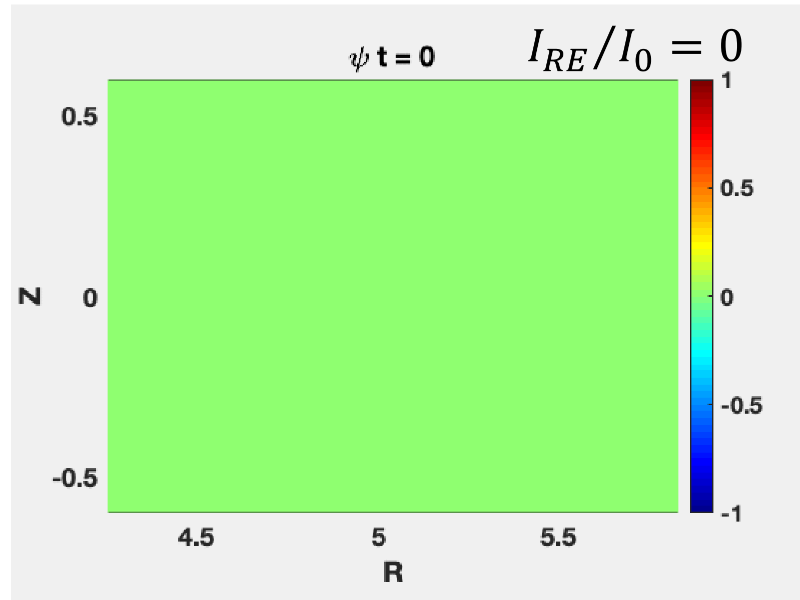
▪ Linear growth rate of 1/1 resistive kink mode with different resistivity



- For low resistivity cases, the growth rate of 1/1 kink mode with and without RE obeys the 1/3 law of resistivity.
- For higher resistivity cases, the runaway current has restrained the resistivity correction of kink mode.
- The result form M3D-C^1 is consistent with eigen solver.

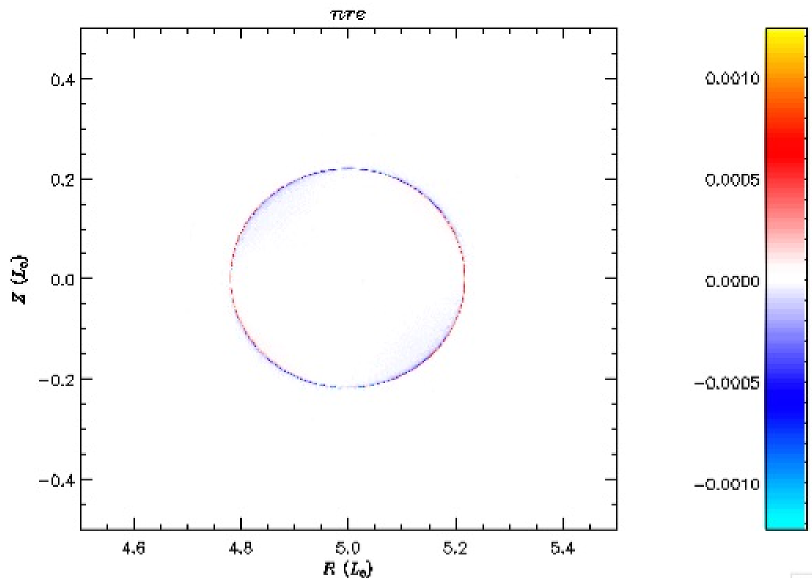
▪ Magnetic island of 2/1 tearing mode

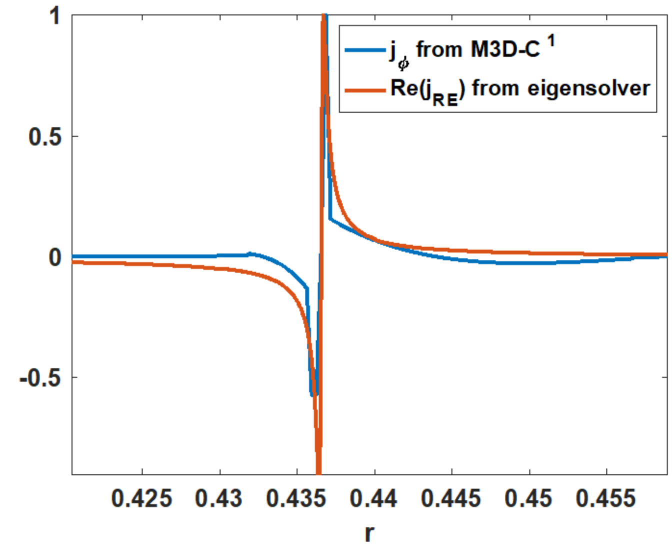




- The mode structure of 2/1 tearing mode with RE is similar with 2/1 tearing mode with out RE.
- The runaways drive the 2/1 tearing mode islands to rotate with a constant frequency.

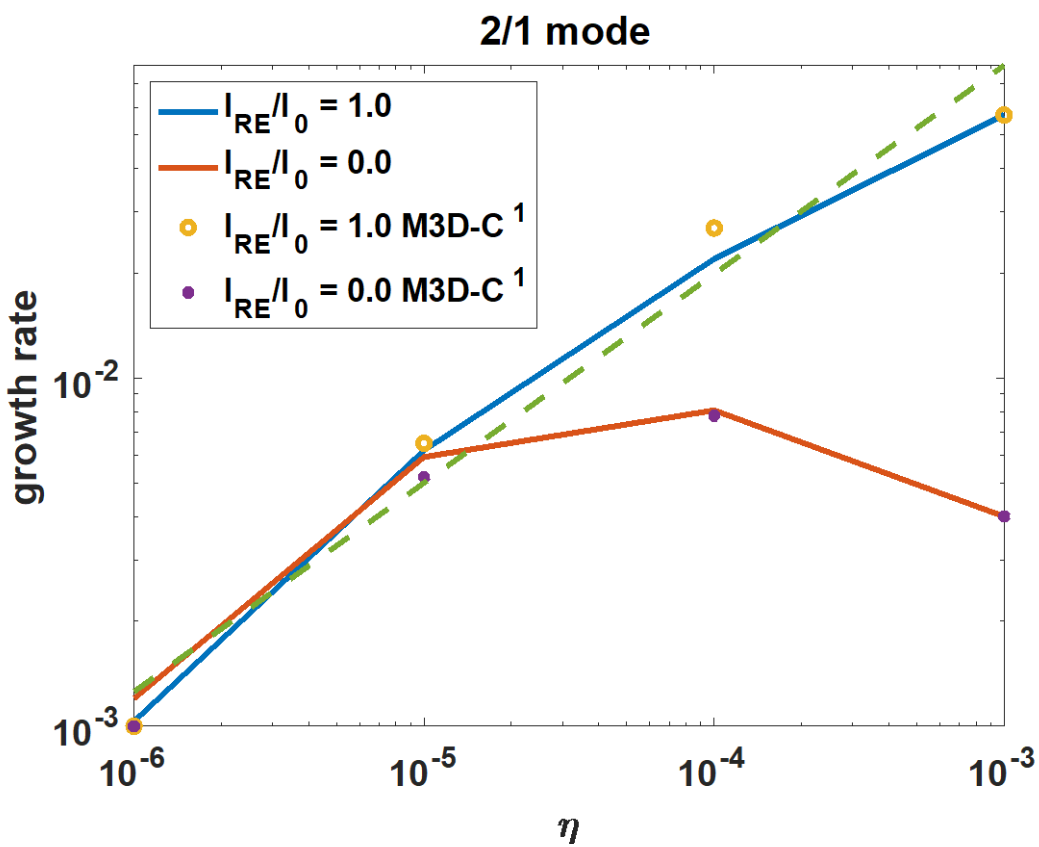
- **RE density and current perturbation of 2/1 tearing mode**





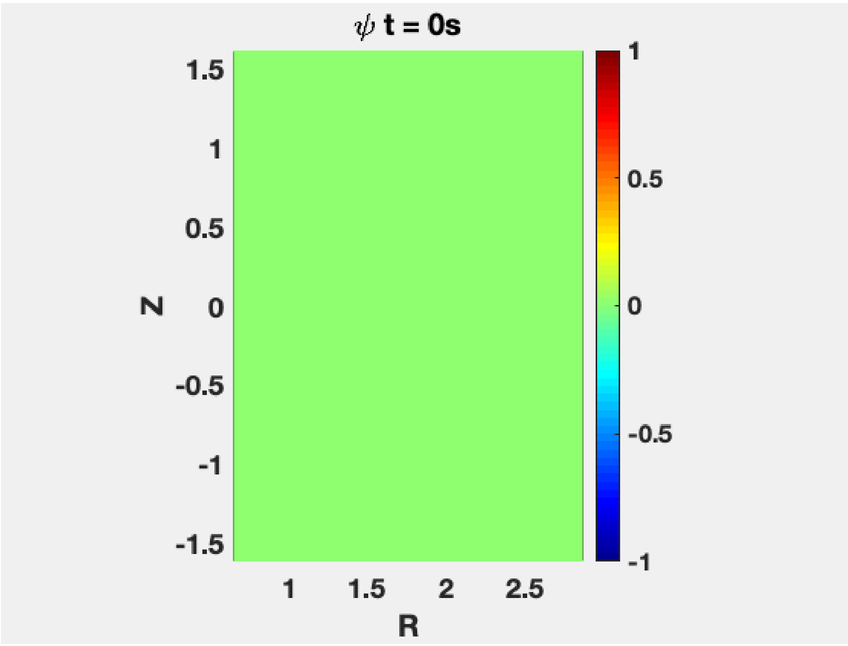
- The perturbed RE density is peaked around the q=2 surface and drives a toroidal current peaked around the rational surface. $\delta J_{\phi} \sim \delta J_{RE} = -e\delta n_{RE} c$
- The structure of RE current result from M3D-C[1] is consistent with eigenvalue solver.

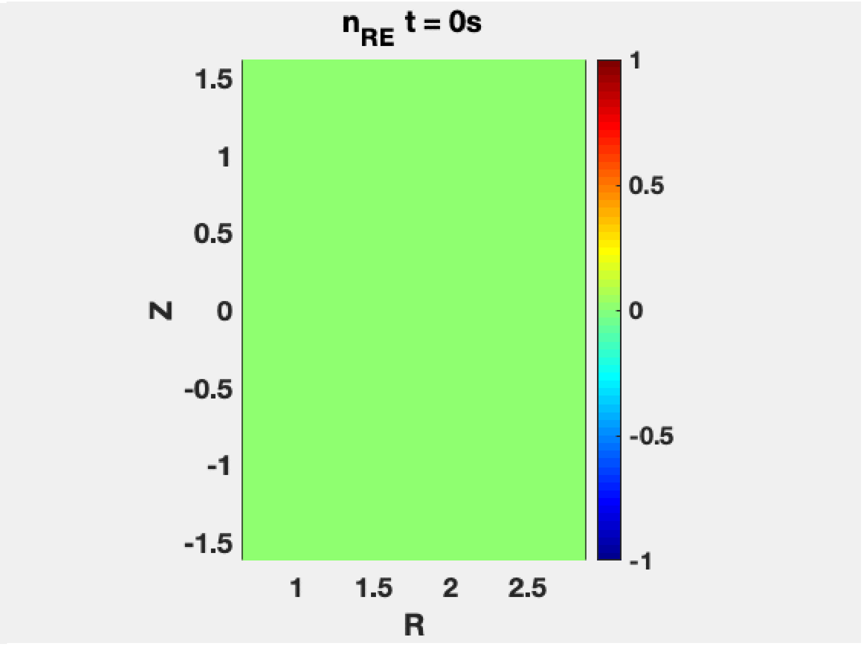
- **Linear growth rate of 2/1 tearing mode with different resistivity**



- For low resistivity cases, the growth rate of 2/1 tearing mode with and without RE obeys the 3/5 law of resistivity.
- For higher resistivity cases, the runaway current has restrained the resistivity correction of kink mode.
- The result form M3D-C^1 is consistent with eigen solver.

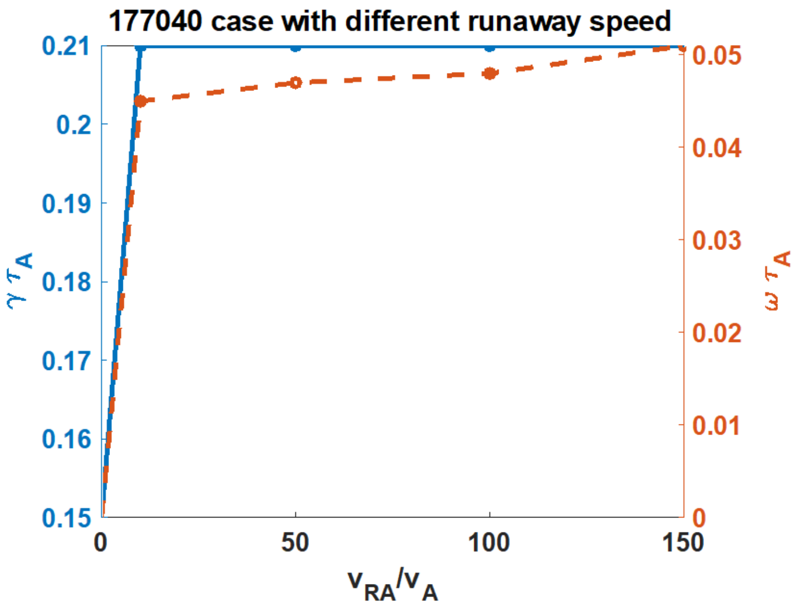
▪ Magnetic field perturbation and runaway electron current of DIII-D 177040 equilibrium

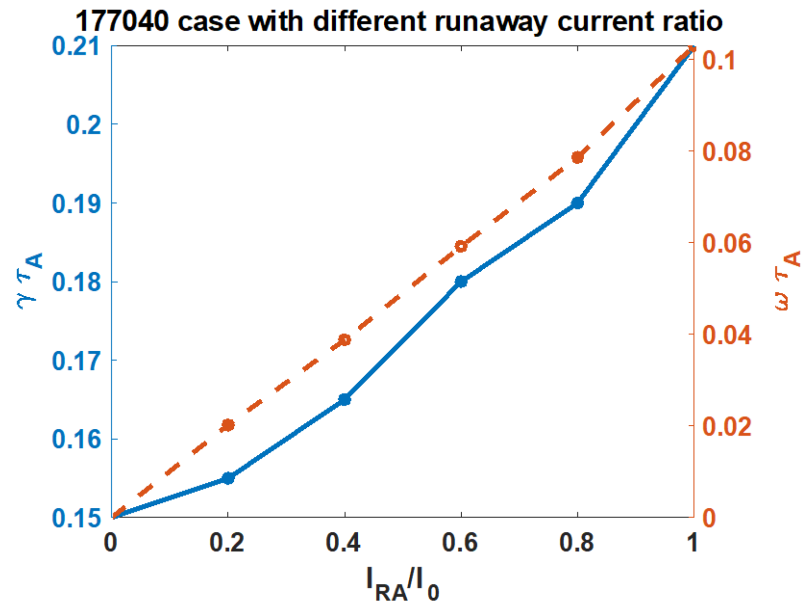




- There was a 2/1 tearing mode with rotation in 177040 equilibrium case

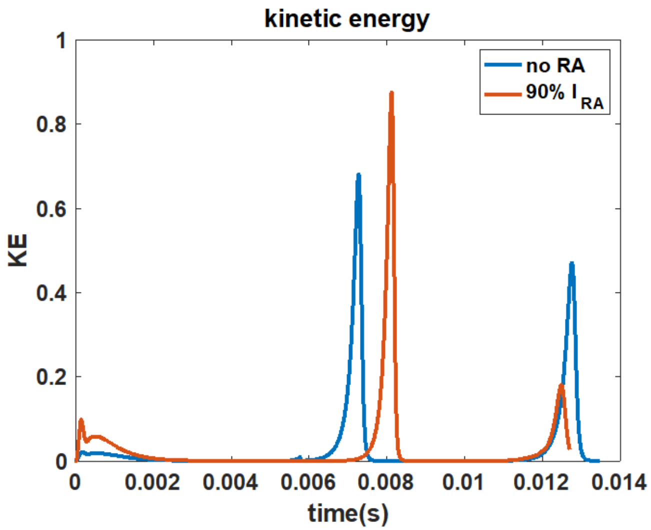
▪ Growth rate and real frequency of 177040 case with different runaway speed and current

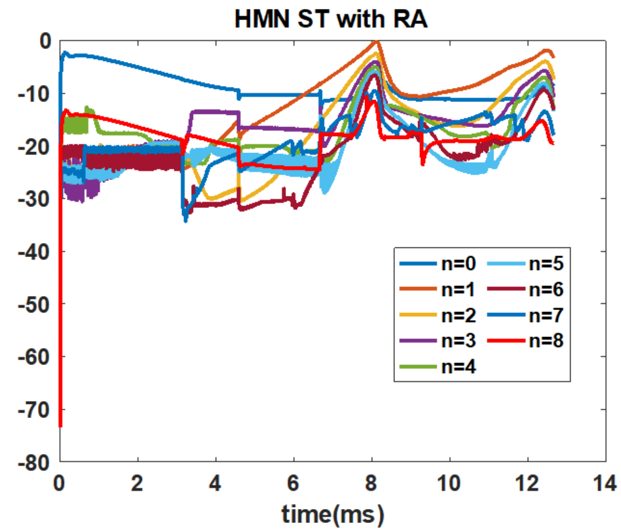




- The real frequency and growth rate do not change very much when runaway speed is larger than 10 Alfven speed.
- The real frequency and growth rate increase when the runaway electron current increases.

- **Sawtooth case with runaway electrons.**

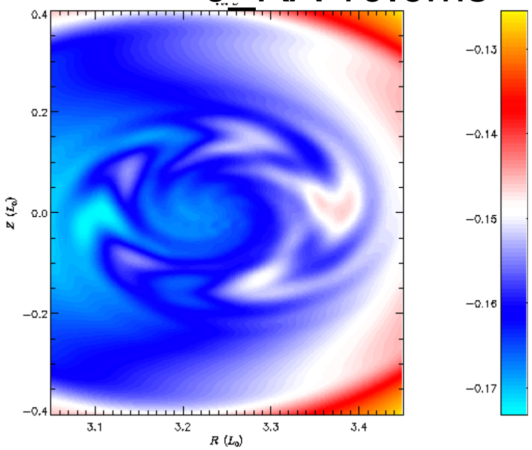




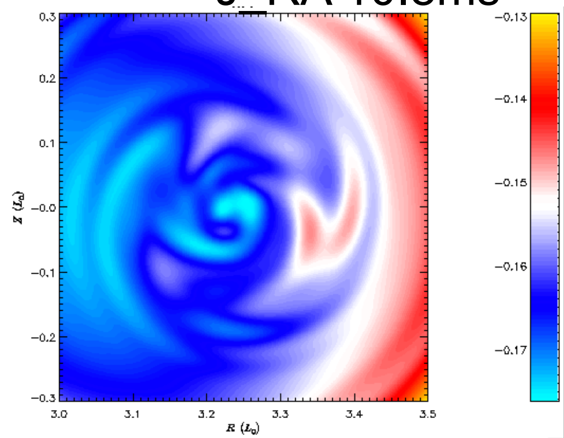
- It needs more time to see if there is only on ST phase with RA
- Most unstable mode is n = 1 with RA

Runaway current during 2^{nd} ST phase

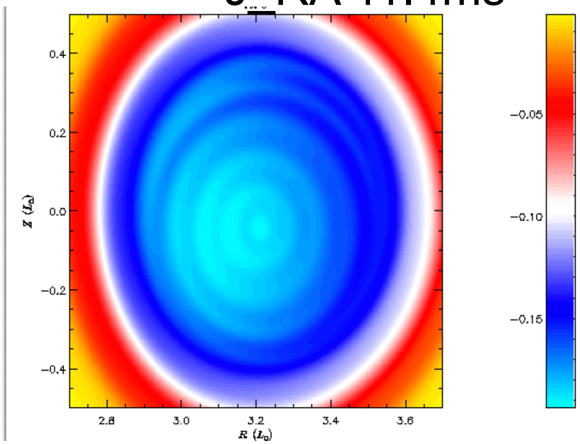
J_RA 10.6ms



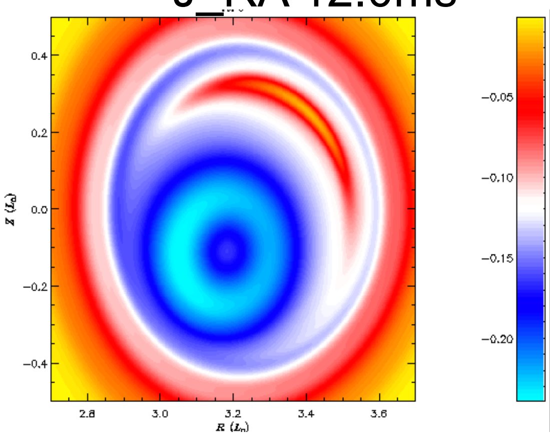
J_RA 10.8ms



J_RA 11.4ms



J_RA 12.0ms



- At 2^{nd} ST phase there is a high m mode reduced to m~1 mode in runaway current density.
- Since the safety factor is about 1 at the center, there maybe a finite $k_{||} = \frac{(nq-m)}{r}$ effect induced by the nonlinear wave interactions with runaway electrons.
- This wavelength changing maybe the main reason we have a much smaller 2^{nd} ST phase which different from the previous work(Cai & Fu 2015).

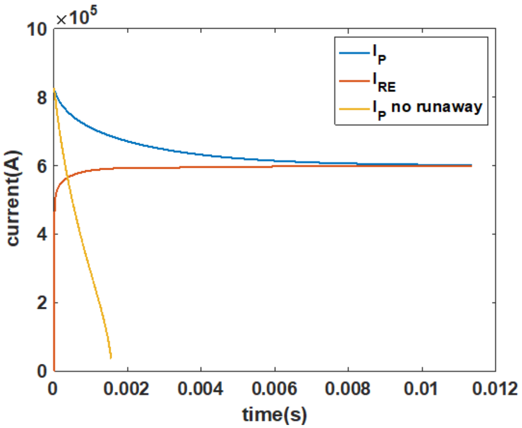
outline

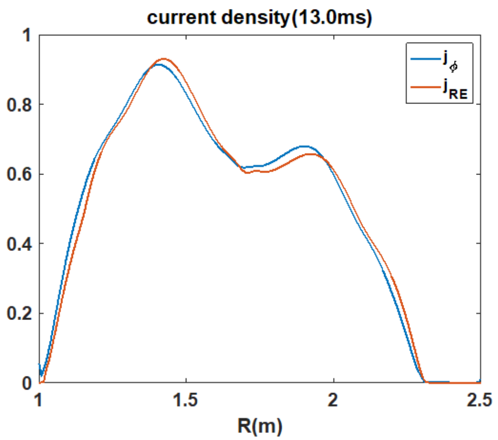
- **Introduction of M3D-C1**
- **MHD instabilities with runaway electron**
- **Runaway electron plateau during the current quench**
- **Summary**

▪ M3D-C1 runaway generation results with DIII-D 177053 equilibrium

- 3% Argon impurity uniformly distribute in the plasma region instead of pellet injection
- Parameters:

$$\beta_0 = 0.15$$
$$\kappa = 0.01$$
$$a = 0.65m$$
$$\mathrm{R} = 1.7m$$
$$B_0 = 1.9T$$
$$\eta = 1.0 \times 10^{-4}$$
$$n_0 = 1.0 \times 10^{20} m^{-3}$$
$$c = 150 v_A$$
$$N_{elements} = 12261$$
$$\Delta t = 1.0\tau_A$$



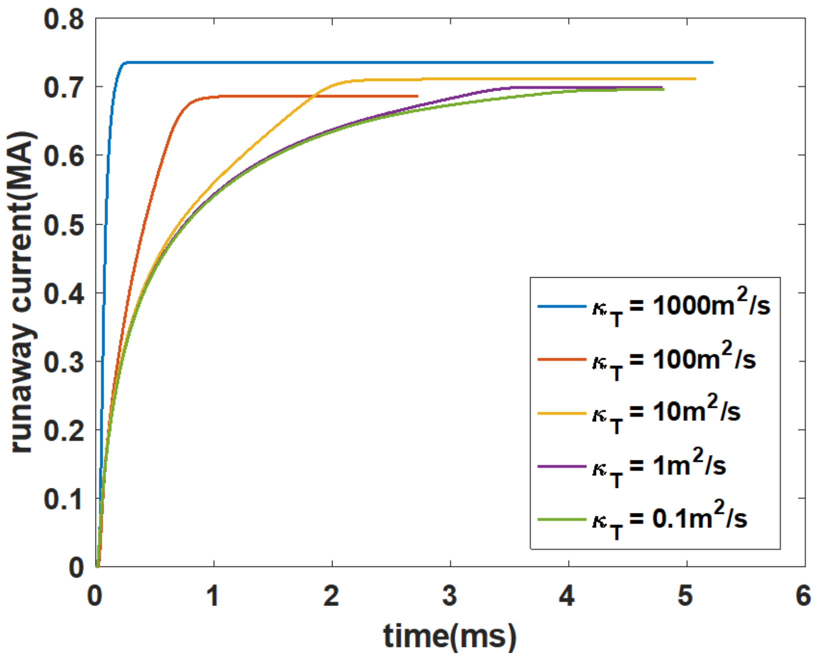


- The final plasma current is equal to the total plasma current which means the total current is fully carried by the runaway electrons.

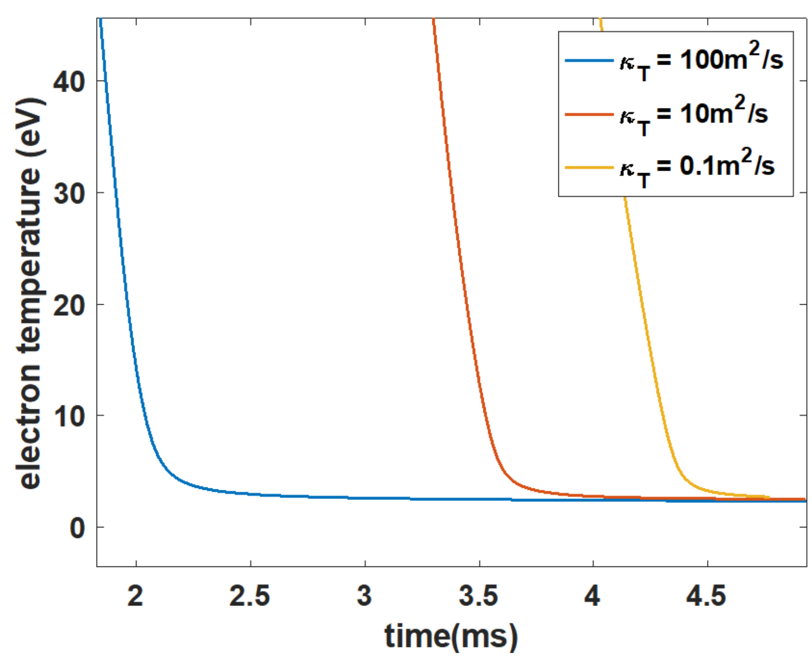
- If no runaways, the plasma current dropped fast to 0.

▪ 177053 case with different thermal conductivity

Runaway Current



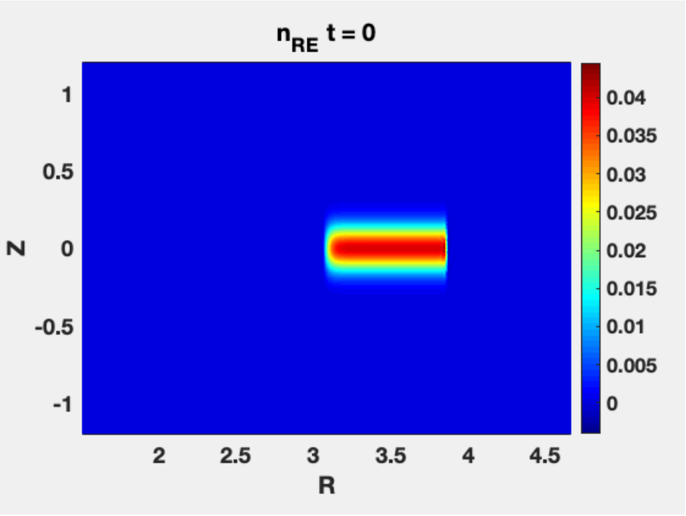
Temperature

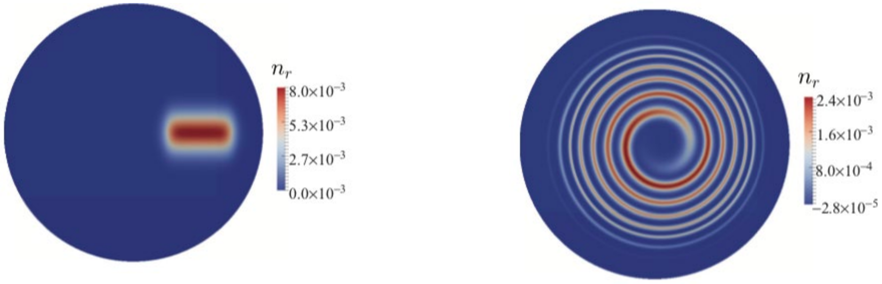


- The cooling time did not affect the runaway current saturation value very much.
- The final temperature also was not affected by the conductivity.
- The final temperature is about 5eV.

▪ Runaway convection benchmark with JOREK

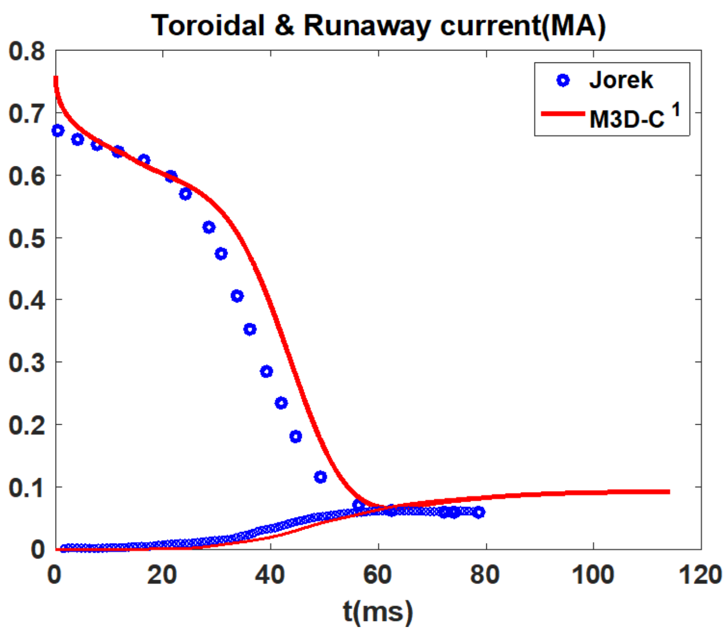
- Circular cross section of ϕ=0
- Initial runaway electron density is similar with Bandaru 2019.
- At the beginning (t < 12 Alfven time) the advection results is similar with JOREC. Then with the diffusion results the runaway density become uniform at theta direction.
- The runaway electron density moves along the magnetic field and has parallel and perpendicular diffusion, so that it become independent of the theta coordinate.

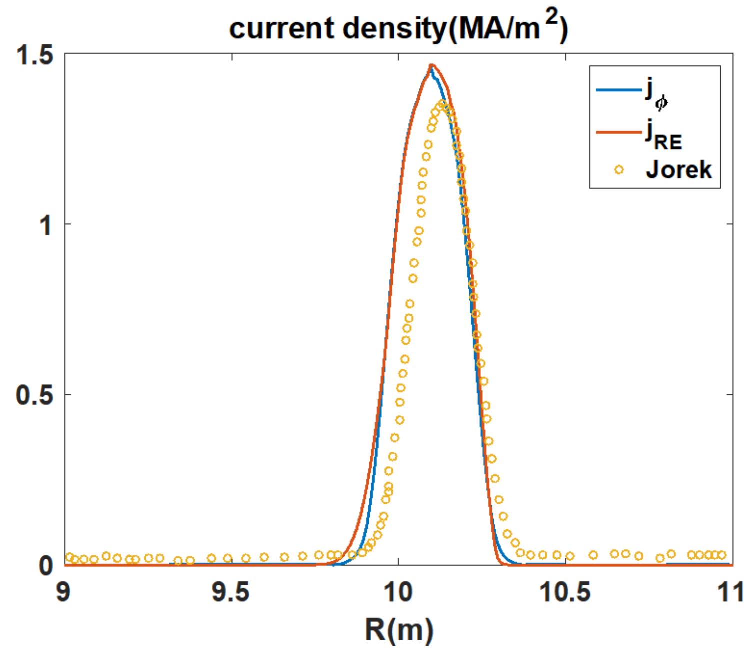




Bandaru 2019

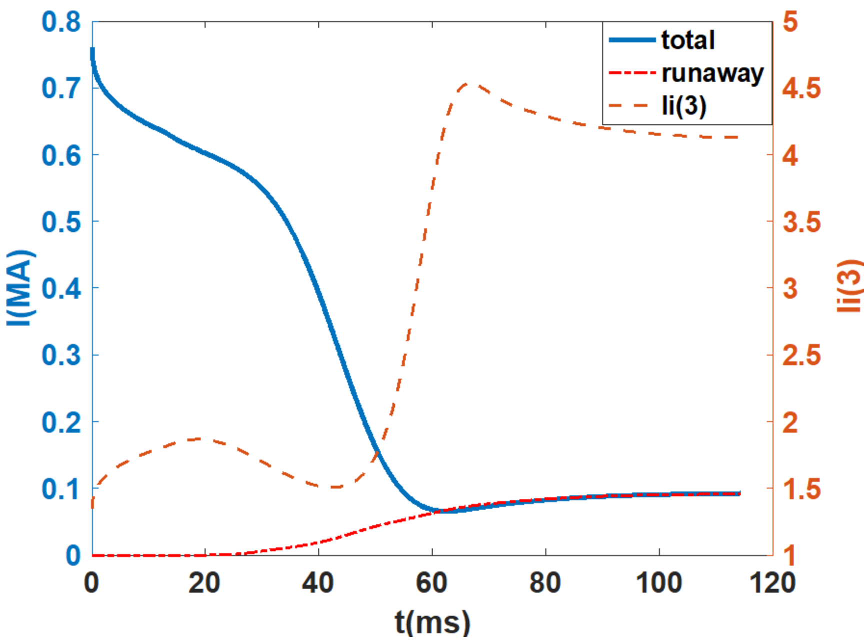
▪ Current quench benchmark with JOREK





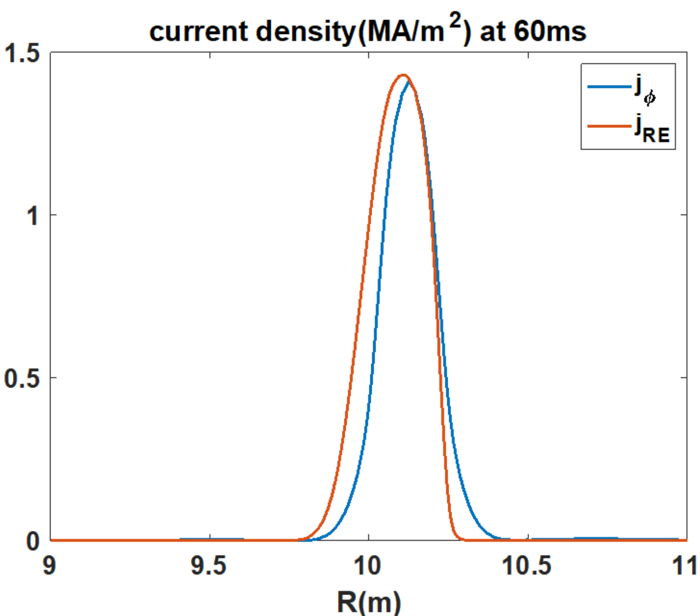
- The total current and runaway current result from M3D-C1 is nearly the same as JOREK.
- The radial profile of the final runaway current density is same as plasma current and nearly the same as JOREK .

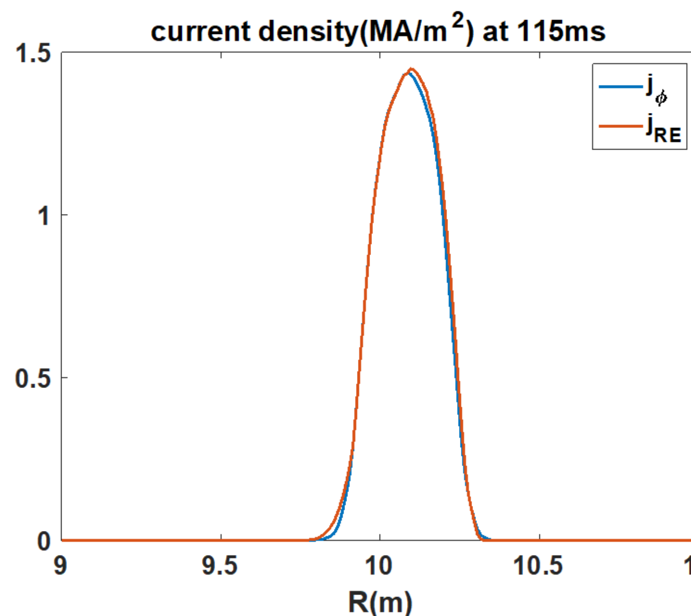
Current & internal inductance in M3D-C1



- The internal inductance increases at about 60ms and makes both the total current and runaway current increase a little.
- The increasing internal inductance maybe caused by the current differences at about 60ms.
- Further detail needs more study.

Current density profile in M3D-C1





outline

- **Gyrokinetic simulations about ECM(edge coherent mode)**
- **MHD simulaitons with runaway electrons**
- **Summary**

▪ Summary

- The perturbed toroidal current of 1/1 and 2/1 mode are peaked around the rational surfaces by the RE current effects. The RE current causes the 1/1 and 2/1 mode to rotate. If the runaway speed is large enough, the growth rate and real frequency do not dependent on the value of the speed. However, the scale length of the runaway current becomes smaller with higher runaway speed. (Zhao et. al. 2020, Liu et. al. 2020)
- The runaway electrons have restrained the resistive correction effect in high resistivity cases.
- We have already developed the source term for runaways in M3D-C1 to study the runaway generation in experiments, and the result using DIII-D parameters shows that the plasma current becomes fully carried by runaway electrons, which is similar to what is seen experiment.
- The final runaway current and temperature value of the DIII-D 177053 case seems independent on the plasma thermal conductivity over a wide range.
- The M3D-C1 code is benchmarked with JOREK code for the runaway source terms.

Thank you!